

Clustering Algorithms in Process Monitoring and Control Application to Continuous Digesters

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In this study, the controllability of the Kappa number in two cooking applications is investigated. The Kappa number is one of the quality measures in the pulp cooking process and usually the only on-line measurement. It is a measure of the residual lignin content in the pulp. The cooking of the pulp mainly takes place in the digester, where the significant part of the lignin is removed from the chips. The control of the Kappa number is mainly carried out by temperature before the cooking zone, therefore it is important to get some indication of the quality (Kappa number) beforehand. The Kappa number is predicted before the cooking zone in two different cooking applications with the main variables affecting the Kappa number using a clustering and fault diagnosis system (SOM and fuzzy clustering). The clustering and fault diagnosis system is used also for a monitoring of the input variables. The data is collected from industrial conventional and Downflow Lo-Solids continuous cooking digesters. Good results were achieved using the clustering and fault diagnosis system.

Povzetek:

1 Introduction

Industrial processes generate a lot of information for operators. The operators have many measurements to observe and control at the same time. This can be helped by combining the knowledge. Clustering (see e.g. [1] and [2]) is one of the methods for combination, because the information is saved in databases and it is available. Industrial processes are usually highly non-linear and it is very difficult or impossible to make accurate models with conventional modeling techniques. These kinds of systems can be called complex systems. Pulp and paper processes are examples of this kind of systems. Due to the nonlinearities and insufficient measurements for physical modeling, neural networks ([3] and [4]) and fuzzy methods ([5], [6], [7],[8], [9], [10], [11], [12] and [13]) have been applied for the modeling and clustering purposes of the industrial systems.

The processes studied are continuous cooking applications. Most of the kraft pulp is produced in the continuous digesters [14]. In a typical chemical pulping process, the pre-treated and penetrated wood chips are fed into the impregnation vessel and pulp digester where lignin is removed from the chips with the aid of chemical reactions. Thus the wood fibres are separated from each other. The kraft pulping process has been widely investigated during recent years (see e.g. [15], [16] and [17]) and the optimal cooking conditions at the single chip scale are well known. The usual problem, however, is that the optimal conditions

at the digester scale cannot be ensured. Reasons for this are the large dimensions of the process equipment, inadequate measurements and a residence time of several hours.

The quality of the pulping is characterized e.g. by the pulp's strength, viscosity, yield and Kappa number. The Kappa number indicates the residual lignin content of the pulp in the blow line. The control of the Kappa number is a very important part of the continuous cooking process. A steady blow line Kappa number enables an optimized chemical consumption in the subsequent parts of the fibre line. The quality of the pulp has a major effect on the final paper quality. [18]

The Kappa number is one of the most important quality indicators in the cooking process. Therefore, the control of the Kappa number is important. In conventional cooking, the main control actions are performed in the top of the digester, but the on-line measurement of the Kappa number is in the bottom of the digester. The residence time between these points is about four hours. It is obvious that with a prediction of the Kappa number in the top of the digester, more information is achieved and control actions can be executed earlier than without any prediction. The prediction can give new information of the change in the process state and the direction of a change earlier. In the Downflow Lo-Solids cooking process, the cooking zone begins at the middle of the digester and the cooking temperature is the main variable for Kappa number control.

The main active variables for the Kappa number are

temperature, alkali concentration, cooking (residence) time and the wood species. The temperature is controlled prior to the cooking zone. The alkali (white liquor) is added into the several parts of the process, depending on the application. The alkali is impregnated into the chips in the impregnation vessel, before the cooking operation occurs in the digester. The air is removed from the chips before the impregnation vessel in order to ensure the impregnation of the chips with the alkali. The lignin is partly removed in the impregnation vessel, due to the high temperature and alkali addition in the feed of the impregnation vessel. The main lignin removal takes place in the cooking zone, where the temperature is significantly higher than in the impregnation vessel. The pulp is washed in the counter-current washing zone.

