A Temporal Perspective on the Paradox of Pinocchio's Nose

Matjaž Gams
Jozef Stefan Institute, Jamova 39, Ljubljana, Slovenia

Eva Černčič
University of Southampton, Southampton SO17 1BJ, United Kingdom

Angelo Montanari
Università degli Studi di Udine, Via delle Scienze, 206, 33100 Udine, Italy

Keywords: logical paradoxes, Pinocchio, nose growth

Received: March 15, 2016

The paradox in the question originates from the well-known children story of Pinocchio, starring a wooden boy whose nose grows whenever he is lying. The paradox stems from his statement: "My nose is growing". In real life, a statement that a person's nose grows should not cause any problem, since noses have the property of growing; however, the statement refers to a specific growth, associated with lying and logic. In this paper, we present yet another view, based on artificial-intelligence agents.

Povzetek: Prispevek obravnava paradoks Ostržkovega nosa in v nasprotju s prevladujočim svetovnim mnenjem pokaže pravilnost Eldridge-Smithovega paradoksa, čeprav velja le kratek čas.

The Paradox of Pinocchio's Nose was first proposed on February 2001 by 11-year-old Veronique Eldridge-Smith, the daughter of Peter Eldridge-Smith, who wrote an article in the journal Analysis. It was often proclaimed to be a liar paradox, but this paper finds the paradox valid. However, as with most logic paradoxes when faced with real life and AI solutions, the paradox turns out valid for only a short period of time.

To formalize the problem as a logical puzzle [1], we treat it as a set of propositions:

The statement: "*Pinocchio's nose is growing*" is true if and only if Pinocchio's nose is growing at the moment.

Pinocchio's nose is growing if and only if he is telling a lie.

Pinocchio is saying: "*My nose is growing*".

We can give the following abstract form to the problem. Let 'P' denote the sentence "*My nose is growing*", and let P be its description in the natural language, i.e., the action of nose growth. Then, if 'A' is any statement by Pinocchio, we are left with the following system:

True('P') iff P

P iff False('A')

A = P

We evaluate the system in the following way. We assume 'P' is true. Thus, (1) implies the truth of P. However, by (2) 'A' is false and (3) yields that so is 'P'. Summarising, we can conclude that True('P') implies False('P'). A similar argument may be conducted to show that False('P') implies True('P'), and, hence, we can conclude that True('P') if and only if False('P').

According to Peter (and Veronique) Eldridge-Smith, who originally formulated the problem in [2] and further elaborated on it in [3], the Pinocchio paradox is an improved version of the classical liar paradox, where any attempt to assign a binary truth value to the statement: "*This sentence is not true*" leads to the conclusion that the statement is true if and only if it is false. The same conclusion is reached in the Pinocchio case, where his nose is growing if and only if it is not growing. However, as Eldridge-Smith points out [2], the Pinocchio paradox differs from the classical Liar paradox in one important feature: *"(the nose) is growing"* is not a synonym for *"(the sentence) is not true",* as having one's nose grow is not a semantic feature. Hence, the way out from the Liar paradox originally proposed by Tarski, explained in [4] (to exclude semantic predicates from the object language), and refined by Kripke [5] (to define a non-strict metalanguage hierarchy that allows non-paradoxical uses of semantic predicates in the object language) does not work here. Unlike the liar paradox, the Pinocchio's nose should represent a "true paradox" as declared by Eldridge-Smith. A discussion followed by Beall and Eldridge-Smith [6, 7, 8], who argued whether the Pinocchio paradox affords no argument against 'simply semantic dialetheism' or not, without mutual agreement.

In this paper, we focus our attention on another distinctive feature of the Pinocchio paradox: its temporal dimension. Eldridge-Smith himself briefly mentions it at the very beginning of his paper [2]. In the original formulation of the paradox, indeed, given by his daughter Veronique, Pinocchio says '*My nose will be growing*'.

Quoting Eldridge-Smith: “The use of a future tense ties in with when Pinocchio's nose is supposed to grow after telling a lie. Philosophers will naturally want to know how soon afterwards. (There is an interesting version of the Epimenides though, if one does not restrict how soon Pinocchio's nose should grow. Imagine Pinocchio says ‘My nose will grow' but everything else Pinocchio says is true. Then, Pinocchio's nose will grow if and only if it does not.)".

In the following, we elaborate on the temporal issues involved in the Pinocchio paradox from an artificial intelligence perspective [9], showing the possibility of another interpretation of the paradox. First, we observe that the temporal length of the utterance cannot be ignored. Utterances are not instantaneous, and any statement can be properly evaluated only at the moment when it is completed. Second, we assume that Pinocchio, despite his fictive nature, has to abide to the laws of our universe (which is evident through other stories about Pinocchio). In this context, some computing mechanism (fictive or real, still obeying laws of the ‘computing universe') must evaluate Pinocchio's statements and possibly ignite the growing of the nose, say in time ε. Afterwards, in the case of untruth, the nose growth is instantaneously triggered for a period of time, say x, independent of the nature of lies.

