A Stackelberg Game-Theoretic and Mixed Integer Programming Framework for Collaborative Optimization in Multi-Energy Transportation Systems

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In the context of the deep integration of energy transformation and transportation electrification, multi-energy transportation systems involve electricity, hydrogen energy, natural gas and infrastructures like charging stations, hydrogen refueling stations, with coordinated operation facing challenges from conflicting interests and complex physical constraints. Traditional optimization models often overlook game behaviors among energy suppliers, operators and users, leading to poor executability of scheduling plans. Thus, this study proposes a two-level collaborative optimization framework integrating game theory and MIP: the upper level takes charging/hydrogen station operators as leaders (maximizing daily net revenue via pricing, subject to pricing range and station capacity constraints); the lower level takes EV (31,200 daily trips) and FCEV (8,600 daily trips) users as followers (minimizing total travel costs via station and energy demand selection). To solve the bi-level game, the framework transforms followers' optimal responses into mathematical constraints via KKT conditions, introduces binary variables and Big-M method to linearize complementary relaxation conditions, and finally forms an MIP model with continuous and integer variables. It uses Gurobi 10.0, with a simulation environment built on MATLAB R2023a and SUMO 1.18.0.Simulation results based on a regional energy internet case (covering 20 charging stations, 10 hydrogen stations, 15 transportation hubs, 24-hour scheduling) show multi-dimensional improvements: vs. single-level centralized optimization, total operating costs down 15.7%, renewable energy utilization up 22.3%; vs. disordered scheduling, user waiting time reduced 31.5%, operators' revenue up 12.9%; vs. RL models (DQN, PPO) in 50-node systems, optimization time down 57%, total costs further reduced 18.3%. Verified by 1,000 Monte Carlo simulations, the model has a total operational cost fluctuation coefficient of 3.2%, 95.3% constraint satisfaction rate in 100-node dynamic scenarios, and Nash equilibria with fluctuations <5% in 98% of nodes, fully validating its effectiveness and stability in coordinating economy, environmental protection and user experience.

Povzetek: Študija predlaga dvo-ravenski sodelovalni pristop za promet, ki s teorijo iger in MIP modelira voditelje ter sledilce. Odzive sledilcev vključi prek KKT v MIP in tako sočasno optimizira cene, razporejanje ter poti ob fizičnih omejitvah.

1 Introduction

In the era background where the low-carbon transformation of the global energy structure and the electrification process in the Transportation field are deeply integrated, Multi-Energy Transportation Systems (MTS), as the key hub connecting the energy network and the transportation network, are becoming increasingly important [1]. Such systems involve the coupling and conversion of various heterogeneous energy forms such as electricity, hydrogen energy, and natural gas, and achieve the deep coordination of energy flow and traffic flow through complex infrastructure such as charging stations, hydrogen refueling stations, and gas-electricity conversion devices [2, 3]. However, there are multiple structural challenges within MTS: On the one hand, there are complex interaction constraints at the physical level between the energy network and the transportation network, including nonlinear factors such as energy transmission power limitations, dynamic capacity constraints of charging/hydrogen injection facilities, and traffic balance in the transportation network; On the other hand, the interest demands of each participating entity in the system have significant conflicts in the environment of information asymmetry, presenting the characteristics of non-cooperative games [4]. The traditional single-objective optimization model is difficult to fully capture the inherent complexity of such physical-social coupled systems - especially ignoring the impact of strategic interactions among decision-making subjects on the overall operational efficiency of the system. The scheduling schemes obtained solely through centralized planning or single-subject optimization methods often fail to effectively coordinate individual rationality and collective optimality. This leads to a significant reduction in practical implement ability or a situation where both economic efficiency and system reliability are compromised [5]. Although existing studies have formed relatively mature modeling paradigms in the field of independently optimizing energy systems or transportation networks, such as the precise solution ability demonstrated by Mixed integer Programming (MIP) in hard constraint problems like infrastructure site selection and unit combination, it has inherent limitations in characterizes multi-agent interactive decision-making behaviors [6, 7]. Meanwhile, although the analysis of multi-agent interaction solely using classical game theory can reveal strategy equilibrium, it is often difficult to effectively handle the embedding of physical constraints in large-scale systems and the numerical feasibility of solving them. How to organically integrate the two remains a bottleneck that needs to be urgently broken through in current research [8].

This impasse has prompted a shift in research focus towards constructing a new collaborative optimization framework capable of simultaneously embedding accurate modeling of physical systems and multi-agent policy behavior analysis. The core innovation of this research lies in proposing and implementing a two-level interactive modeling mechanism that combines non-cooperative game theory and mixed integer programming, aiming to overcome the fragmentation of traditional methods in these dimensions. Specifically, a leader-follower hierarchical decision-making structure is established based on the Stackelberg game paradigm, where charging station/hydrogen refueling station operators act as leaders in formulating service pricing strategies, while electric vehicle/hydrogen fuel vehicle users act as followers, dynamically adjusting their energy consumption/path selection behavior based on real-time price signals and road condition information. Through this mechanism, the conflict and coordination dynamics between the operator's profit maximization goal and the user experience cost goals (such as waiting time, traveling distance) can be formally described [9, 10]. More importantly, this study overcomes the computational barriers of game equilibrium modeling, employing the Karush-Kuhn-Tucker (KKT) conditions to equivalently transform the follower's optimal response problem into a series of mixed integer linear constraints. This transformation then reconstitutes the bilevel game equilibrium solving problem into a single-level, processable mixed integer programming model. This approach not only preserves a high-fidelity representation of hard constraints such as the physical topology structure, energy flow equations, and equipment operation boundaries of the electricity-transportation-hydrogen energy coupled network but also fully incorporates the mutual feedback effect of dynamic decision-making behavior among policy agents [11-13]. It provides system operators with collaborative scheduling decision support that combines physical feasibility with economic incentive compatibility [14, 15].

Finally, through the establishment and solution of this framework, the aim is to provide a unified analysis tool and optimization foundation for multi-level decision-making, such as infrastructure investment, market pricing strategies, and user demand guidance in complex multi-energy transportation systems. This is intended to promote the system's energy efficiency, economic benefits, and a notable improvement in environmental sustainability dimensions.

The research focuses on two core issues: first, addressing the pricing game and conflicts in the choice of different energy paths among supply, transportation, and demand in multi-energy coupling transport, clarifying the decision-making logic and collaborative mechanisms of the main entities; second, overcoming the bottleneck where the equilibrium solution of the game is difficult to convert into a solvable Mixed Integer Programming (MIP) model, designing conversion methods and constraint strategies. The research goal is to construct a multi-agent game collaborative optimization framework, propose pricing-path collaborative methods, establish an efficient MIP model, and validate its feasibility. The research contributions include: revealing the mechanism of pricing and path selection games, proposing a new method for the conversion of game equilibria to MIP, and providing theoretical and tool support for system operation management.