The Kappa number is modeled or predicted in several studies, e.g. [19],[20],[21],[22] and [23]. A neural network trained with a back propagation learning rule was used in Dayal [19]. In Musavi *et al.* [20] a radial basis function neural network model was constructed. In Musavi *et al.* [21] a neuro-fuzzy system is utilized in the Kappa number prediction. Gustafson's Kappa number model [23] is used in the real-time Kappa number modeling in the conventional cooking process in [24] and in the Downflow Lo-Solids cooking process in [25].

In this study, the Kappa number is predicted in the two cooking applications before the cooking zone using the main variables affecting the Kappa number. The inputs before the cooking zone (BCZ) and the output (blow line Kappa number) of the model are presented in the Table I.

Table I. Variables of the system.

Variable	Unit
Alkali concentration BCZ	g/l (Na ₂ O, EA)
Temperature BCZ	K
Production rate BCZ	adt/d
Kappa number BCZ	
Blow line Kappa number	

The prediction model for the Kappa number is constructed by a combination of the SOM [4] and fuzzy clustering [10]. The system is used for prediction and monitoring purposes. SOM is the first clustering tool and also a fault diagnosis system. SOM has been used for monitoring and prediction purposes in [26]. The quantization error is calculated, and if the error is notable, information is given that the prediction can be faulty. This signal is given with color codes. The colors of the traffic light are used as in Ahvenlampi *et al.* [27]. If the system is in a good process state, the signal is green. A slight deviation from the normal process state is indicated using a yellow color, and very significant changes are colored with red. This color code is a very useful tool for the operators. The final prediction model is done with a fuzzy clustering model. In this study, the Gustafson-Kessel [28] fuzzy clustering model, which is a modification of the fuzzy c-means [29] algorithm, is used. The inputs are the main active variables of the Kappa number: the effective alkali, the temperature and the residence time of the chips in the cooking, which is, in

our case, the production rate. Also, the Kappa number before the cooking zone was used as an input to the system. The Kappa number was modeled using Gustafson's [23] Kappa number model. The input variables are the same as in Gustafson's Kappa number model. The results for the conventional cooking process are presented for the first time in [30]. In this study, the results for the Downflow Lo-Solids cooking process are also presented. Good results were achieved in both processes using the clustering and fault diagnosis system.

The structure of the paper is as following. The methods used are presented in chapter 2. Results are considered in chapter 3 and discussion and conclusions are displayed in chapters 4 and 5.

2 Methods used

In this chapter, methods used are presented. Empirical and experimental methods were applied. Gustafson's [23] Kappa number model is an empirical model for delignification. Clustering methods, such as fuzzy clustering (see e.g. [9] and [10]) and SOM [4] are also presented. The clustering and fault diagnosis system is formulated using the combination of SOM and the fuzzy clustering model.

2.1 Gustafson's Kappa number model

Gustafson *et al.* [23] have derived a mathematical model consisting of a series of differential equations describing the combined diffusion and kinetics within a wood chip during the kraft pulping process. The model development has been based on the studies of several researchers (see, e.g. [23], [31] and [15]). The results are compared with data from cooks, in which the pulping rates were kinetically controlled, and in which the pulping rates were partially mass transfer controlled.

The lignin removal in the impregnation vessel can be calculated using Gustafson's Kappa number model for the initial phase. The rate equation for the initial phase delignification, is:

$$\frac{dL}{dt} = k_{il} e^{(17.5 - 8760/T)} L \quad (1)$$

where L is the lignin content at time t ,
 k_{il} is a species specific constant and
 T is temperature.

The species specific parameter k_{il} in the rate equation in the initial phase is 1. The initial phase seems to be independent of the OH^- concentration. This does not mean one can proceed through this phase without alkali, but only indicates that alkali concentration does not influence the rate.