Now, we can consider the most basic case, in which prior to saying ‘P’, where P indicates a lie, Pinocchio was quiet, e.g., at sleep. Timing of ‘P’ on the time line corresponds to the point in time t_0 when the sentence was completed.

Figure 1: Simple nose growth.

Timing of the nose growth is depicted in Figure 1. In case of a lie, Pinocchio's nose grows between $t_0 + \varepsilon$ and $t_0 + \varepsilon + x$. If ‘P’ indicates the sentence “*My nose is growing now*", then the mechanism evaluates P at t_0, which turns out to be a lie, and Figure 1 correctly displays the process of nose growing, since at the moment of speaking his nose was not growing.

Let us consider now a more complicated situation where Pinocchio did tell some lie in the past, with the consequence of growing nose in the present, or will tell some lie in the near future, interacting with the time schema of nose growing. Consider the case of two sequential lies, completed at times t_0 and t_1, respectively, where at time t_1 the sentence “*My nose is growing now*" is uttered. Depending on the timing of t_1, we distinguish two non-trivial cases:

$t_0 < t_1 < t_0 + \varepsilon$: the first one is of purely theoretical interest, as it involves completing the sentence in a time frame smaller than is physically possible. Nevertheless, as his nose is not yet in the growth phase, the total growth time is going to be extended, as displayed in Figure 2.

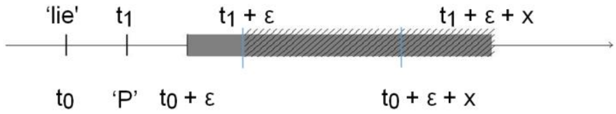

Figure 2: Nose growth in case of overlapping lies.

$t_0 + \varepsilon < t_1 < t_0 + \varepsilon + x$: in that period his nose is already growing, due to the previous lie, and thus no further action is taken.

Cases with a slightly different formulation of the sentence may also be of interest. “*My nose will be growing*" is obviously a true statement, since Pinocchio will sooner or later utter another lie (notice that since evaluating Pinocchio's statements takes some time, there is in any case a delay between the completion of the utterance and the possible start of the process of nose growing, thus avoiding the variant of the Epimenides mentioned by Eldridge-Smith in [2]).

But what about: “*My nose will be growing at time t*", formally denoted by ‘P(t)'? This sentence is unproblematic for $t < t_0 + \varepsilon$ since it is obviously a lie and consequently his nose will grow, starting at $t_0 + \varepsilon$. But for $t \geq t_0 + \varepsilon$ we obtain the original formulation of the problem, resulting in the paradoxical state, which cannot be avoided as in the case of the liar paradox. However, as mentioned we can elaborate the problem from the point of view of artificial intelligence [9, 10], by assuming that there exists an agent that computes the truth of Pinocchio's statement continuously over time. For the sake of simplicity, we assume that the computing time is ε (as we did before) and that the shortest observable/measurable time interval is Δ. The behaviour of the agent responsible for the nose growth can be defined as follows:

for every time step Δ*, compute: if False(‘P(t)') holds at time* $t = t_c$*, where* t_c *represent the current time of the evaluation, then trigger nose growth for a period of time x, starting at* $t_c + \varepsilon$

Let us consider the statement ‘P(t)’ for some meaningful values of t.

$t = t_0$: we have the original formulation of the problem, which was already discussed before. The same evaluation applies to all times $t < t_0 + \varepsilon$ as the agent knows no growth mechanism could be triggered resulting in growth at time t.

$t = t_0 + \varepsilon$: ‘P(t)’ is processed and evaluated at t_0 (the evaluation process takes time ε). As the system cannot decide whether the statement is true or false for time $t_0 + \varepsilon$, the paradoxical state appears in the time interval $[t_0 + \varepsilon; t_0 + \varepsilon + \Delta]$, and the agent cannot cause growing the nose at time $t_0 + \varepsilon$. But the growth mechanism can be triggered already for the growth at time $t_0 + \varepsilon + \Delta$, as an intelligent agent knows there was no mechanism triggered that would result in the nose growth at time t ($= t_0 + \varepsilon$). Notice the way we assimilated a paradoxical state to a state with no information (undecided) and we assumed that if an agent has no information about the fact that statement ‘P’ is false at time t, it makes no action. This may look, to some extent, arbitrary, but we consider such a choice the most reasonable one. Moreover, at the time of decision there are in fact only two possibilities: the nose starts growing or not. A short time later, the agent can conclude something in both cases. Therefore, the true paradox actually disappears after a short time in any sensible interpretation, not only in the above-mentioned one.