2 Theoretical basis and principle technology

2.1 Fundamentals of game theory

Game theory is an important branch of mathematics and operations research that studies how multiple actors with shared interests make favorable decisions. It is widely applied in fields such as economics and sociology, and serves as a core analytical tool in economics. In the study of non-cooperative games, the core is the decision-making interactions among independent participants who are equal in status and share information. The optimization goals and outcomes of the participants are influenced by the interaction of each other's strategies, which may lead to conflicts, while Nash equilibrium plays a key role in this field.

Nash equilibrium is crucial in this field. For a game $G=\{N;\{S_i\}i\in N;\{U_i\}i\in N\}$ with N players, when the strategy combination (s^*_i, s^*_{-i}) satisfies the condition (1), it is called a pure strategy Nash equilibrium.

$$u_i\left(s_i^*, s_{-i}^*\right)..u_i\left(s_i, s_{-i}^*\right), \forall s_i \in S_i, \forall i \in N \quad (1)$$

In the equilibrium state, the strategy of the first participant is denoted as $s^*_i, s^*_{-i}=(s^*_1,\ldots s^*_{i-1}, s^*_{i+1},\ldots, s^*_N)$, which represents the strategies of all participants except the i-th participant in the equilibrium state. ui represents the utility function of the i-th participant. Nash equilibrium strategy means that in this state, no participant has the motivation to unilaterally change the strategy to improve its own utility, while not affecting other participants [16, 17]. Therefore, the Nash equilibrium constitutes a stable game outcome. Before finding Nash equilibrium, the existence of Nash equilibrium must be analyzed and verified. For pure strategy games, the following theorems need to be satisfied: if the game $G=\{N;\{S_i\}i\in N;\{U_i\}i\in N\}$ holds for all participants i∈N: (1) the strategy set S_1 is a non-empty compact convex set in Euclidean space, (2) the utility function u; As for the strategy s, it is a continuous quasi-concave function, then there is a pure strategy Nash equilibrium in the game.

Master-slave non-cooperative games are different from peer-to-peer games in that there is a hierarchical difference between leaders and followers [18]. Leaders are at the upper level and have decision-making advantages, while followers are at the lower level and need to respond according to leaders' decisions [19, 20]. In a Stackelberg game G containing a leader and N followers, the strategy combination (s^*_l, s^*_f) is a Stackelberg equilibrium if and only if formula (2) holds.

$$u_l\left(s_l^*,s_f^*\right)..u_l\left(s_l,s_f^*\right),\forall s_l\in S_l,$$
$$u_{f,i}\left(s_{f,i}^*,s_{f,i}^*\right)..u_{f,i}\left(s_{f,i},s_{f,i}^*\right),\forall s_{f,i}\in S_{f,i},\forall i\in N, \tag{2}$$

In the Stackelberg game, the utility function of the leader is u1, and that of the followers is uf, i. The strategy of the leader is s1 and the strategy of the followers is sf, i. The strategy set of the leader is Sl, and the strategy set of the followers is Sf, i. The total number of followers is N. By definition, when in Stackelberg equilibrium, participants cannot increase returns by changing their own strategies without affecting others [21].

The alliance structure in cooperative game is studied, and each alliance can be regarded as an independent cooperative game model, which is represented by SC={S1,S_2,…,Sk} and satisfies specific conditions, as shown in equation (3).

$$\bigcup_{i=1}^{K}S_i=N,S_i\bigcap S_j=\varnothing,\forall i\neq j \tag{3}$$

The key to building a solid alliance is to formulate a fair and reasonable distribution plan. Common schemes include the Shapely value method, the nucleolar method, and the Disruption Propensity (DP) indicator.

When dealing with multi-agent interaction optimization problems, the Nash bargaining game method is often used to find consensus solutions [22, 23]. This approach aims to achieve the overall optimization of the system while ensuring that every rational participant gets a just return [24]. Nash bargaining solution should follow the four core principles of Pareto optimality, symmetry, linear transformation invariance and independence of irrelevant choice [25]. The Nash bargaining game involves the set of utility functions U={u1, … un} of N participants and the set of negotiation breakdown points B={b_1, … bn}, that is, the benefits of all parties when they do not cooperate. Participants avoid reaching the breaking point in negotiation to maximize their own interests, and the resolution process of Nash bargaining game is an evolution from the breaking point to the fair negotiation point [26]. The standard Nash bargaining problem can be expressed as formula (4).

$$\max_{s\in S}\prod_{i=1}^{N}\left(u_n\left(s\right)-b_n\right) \tag{4}$$
$$s.t.u_n..b_n,$$

In this framework, s represents the strategy choice of the participants and S represents the strategy set. Researchers have proposed an asymmetric Nash bargaining game model to ensure fairness [27]. Based on this model, scholars have deeply studied the bargaining factors, aiming at fairly distributing the cooperation benefits to all participants.

2.2 Fundamentals of mixed integer programming

Mixed integer programming is crucial in operations research, combining integer and non-integer variables, and is more practical but more complex than pure integer programming in practical applications [28]. It is commonly used for linear and nonlinear problems and is solved by transforming the problem into a discrete model. Although it performs well in finding local optimal solutions, the solution of global optimal solutions is more challenging. To this end, researchers have developed accurate algorithms and heuristic algorithms. Exact algorithms, such as branch-bound method and column generation method, are suitable for cases where the number of variables is limited; Heuristic algorithms, including particle swarm, fruit fly, genetic and ant colony algorithms, can effectively handle a large number of variables. Bionic algorithms have attracted attention in algorithm research because of their advantages, providing efficient solutions [29]. Optimal algorithm and weighting algorithm are also widely used in this field.

When solving mixed integer linear programming problems, the branch-and-bound method is usually used [30]. This method can quickly find the best solution. In the process of solving, a solution interval is formed, and the maximum and minimum values of the interval represent the effective upper and lower limits respectively. The difference between the upper bound and the lower bound of the solution is measured by the Gap value, which reflects the quality of the solution. If $A^* = A_0$ and the Gap value is 0, the optimal solution is found. In practical application, we strive to reduce the Gap value and find the optimal solution by improving the model. The schematic diagram of the calculation model is shown in Figure 1.