2.2 Clustering methods

Fuzzy clustering methods can be used in modeling, identification and pattern recognition [29]. In this chapter, several objective functions used for Takagi-Sugeno[6] model

identification, usually minimized by fuzzy clustering methods, are presented. SOM [4] is also presented in this chapter. Classified data in c clusters is arranged in a vector $Z = \{z_1, z_2, \dots, z_N\}$. In this study, the consequent parameters for Sugeno models are estimated using weighted least squares.

2.2.1 Fuzzy c-means

Fuzzy c-means is a widely used algorithm for fuzzy identification. The FCM cost function is usually formulated as [29]:

$$J(Z; U, C) = \sum_{i=1}^c \sum_{k=1}^N (\mu_{ik})^m D_{ik}^2 \quad (2)$$

where $C = \{c_1, \dots, c_c\}$. $\{c_1, \dots, c_c\}$ are the cluster centers (prototypes) to be determined, $U = [\mu_{ik}]$ is a fuzzy partition matrix [29] and

$$D_{ik}^2 = (z_k - c_i)^T B (z_k - c_i) \quad (3)$$

is a distance (norm) defined by matrix B (usually the identity matrix), and m is a weighting exponent which determines the fuzziness of the resulting clusters.

2.2.2 Gustafson-Kessel algorithm

Gustafson-Kessel algorithm [28] (Appendix A) is the extension most used by the FCM for identification [9]. In this method, norm can be different with every cluster, and the method has the advantage of looking for variable size hyper ellipsoids. The new distance to be used in (2) becomes:

$$D_{ikB_i}^2 = (z_k - c_i)^T B_i (z_k - c_i) \quad (4)$$

In this way, quasi-linear behaviors of the existing operating regimes are detected quite correctly. Improved covariance estimation for Gustafson-Kessel algorithm has been introduced in [32].

2.2.3 Number of the clusters

The decision of the number of the clusters is perhaps the most critical point in fuzzy clustering. Many methods have been introduced for the selection of the clusters, see e.g. [9] and [10].

In this study, fuzzy hypervolume [33] is used in deciding of the clusters. Fuzzy hypervolume is calculated using equation (5)

$$F_{hv} = \sum_{i=1}^c [\det (F_i)]^{1/2} \quad (5)$$

where F_i is a fuzzy covariance matrix.

2.2.4 SOM

The SOM [4] (Appendix B) is an unsupervised artificial neural network. The network is normally a two-dimensional mapping / projection of the data group. The visualization of the map is easier with a two-dimensional map. In the training of the SOM network, data points are sequentially introduced to the SOM. In each iteration, the SOM neuron which is closest to the input unit is selected by the equation (6). This unit is the Best Matching Unit (BMU) or winner.

$$\|z - c_c\| = \min_i \{\|z - c_i\|\} \quad (6)$$

The weight vectors are updated using the following formula. Only the weight vectors which are inside the neighborhood radius, are updated.

$$c_i(t+1) = c_i(t) + h_{ci}(t) [z(t) - c_i(t)] \quad (7)$$

2.3 Clustering and fault diagnosis system

The clustering and fault diagnosis system is formulated with the combination of SOM and the fuzzy clustering algorithm. The SOM is used as a first clustering method [34] and a fault diagnosis tool in the system. The SOM is trained with the normal operation data which is normalized. The inputs to the system are the temperature, alkali concentration, production rate and Kappa number before the cooking zone. The output is the Kappa number at the blow line of the digester. The SOM codebook matrix (50 times 40 matrix) is used as input data for fuzzy clustering identification. When the clustering and fault diagnosis system is formulated, the validation data is put through the SOM network and the best matching unit is found. The best matching units are used with the fuzzy clustering model. The quantization errors are used in the coloring of the trends of the measured inputs and the predicted output. The value of the error is used in the color-coding. In the normal process state, the color code is green. Yellow color is used for slight deviations from the normal operation and major changes are colored with a red color code. The structure of the clustering and fault diagnosis system is illustrated in the Fig. 1.