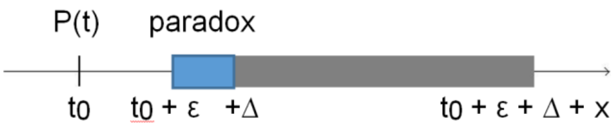

Figure 3: Nose growth in case of Pinocchio's statement: “My nose will be growing at the time $t_0 + \varepsilon$ ".

$t > t_0 + \varepsilon$: ‘P(t)’ is first processed at t_0 and then successively at each step Δ later, until the evaluation at time $t - \varepsilon + \Delta$, where it becomes clear if nose will be growing at time *t*. If the growth mechanism was not triggered by some previous lie, the scenario presented in Figure 3 repeats with a short contradicting moment (the paradoxical state) followed by the nose growth as shown in Figure 4.

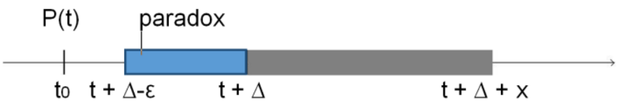

Figure 4: Nose growth in case of Pinocchio's statement: “My nose will be growing at the time greater than $t_0 + \varepsilon$ (iii) ".

Finally, Pinocchio can proclaim that his nose will grow in some intervals, say, from $t1$ to $t2$. Let $t = (t1;t2)$. Analyses depend on the length of the interval. In particular, we distinguish the following cases:

$t2 < t1 + \Delta$: due to the length scale of the interval, the situation is treated in the same manner as if interval was a point, as discussed above.

$t1 + \Delta < t2 < t1 + \Delta + x$: the agent evaluates ‘P(t)’ in time intervals Δ. The paradoxical state appears for a short moment at $t1 - \varepsilon + \Delta$, then the growth mechanism is triggered. At all later evaluations, a truth value is assigned to ‘P’, due to triggered nose growth. Thus, the situation is identical as in Figure 4.

$t_2 > t_1 + \Delta + x$: the treatment of the problem is the same as in the previous case until time $t_1 + \Delta + x$, where a paradoxical state appears again and another nose growth is triggered. This process can repeat itself numerous times until the evaluation at time t_2 yields the truth value. The situation is depicted in Figure 5.

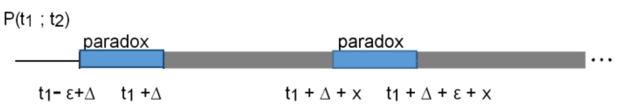

Figure 5: Nose growth: interval evaluation.

The contribution of the paper can be summarized as follows. In [2, 3], Peter Eldridge-Smith introduced the paradox of Pinocchio's nose growth and pointed out that, unlike the liar paradox, it is a true paradox, which cannot be solved by disallowing problematic uses of semantic predicates in the object language. In this paper, we argued that Pinocchio paradox can be suitably confined to some very small time intervals.

Unlike the liar paradox, which is based on a timeless truth statement, the treatment of Pinocchio paradox is a time-dependent truth statement. It is not new that time and paradoxes can be intertwined: for example, the Ross-Littlewood paradox [11] deals with time and infinity, but it is not directly related to this paper.

We showed that the Pinocchio paradox can be analysed using a temporal approach, which takes time aspects, such as the temporal length of utterances as well as the duration of the process of statement evaluation, into consideration, and consequently makes it possible to significantly reduce the (temporal) extent of the paradox. On the other hand, for that particular period of time, Eldridge-Smith's claim that the Pinocchio paradox differs from the classical Liar paradox seems to be valid.

References

[1] Kremer, P. (2009) The Revision Theory of Truth. The Stanford Encyclopedia of Philosophy (Spring 2009 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/spr2009/entries/truth-revision/.

[2] Eldridge-Smith, P., Eldridge-Smith, V. (2010) The Pinocchio paradox. Analysis 70:212-215.

[3] Eldridge-Smith, P. (2011) Pinocchio against the dialetheists. Analysis 71:306-308.

[4] Hodges, W. (2013) Tarski's Truth Definitions. The Stanford Encyclopedia of Philosophy. (Spring 2009 Edition), Edward N. Zalta (ed.).

[5] Kripke, S. (1975) Outline of a Theory of Truth. The Journal of Philosophy, 72(19):690-716.

[6] Beall, J. C. (2011) Dialetheists against Pinocchio. Analysis 71 (4):689-691.

[7] Eldridge-Smith, P. (2012) Pinocchio beards the Barber. Analysis 72:749-752.

[8] Beall, J. C. (2014) Rapunzel Shaves Pinocchio’s Beard. Contradictions: Logic, History, Actuality. 6:27-30.

[9] Russel, S., Norvig, P. (2010) Artificial intelligence: a modern approach (third edition), Prentice Hall.

[10] Wooldridge, M. J., Jennings N. R. (2014) Intelligent Agents, Springer.

[11] Littlewood, J. E. (1953) A Mathematical Miscellany. Methuen; reprinted as Littlewood's Miscellany, B. Bollobas (ed.), Cambridge: Cambridge University Press, 1986.