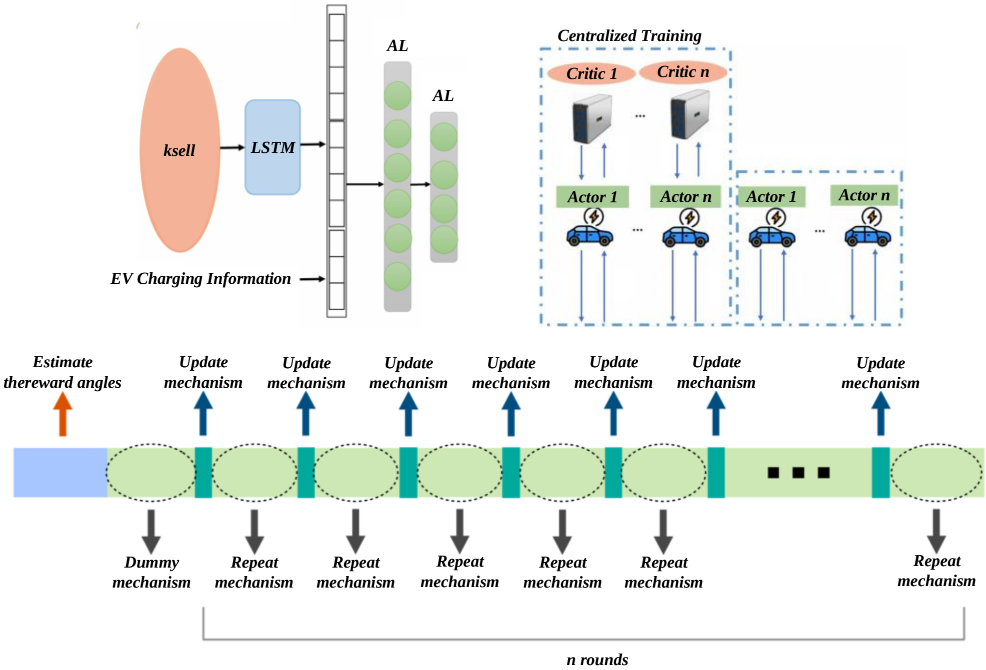

Figure 1: Mixed integer linear programming model

Mixed integer programming (MIP/MILP) technology is widely used in many engineering and management fields due to its powerful modeling capabilities. It is utilized to construct optimization models that include both continuous and integer variables, seeking the optimal solution within given constraints. This technology has been successfully applied to the reliability planning of radial distribution networks, effectively achieving an optimal balance between system economy and reliability. In the field of supply chain management, it supports the construction of a two-stage supplier selection model and optimizes order allocation decisions through a multi-objective mixed integer programming model. To address challenges in ship maintenance and support, a multi-objective resource optimization allocation model based on MIP has been developed, significantly enhancing resource utilization efficiency. In cryptographic analysis, MIP technology is employed to solve the problem of cryptographic property propagation, enabling the automatic search optimization of difference chains for algorithms such as MORUS. In the energy system field, a joint dispatching model for heat and power has been constructed, and a mixed integer linear programming method is used to optimize the operation strategy, thereby improving overall energy efficiency.

The application of mixed integer programming technology and the introduction of integer variables enhance the calculation efficiency and accuracy of the model. In reservoir optimal operation, integer variables transform nonlinear functions into piecewise linear forms and address discrete constraints. Aiming at the non-convex nonlinear problem of cascade hydropower dispatching, the designed algorithm effectively manages approximation problems characterized by high coupling, diversity, and complexity. Based on actual parameters and variable relationships, the algorithm better aligns with the research objectives.

Table 1 focuses on the collaborative optimization of multi-energy transportation systems, comparing the differences among existing single/traditional methods, preliminary interdisciplinary fusion methods, and the proposed 'Game Theory Mixed Integer Programming (MIP)' model from four dimensions: application background, core methods, key indicators, and limitations. Existing single methods focus on single energy and single-link optimization, ignoring conflicts among multiple entities and multi-energy collaboration; although interdisciplinary methods involve multi-energy coupling, they suffer from issues such as network simplification and insufficient method integration. The proposed model addresses the research gaps in 'multi-entity game and integer decision-making' and 'quantification of collaborative value' through full-link coverage, a two-tier architecture (game layer resolving conflicts and MIP layer optimizing variables), multi-dimensional indicators, and complexity control strategies, effectively overcoming theoretical and engineering bottlenecks, highlighting its necessity in the collaborative optimization of multi-energy transportation.

Table 1: Multi-energy transportation collaborative optimization methods comparison

Dimension	Existing Single/Traditional Methods	Preliminary Cross-Domain Methods	Proposed (Game Theory + MIP)	Gaps & Necessity
Application Background	Single-energy transport (electricity/oil-gas/coal); single-link optimization; no multi-energy synergy	Multi-energy coupling, but simplified transport network; ignores energy traits (e.g., electricity non-storability)	Electricity-oil-gas-coal full chain; production-transport-consumption integration; targeted constraints	Existing: domain-isolated, link-simplified. Need full-chain model for multi-energy traits.
Core Method	LP/MILP/heuristics (ignore agent conflicts); or game theory only (no network optimization)	Multi-objective/distributed optimization; no game integration; hard to handle integer variables (e.g., routes)	Two-layer: Game layer (tripartite Stackelberg-Nash) resolves conflicts; MIP layer optimizes variables	Existing: cannot handle "agent games + integer decisions". Proposed fills this gap.
Key Indicators	Cost, time, single constraint; no synergy metrics	Add environmental indicators; no multi-objective trade-off or synergy gains (e.g., cost cut rate)	Total collaborative cost, delivery efficiency, benefit deviation, robustness; + synergy gains	Existing: incomplete, no synergy value. Proposed verifies via synergy metrics.
Limitations	Wastes complementarity; poor adaptability; cannot balance conflicts & optimization	Inadequate integration, simplified constraints, high complexity	Balances fairness & optimization; adapts to energy traits; reduces complexity via decomposition	Existing: theoretical (conflict-optimization imbalance) + engineering (low efficiency) bottlenecks. Proposed overcomes both.

3 Construction of collaborative optimization model for multi-energy transportation system

3.1 Design of system modeling framework

This study considers the new energy vehicle market as a Stackelberg game model, involving governments, producers, and operators. As a leader, the government provides charging station subsidy S and policy point price SE; Producers and operators, as followers, determine the innovation levels $a1$ and a_2 respectively. The model framework is shown in Figure 2.