3 Case studies

In this study, clustering and fault diagnosis system is used for monitoring and prediction purposes in conventional and Downflow Lo-Solids continuous cooking digesters. The inputs to the system are monitored and the Kappa number in the blow line is predicted. The clustering and fault diagnosis system is a combination of SOM and fuzzy clustering methods. The modeling data (about 30 000 data points for both applications) was normal operation data from the industrial continuous digesters. The outliers and faulty measurements are filtered out from the data. The inputs are

Figure 1: Clustering and fault diagnosis system

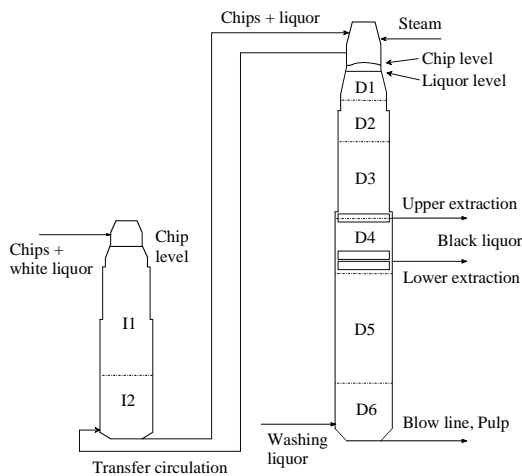


Figure 2: Impregnation vessel and continuous Kamyr digester in conventional cooking process

temperature, alkali, Kappa number and production rate before the cooking zone. The output is the predicted Kappa number at the bottom of the digester. The system is validated with the data from the same industrial digester, but from the different time periods.

3.1 Case 1

Case one is a conventional Kamyr process consisting of an impregnation vessel and a steam/liquor phase digester (Fig. 2). The process has been simplified by removing almost all of the original liquor circulations, thus only the upper and lower extraction screens in the end part of the cooking zone are used. A characteristic of this process is the grade changes between softwood and hardwood performed almost every other day. The active alkali concentrations of the white liquor, the digester feed circulation liquor and the two black liquor circulations from the end of the cooking zone are measured. The sulphide concentration of the white liquor is also measured and it is assumed to stay constant during the cooking. Before the latest simplifications

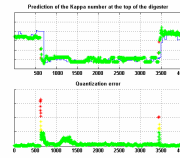


Figure 3: The coloring of the grade changes in the validation period 1.

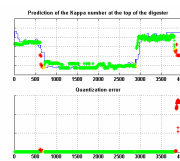


Figure 4: The coloring of the grade changes and shutdown in the validation period 2.

of the process, alkali measurements were taken from the extraction screens in the upper part of the digester’s cooking zone. These measurements have been utilized in the alkali profile. Temperatures are measured from the various parts of the digester.

The size of the SOM network structure was 50 times 40. The SOM codebook vector (2000 neurons) was an input data for the fuzzy clustering model. The fuzzy clustering method used was the Gustafson-Kessel algorithm. The fuzzy clustering model was divided into 4 local models (clusters) according to the fuzzy hypervolume [33]. The premise membership functions (bell-typed) are projected from the clusters and the local models are obtained using weighted least squares. The fine tuning of the parameters is performed by gradient descent algorithm, see e.g. [8].

The fault diagnosis phase uses different size quantization errors to indicate the deviations from the normal operation points. Thus, this information is used in the coloring of the Kappa number prediction trend with the colors (green, yellow, red). In the Figs. 3 and 4, the situations where the errors deviate and the trends have changing colors are

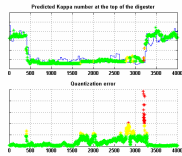


Figure 5: Predicted Kappa number and the quantization error in the validation period 3.

shown. In these process states, the deviations are caused by the grade changes and shutdown. In Fig. 3, there are grade changes at the points of 700 and 3500. In Fig. 4, the grade changes are at the points of about 600 and 2900. The shutdown can be seen in Fig. 4 at the point of 3800.