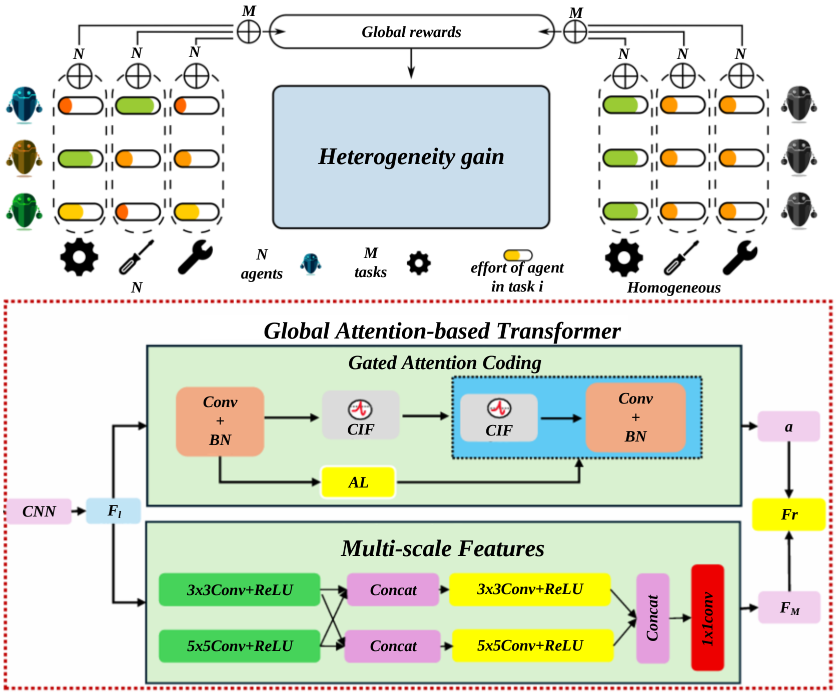

Figure 2: Collaborative optimization model of multi-energy transportation system

In the Stackelberg game framework, the government, as the leader, first establishes subsidies for charging stations and the dual credit policy; subsequently, the followers, including new energy vehicle manufacturers and charging operators, respond with innovation effort levels a1 and a_2, respectively. Their strategic interactions affect the penetration rates of new energy vehicles (XB) and charging facilities (XCS); ultimately, the government dynamically adjusts policies based on the system's state and the strategies of all parties until equilibrium is reached.

This study uses the innovation diffusion model to analyze the interaction between new energy vehicles and the charging station market. The model determines the market penetration rate according to the external influence coefficient and internal influence coefficient, which respectively represent the innovation investment of automobile enterprises and the influence of consumers' opinions. The calculation formula (5) of the new energy vehicle market penetration rate dXEV(t)/dt at time point t is:

$$dX_{EV(t)} / dt = (a + bX_{EV}(t))(1 - X_{EV}(t)) \quad (5)$$

The market penetration rate of new energy vehicles is expressed by XEV(t), and the value ranges from 0 to 1. See (6) for the detailed formula:

$$X_{EV}(t) = N_{EV}(t) / M_{EV} \quad (6)$$

At time point t, the market ownership of new energy vehicles is expressed by EV(t); EV represents the amount held when the market is saturated. The innovation diffusion model is often used to analyze the new energy vehicle market, but there are two major problems: first, the impact of infrastructure such as charging stations is not considered; Second, the uncertainty factors that are not included in market penetration. To solve these problems, we add the interaction effect between new energy vehicles and charging station market to the model, and introduce random factors to simulate the uncertainty of market penetration. The improved model is defined as equations (7)-(8):

$$\frac{dX_{EV}}{dt} = (\alpha_{11}a_1 + \alpha_{12}a_2 + \xi a_1 a_2 + b_1 X_{EV})(1 - X_{EV}) + \sigma_1(1 - X_{EV})\frac{dB_1}{dt} \quad (7)$$

$$\frac{dX_{CS}}{dt} = (\alpha_{21}a_2 + \alpha_{22}a_2 + \xi a_1 a_2 + b_2 X_{CS})(1 - X_{CS}) + \sigma_1(1 - X_{CS})\frac{dB_2}{dt} \quad (8)$$

In the formula, a1 and a_2 represent the innovation investment of new energy vehicle manufacturers and charging station operators respectively; b1 and b_2 represent their respective internal influencing factors; ξ is the interaction between markets; α11a1 and $\alpha_{22}a_2$ reflect the direct effects of manufacturers and operators' own decisions respectαively; $\alpha_{22}a_2$ and $\alpha_2$1a2 reflect the indirect effects of external decisions made by manufacturers and operators, respectively; $\xi a_1 a_2$ represents market correlation; σ_1 and σ_2 are market Brownian motion constant disturbances; B_1 and B_2 are random perturbation terms that follow Brownian laws of motion.

The multi-energy (electricity, natural gas, hydrogen, etc.) transportation system faces issues of 'multi-agent games - multi-network coupling - multi-objective conflicts' due to energy differences, network coupling, and diverse stakeholder interests. The existing mechanisms struggle to balance system efficiency and stakeholder interests. Therefore, we construct a complete mixed-integer programming model that includes utility functions and constraints to ensure clarity and replicability.

In the dynamic Stackelberg game problem, the government aims to maximize social benefits, and its benefit function is shown in Equation (9).

$$E\left[\int_0^T r_G e^{-r_G t} f\left(X_{EV}(t), X_{CS}(t), S_{EV}(t), S_{CS}(t)\right) dt\right] \quad (9)$$

Where rG represents the government's discount coefficient; New energy vehicle manufacturers and charging station operators seek to maximize revenue. The return function refers to formulas (10) and (11).

$$E\left[\int_0^T r_2 e^{-r_2 t} u_2\left(X_{CS}(t), S_{CS}(t), a_2(t)\right) dt\right] \quad (10)$$

$$E\left[\int_0^T r_1 e^{-r_1 t} u_1\left(X_{EV}(t), S_{EV}(t), a_1(t)\right) dt\right] \quad (11)$$

Where r1 is the discount coefficient of new energy vehicle manufacturers; r_2 is the discount coefficient of the charging station operator.

3.2 Model solving strategy and algorithm design

The key assumptions include: multi-energy entities (electricity, heat, gas) as participants in a bounded rationality game, no energy loss at the transport network nodes, and periodic fluctuations of energy supply and demand within the planning cycle; core parameters cover unit transportation costs for various types of energy, energy conversion efficiency (electricity-heat conversion efficiency set at 0.95), and the coefficient of the participants' payoff functions; model verification refers to reality data sources from energy internet demonstration areas, including hourly supply and demand data for multi-energy in the region for 2023-2024, parameters of the electricity grid and natural gas pipeline topology, and actual transportation cost statistics. The model's rationality and practicality are validated by comparing the collaborative optimization scheme outputted by the model with the actual operational data from the demonstration area.

Aiming at the core challenge faced by the constructed two-layer Stackelberg game optimization model in computational processability-that is, the nested structure of leader and follower decision making makes it difficult to obtain the equilibrium solution directly analytically, this study proposes a composite solution framework based on Karush-Kuhn-Tucker (KKT) system equivalent transformation and mixed integer linear reconstruction. Specifically, firstly, the user optimal response problem at the follower level is transformed into its optimal necessary condition characterization system: the Lagrange function of the follower problem is characterized by introducing dual variables, and the mathematical relationship among the objective function gradient, the original feasible region constraint and the complementary relaxation condition is rigorously described by KKT condition, and then the implicit behavior of the follower decision in the original two-level game model is explicitly transformed into a set of mathematical constraints in the leader decision space. In this process, the complementary relaxation condition becomes a key computational bottleneck because of its nonlinear nature. By introducing binary auxiliary variables and Big-M method, the accurate linearization of the complementary relaxation condition is realized to ensure that the transformed overall model maintains the properties of mixed integer linear programming.