In Figs. 5 and 6, a faulty process state where the system is not normal can be observed. There are grade changes at the points of about 450 and 3250. A slight deviation can be seen at the point of about 2750, and it can be observed from both Figs. 5 and 6 as a yellow and red trend color. The same kind of example is illustrated in Figs. 7 and 8, where the grade changes are at the points of about 500 and 3750. The operational failure is at the point of 2450, which has been identified by the clustering and fault diagnosis system. In these figures the only significant deviations are colored. Thus, the system is not too sensitive to small deviations. The error size can be used as a tuning factor to the system. The color changing value can be small, if every deviation is desired to be shown, and if only notable disturbances are needed to be shown, the tuning factor can be big.

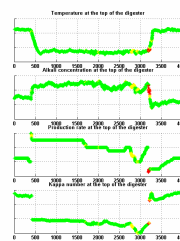


Figure 6: Inputs to the system in validation period 3

In Figs. 3-8, the validation results of the clustering and fault diagnosis system in the conventional cooking process are shown. The time period in the figures is one minute. It is the same time period as for the history database in this industrial plant. As it can be seen from the figures, the combined clustering model is accurate and it is able to observe the changes in the process. The clustering and fault diagnosis system can be used in fault diagnosis and for Kappa number prediction purposes.

3.2 Case 2

Case two is a Downflow Lo-Solids [35] cooking process (Fig. 9). The chips are impregnated in the impregnation vessel (I1-I2) and in the first zone (D1) of the digester. Between upper extraction and cooking circulation there is a counter-current washing zone (D2). In this zone, black liquor is displaced with cooking circulation liquor which

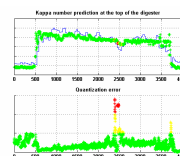


Figure 7: Prediction in validation period 4.

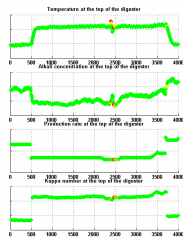


Figure 8: Inputs in the validation period 4.

temperature and alkali concentration are high. The lignin is mainly removed in the comparatively long co-current cooking zone (D3). At the bottom of the digester there is a short washing zone. Softwood chips mainly consist of pine chips with a small amount of spruce chips. Hardwood chips consist mainly of birch chips with a small addition of aspen chips.

The effective alkali concentrations of the white liquor, digester feed circulation liquor, two black liquor extractions and cooking circulation are measured. The white liquor is added to the impregnation vessel's feed circulation, to the digester's feed circulation and to the cooking circulation. The sulphide concentration of the white liquor is measured, and it is assumed to stay constant during the cooking. Temperatures are measured from the liquor circulations and from the heating steam at the top of the digester. A temperature profile from the top of the digester to the cooking circulation is constructed emphasizing the measured temperatures suitably. The temperature profile from the cooking circulation to the blow line is based on the temperature of cooking circulation.

In Downflow Lo-Solids cooking, the Kappa number control is mainly performed by the cooking zone temperature in the middle of the digester (before D3).

The Kappa number prediction is shown in Fig. 10 and the inputs to the system in Fig. 11. The prediction is quite accurate in both species (hardwood at 0-1750 and softwood at 1750-3500). A grade change has occurred at 1750, where the system indicates a disturbance. Another faulty period is between 1850-2100, where the system has turned to a yellow signal.