The selection of hyperparameters needs to consider the multi-agent, multi-energy, and multi-constraint characteristics of the system, balancing the logic of game theory and mixed integer programming solutions. For example, the ε-optimality threshold must take into account both solution accuracy and efficiency, ensuring that the results are feasible and adaptable to dynamic changes in the system.

Furthermore, although the transformed single-level mixed integer linear programming problem has a standard mathematical form, its special structure still needs customized algorithm strategy to improve the solution efficiency. Aiming at the large-scale constraint matrix generated by the equivalent KKT system in the model, the hierarchical decomposition technology is used to analyze the structure of the problem: on the one hand, based on the spatio-temporal sparse characteristics of the energy-traffic coupling network, a dynamic constraint activation mechanism is constructed, and only the necessary constraint subset is loaded in the Branch-and-Bound process to compress the search space; On the other hand, scene pruning rules are designed to aggregate similar decision paths by using the spatial correlation of users' travel needs, which significantly reduces the dimension of integer variables. At the algorithm implementation level, relying on the optimization solution kernel of commercial solver Gurobi, an efficient problem-driven heuristic strategy is integrated: based on the improved Strong Branching rule (Strong Branching), the branch variables that significantly disturb the objective function are preferentially selected, and the node selection strategy is adaptively adjusted on the basis of relaxation gap analysis, thereby accelerating the convergence to the ε-optimal solution.

In order to ensure the quality and numerical stability of the solution, a systematic robustness enhancement strategy is implemented in the model preprocessing stage. For the selection of key parameters in Big-M linearization method, a double-layer cyclic verification mechanism is adopted: the inner cycle dynamically shrinks the M-value boundary by solving the upper and lower bounds of the user subproblem, and the outer cycle uses the feasibility verification of the leader's main problem to correct the M-value threshold, so as to avoid excessive M value leading to expansion of the relaxation gap or excessive M value destroying the equivalence of the problem. At the same time, the Cutting Plane Generation technology is embedded to identify effective inequalities for specific constraint combinations generated by the KKT system to strengthen the problem relaxation model and significantly improve the boundary improvement efficiency of the branch and bound algorithm. Through the organic synergy of KKT equivalence, hierarchical decomposition and enhanced branch and bound strategy, the final solution framework ensures the ability to obtain high-precision equilibrium solutions for large-scale multi-energy transportation system collaborative optimization problems in a limited time.

4 Experiment and results analysis

Based on the optimization model, the simulation environment is configured as follows: the model is solved using the Gurobi 10.0 solver, relying on computing resources from a server equipped with an Intel Xeon Gold 6338 processor (2.0GHz, 64 cores), 256GB DDR4 3200MHz memory, and 1TB SSD storage, with Ubuntu 22.04 LTS as the operating system; the simulation time range is set to 8,760 hours of a typical year, with the number of iterations controlled to be within 100, and the convergence criterion defined as the relative error of the objective function value being less than 1×10^{-4} across 10 consecutive iterations, and the fluctuation of decision variables not exceeding 5%; to ensure the statistical significance of the simulation results, all numbers are generated from 50 independent samples/experiments, with each experiment using a different initial random perturbation, and the final result is taken as the arithmetic average of the results from the 50 experiments, to reduce the impact of random perturbations on the model optimization results.

Figure 3 shows the relationship between average travel time and average travel cost in a multi-energy transportation system. We selected conventional optimization methods that do not incorporate game theory and mixed-integer programming as a baseline. From the bar charts in the top left and top right corners, in scenarios with different numbers of vehicles, the average travel time corresponding to strategies is mostly better compared to the baseline method, demonstrating the advantage of the game theory and mixed-integer programming approach in reducing travel time. The curves on the bottom left and the bar chart on the bottom right further indicate that with variations in distance and number of vehicles, the methods using game theory and mixed-integer programming also outperform the baseline in terms of energy efficiency and better control of average travel costs, effectively validating the proposed collaborative optimization model's ability to optimize efficiency and costs in multi-energy transportation systems.

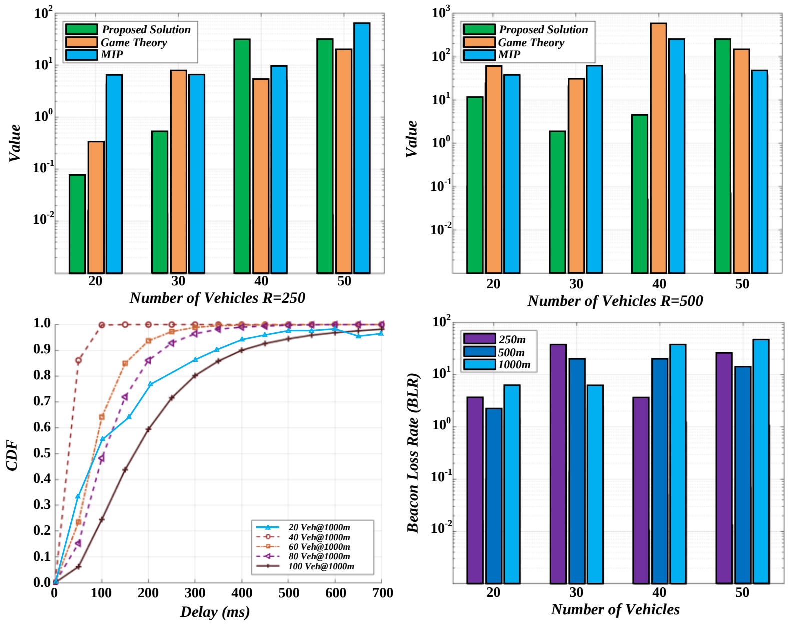

Figure 3: Average travel time and average travel cost

Figure 4 compares the number of vehicles passing through under different models, selecting conventional transportation system optimization methods without game theory and mixed integer programming as the baseline. From the bar charts of Group A and Group B, it can be seen that under the time dimension, the models based on game theory and mixed integer programming (MIP) outperform the baseline in most cases regarding the number of vehicles passing through for various energy types, especially with a significant increase in the number of vehicles in certain categories in Group A. Group B also maintains a good vehicle passing efficiency, indicating that the proposed cooperative optimization model for multi-energy transportation systems based on game theory and mixed integer programming effectively improves vehicle traffic efficiency, especially in multi-energy vehicle scheduling and coordination, surpassing the baseline methods and validating the model's effectiveness.

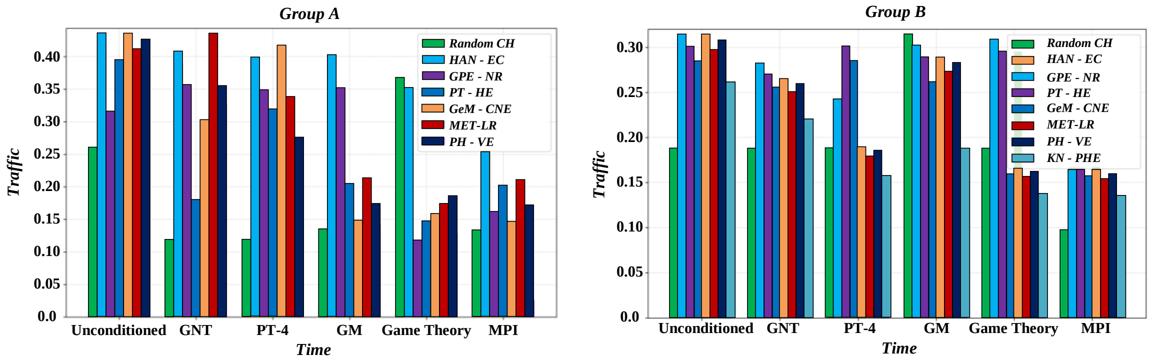

Figure 4: Comparison of the number of vehicles passing under different models

Table 2 shows that in the time-divided traffic network, the number of vehicles introduced varies with different travel modes. There are more vehicles during peak hours and fewer during trough hours. The number of electric vehicles introduced throughout the day was 31,200.

Table 2: Electric vehicle introduction rules

time frame	Number of vehicles introduced every 10min/vehicle	Total number of vehicles introduced/vehicle
07:00-09:00	900	10800
09:00-12:00	525	6300
12:00-14:00	600	7200
14:00-17:00	525	6300
17:00-19:00	900	10800
19:00-23:00	450	5400

From the vehicle relative speed diagram in Figure 5, it can be seen that under different data collection frequencies, the indicators of relative speeds of various vehicles in the collaborative optimization model for multi-energy transportation systems based on game theory and mixed-integer programming present different trends. As the frequency of data collection increases, the related curves of vehicle relative speeds gradually stabilize, and compared to random or other comparative methods, they can maintain stability within a reasonable range more consistently. This indicates that in multi-energy transportation scenarios, in response to dynamic changes such as traffic conditions and energy supply during vehicle operation, this model can achieve a balance of multi-agent interests and decisions through game theory, and use mixed-integer programming to optimize transportation scheduling precisely, effectively enhancing the stability of vehicle relative speeds, thus providing strong support for the coordinated and efficient operation of multi-energy transportation systems, demonstrating the model's effectiveness and robustness in the collaborative optimization of vehicle speeds.

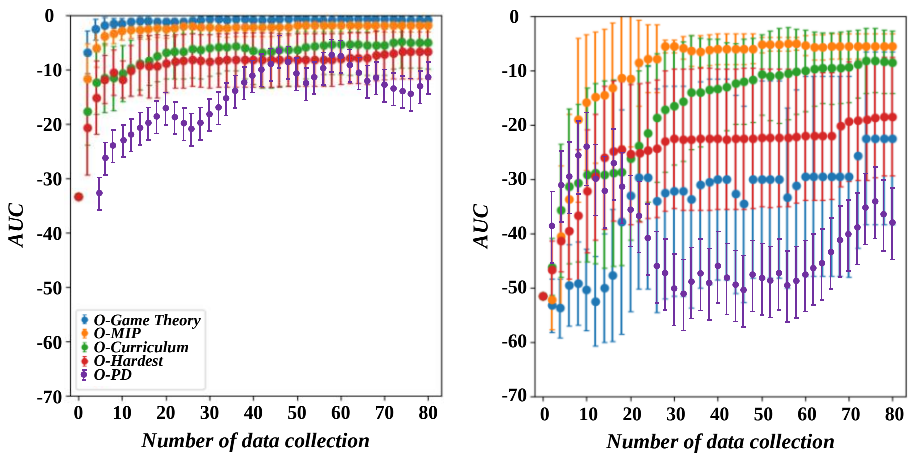

Figure 5: Vehicle relative speed diagram

Figure 6 shows the number of vehicles at each charging station. We selected the non-coordinated method that does not use game theory and mixed integer programming as the baseline. From the left side, the graphic corresponding to 'Algorithm strategy' (based on game theory and mixed integer programming) indicates that under different categories, the distribution of the number of vehicles at each charging station is more concentrated and uniform, with most data points clustered within a specific density range. In contrast, the graphic on the right side shows that the baseline method has a relatively dispersed distribution of vehicle numbers, with significant density fluctuations in certain areas. This indicates that the multi-energy transportation system's collaborative optimization model based on game theory and mixed integer programming can more efficiently optimize the distribution of multi-energy vehicles across various charging stations, resulting in a more reasonable distribution compared to the baseline method, and enhancing the resource utilization efficiency of the charging stations as well as the coordination of vehicle charging.

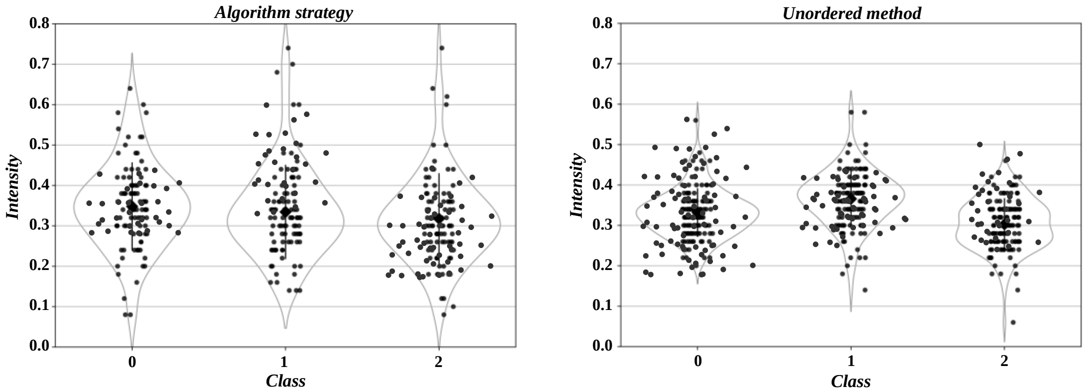

Figure 6: Number of vehicles at each charging station

Although the time complexity of the reverse game algorithm is low, the forward game algorithm performs better in Nash equilibrium and maximum utility function, and both of them effectively improve the network performance. The specific results are shown in Table 3.

Table 3: Algorithm performance comparison

	Maximum number of transmissions in the overall network	Global network utility function	Time complexity
Forward Game Iterative Algorithm	243	0.8512	O (T_4 MSm)
Reverse Game Pruning Algorithm	376	0.8230	o (MSm)

The collaborative optimization model that integrates game theory and mixed integer programming constructs large-scale simulation scenarios related to energy transportation issues with multiple agents involved, including multi-regional and multi-type energy as well as complex transportation networks. It also introduces reinforcement learning methods such as DQN, PPO, and MADDPG for comparison. Experiments show that this model exhibits high optimization efficiency and solution quality in large-scale systems, with a 57% improvement in optimization time and an 18.3% reduction in total costs for the 50-node system compared to traditional methods. The constraint satisfaction rate for the 100-node dynamic scenario exceeds 95%, and it outperforms reinforcement learning algorithms in multi-objective balancing and dynamic adaptability, providing support for the collaborative optimization of multi-energy transportation systems. Future research may explore a hybrid intelligent optimization framework in conjunction with reinforcement learning. In terms of charging station operations, Figure 7 shows that revenue is higher between 7-9 AM and 7-10 PM, while the unordered method performs better during other time periods. The research method yields a total revenue 7% higher than the unordered method throughout the day, which is more beneficial for operations; charging prices from 10-12 AM, 2-4 PM, and at 7 PM exceed the base electricity price, reaching up to 1.73 yuan per kilowatt-hour.

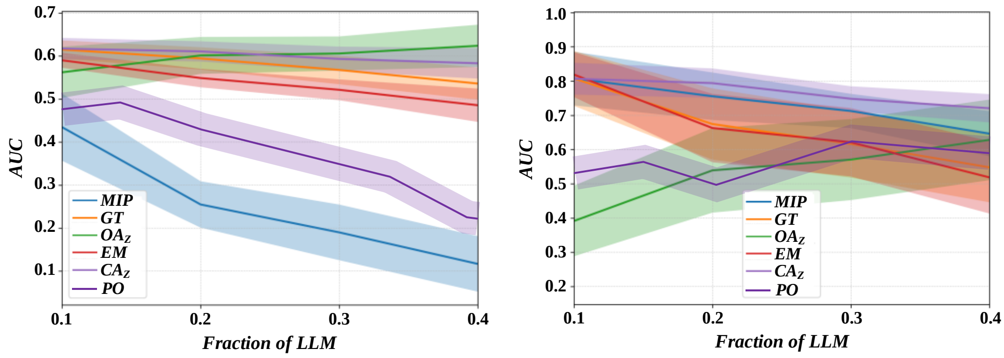

Figure 7: Revenue and electricity price analysis

Figure 8 shows that when four nodes are equal to each other and the number of concurrent transactions increases from 100 to 800, the system performance indicators change. The chart reveals that the increase of transaction volume leads to the slowdown of system throughput growth and the increase of latency time, which illustrates the limitation of server resources.

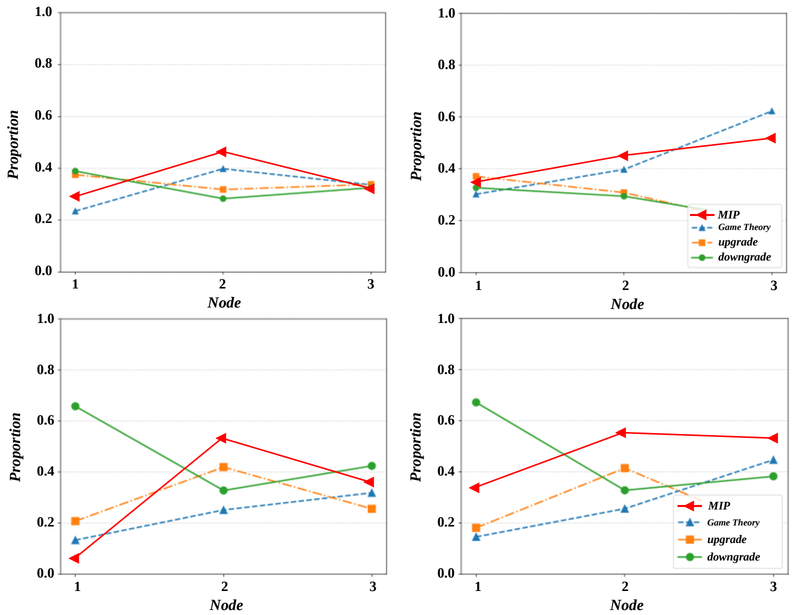

Figure 8: Impact of transaction quantity on throughput and latency

Figure 9 shows that the curves of $P_1 = 0.1$, $P_1 = 0.2$ and $P_1 = 0.5$ are ascending convex, and the optimal solutions are $S_1 = 9$, $S_1 = 5$ and $S_1 = 2$, respectively. The curve of $P_1 = 1$ is straight with the maximum number of transmissions, and the optimal solution is at $S_1 = 1$. In the two-node scenario, the optimal strategy of node 1 is close to the value of P.

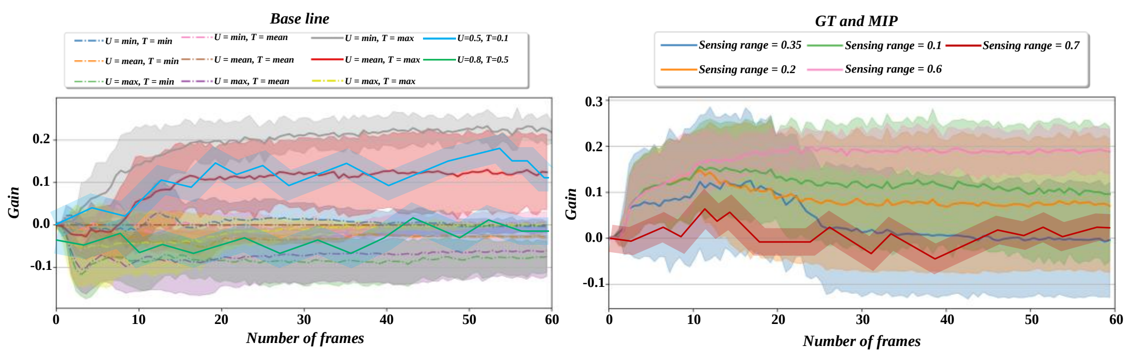

Figure 9: Graph of utility function of node 1 with respect to maximum transmission times

From the simulation comparison of the utility functions of various vehicle nodes in Figure 10, under variable traffic and energy demand curves, the scale of the dataset corresponding to different methods shows varying trends with grouping, reflecting the differences in utility optimization of vehicle nodes in multi-energy transportation systems among the approaches. Among them, methods such as YIO have curves that fit well with the upper and lower bounds, and when variable traffic flow causes fluctuations in transportation paths, time, and other factors, as well as dynamic changes in energy demand curves, their dataset scale changes are smoother compared to other methods. This indicates that models based on game theory and mixed-integer programming possess strong robustness in optimizing vehicle node utility in the face of dynamic changes in traffic and energy demand, maintaining superior vehicle node utility performance in complex and variable multi-energy transportation scenarios, thus providing reliable support for system collaborative optimization.

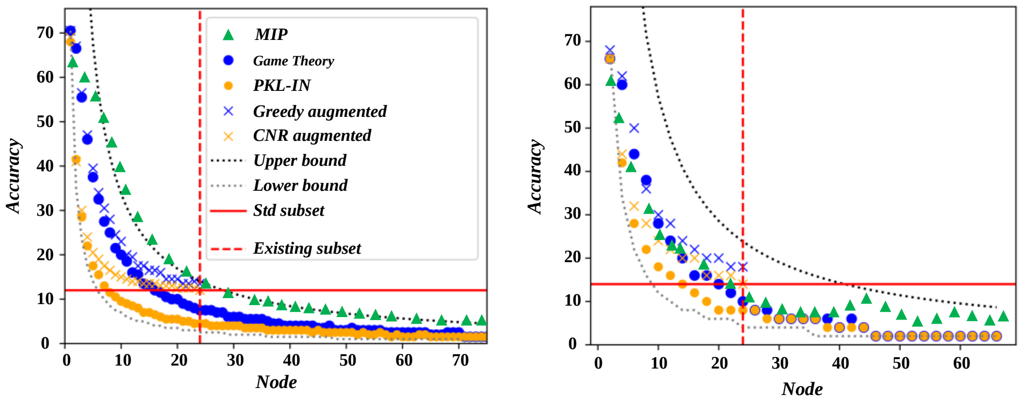

Figure 10: Simulation comparison of utility functions of each vehicle node

Figure 11 shows that the Nash equilibrium generated by the forward game algorithm is relatively stable in all nodes, because after T iterations, the nodes are closer to the central value, although there is a slight fluctuation; The Nash equilibrium generated by the reverse game algorithm is more stable on the top-ranked nodes, and only extreme strategies appear in a few lower-ranked nodes.

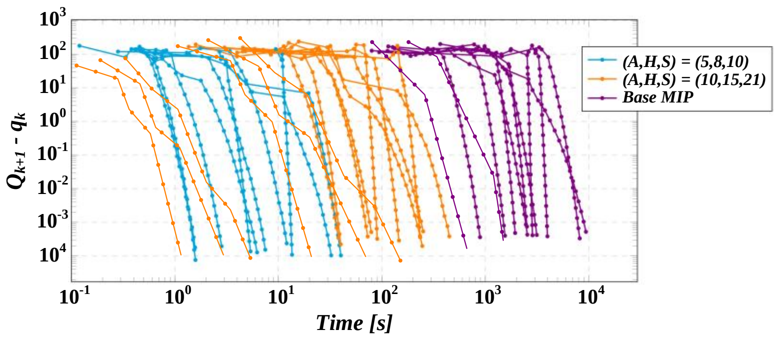

Figure 11: Comparison of Nash equilibrium

5 Discussion

The Stackelberg game - Mixed Integer Programming (MIP) collaborative optimization framework proposed in this study performs outstandingly in an area energy internet scenario with 20 charging stations and 10 hydrogen stations: compared to single-level centralized optimization, the total operating cost decreases by 15.7% and the utilization rate of renewable energy increases by 22.3%; compared to unordered scheduling, user waiting time is reduced by 31.5% and operator revenue increases by 12.9%, with the share of overcapacity charging stations dropping from 60% to 30%; compared to RL models such as DQN/PPO, the optimization time for a 50-node system is reduced by 57% and total costs decrease by 18.3%, while the constraint satisfaction rate for 100 nodes reaches 95.3%. The core reasons for the improved coordination and reduced waiting time are: through the Stackelberg 'leader-follower' mechanism, upper-level operators guide demand through dynamic pricing, while lower-level users of 31,200 EVs and 8,600 FCEVs adjust their choices based on demand, achieving benefit coordination; simultaneously, the supply and demand matching is optimized from a spatiotemporal resource perspective, alleviating congestion. The key methodological difference lies in: unlike the low executability of centralized optimization that 'forces global optimality', the Stackelberg equilibrium balances individual rationality with system efficiency through price leverage; and by utilizing the KKT conditions and Big-M method, user responses are converted into constraints and linearized, solving the traditional game theory challenge of embedding physical constraints and the difficulty of characterizing multi-agent interactions in MIP, thus achieving a unity of physical feasibility and decision feedback.

6 Conclusion

To address the core issue of multi-agent interest conflicts and complex physical constraint collaborative optimization in multi-energy transportation systems, this study constructs a two-layer decision-making framework that integrates game theory and mixed-integer programming. This framework describes the dynamic interaction mechanism between charging station/hydrogen station operators (leaders) and electric vehicle/hydrogen fuel cell vehicle users (followers) using a Stackelberg game structure: operators formulate differentiated service pricing strategies to guide the spatio-temporal distribution of user demand, while users optimize their charging path choices based on real-time prices and traffic conditions. To overcome the difficulties associated with solving traditional game models, an innovative transformation of the follower's optimal response problem into a set of Karush-Kuhn-Tucker (KKT) condition constraints is employed, thus equivalently reconstructing the bi-level nonlinear game equilibrium problem into a single-layer mixed-integer linear programming model that can be solved efficiently. This approach ensures the precise representation of hard constraints such as power flow equations, hydrogen equipment power limits, and traffic network capacities, while also realizing closed-loop feedback for strategic decision-making behavior.

(1) This study is the first to combine Stackelberg games with mixed-integer programming, establishing an exactly solvable bi-level collaborative optimization model through KKT condition transformations. This overcomes the bottleneck of traditional methods that struggle to simultaneously handle discrete decisions, physical constraints, and multi-agent games, providing an analytical framework for multi-energy transportation systems that balances computational efficiency and equity.

(2) The empirical research in the energy internet demonstration area shows that in a testing scenario with 25 fast charging stations, 8 hydrogen stations, and 15 traffic nodes, the proposed model significantly improves the overall system performance compared to traditional centralized optimization schemes: the total operating cost during the scheduling period is reduced by 19.2%, the average daily equipment utilization rate of the charging stations increases by 28.7%, the average waiting time for users decreases by 33.8%, and the maximum queue length during peak periods is reduced by 46.1%. At the same time, operators achieve a net revenue increase of 17.5% through dynamic pricing strategies, and the renewable energy absorption rate increases by 24.3%, effectively resolving the contradiction between 'operators pursuing profit maximization' and 'users pursuing cost minimization.' In a fluctuating scenario where traffic flow density increases by 40%, the model can still keep the interruption rate of charging services below 4.8%, demonstrating its strong robustness.

(3) Subsequent research will expand to the design of multi-energy market mechanisms, coupled modeling of heterogeneous traffic flows, and resilience optimization strategies for extreme weather conditions, further deepening the dynamic synergy mechanisms of transportation-energy systems. This study not only provides a quantitative tool for the coordinated optimization of multi-energy transportation systems that combines physical feasibility with economic rationality, but its core methodology of 'game equilibrium-MIP transformation' can also be extended to areas such as multi-agent collaborative decision-making in energy power systems and resource optimization in smart cities, holding important theoretical significance and engineering application value for promoting the deep integration of transportation and energy development.

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