In Figs. 12 and 13, the results from validation period 2

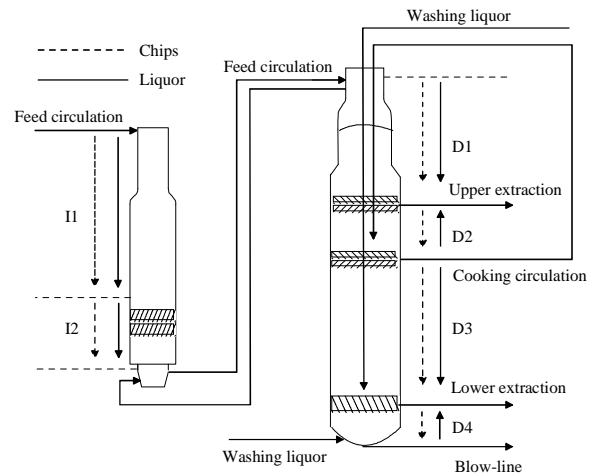


Figure 9: Main flows and flow directions of chips and liquor in impregnation vessel and digester in Downflow Lo-Solids cooking

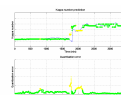


Figure 10: Kappa number prediction in the Downflow Lo-Solids cooking process. (Validation period 1)



Figure 11: The input variables to the system in the Downflow Lo-Solids cooking process. (Validation period 1).

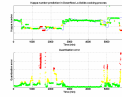


Figure 12: Kappa number prediction in the Downflow Lo-Solids cooking process. (Validation period 2)

are presented. There are grades changes at the points of about 400 and 2700. Shutdown has occurred at the point 5700. There is a faulty process state in both species. In the hardwood, the faulty state is at the period 1300-1600 and in the softwood case, the period is 4700-5100 and after the shutdown at 5700-6000.

4 Discussion

The sampling interval of the on-line Kappa number measurements is about half an hour. Hence, it is useful to also get continuous information about quality properties. The control of the Kappa number is mainly carried out with the cooking temperature, therefore it is important to get an indication of the quality (Kappa number) before the cooking zone in order to execute necessary control actions soon enough.

In this study, a clustering and fault diagnosis system for the monitoring of the process and prediction of the Kappa number in the blow line of the digester is constructed and validated. The system is implemented with a combination of SOM and the fuzzy clustering model.

As shown in Figs. 3-13, the results of the fault diagnosis and clustering system are very accurate. The proposed method is suitable for the optimization and fault diagnosis of the kraft cooking process. In the case of major process changes, the adjustment and verification of the model parameters into the optimal form is quite easy.

Fault diagnosis is carried out using the quantization errors in a coloring of the trends of the input measurements and predicted Kappa numbers. In Figs. 3-13, only significant deviations are colored. Thus, the system is not too sensitive to small deviations. The error size can be used as a tuning factor for the system. The color changing value can be small, if every deviation is desired to be shown, and if only major disturbances are needed to be shown, the tuning factor can be bigger. Color can be used to observe failures in the input measurements or deviation from good operation points. Yellow and red colors indicate also that the

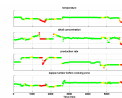


Figure 13: The input variables to the system in Downflow Lo-Solids cooking process. (Validation period 2).

prediction may be inaccurate.

The method has been tested with the conventional and Downflow Lo-Solids continuous cooking digesters and the possibility to implement the system into an automation system is considered. The clustering and fault diagnosis system will be used also as a fault diagnosis and redundant system for Gustafson's Kappa number model.

5 Conclusions

The applicability of SOM and fuzzy clustering approach for the controlability of the Kappa number was considered. The results of the usability of the combined clustering and fault diagnosis system in the monitoring of the conventional and Downflow Lo-Solids continuous cooking processes and the prediction of the Kappa number with the system are shown.

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Appendix A

Process of Gustafson-Kessel algorithm:

Step 1: Compute the cluster centres:

$$c_i^{(l)} = \frac{\sum (\mu_{ik}^{(l-1)})^m z_k}{\sum (\mu_{ik}^{(l-1)})^m}, 1 \leq i \leq C$$

Step 2: Compute fuzzy covariance matrix:

$$F_i = \frac{\sum_{k=1}^N (\mu_{ik}^{(l-1)})^m (z_k - c_i^{(l)}) (z_k - c_i^{(l)})^T}{\sum_{k=1}^N (\mu_{ik}^{(l-1)})^m},$$

$$1 \leq i \leq C$$

Step 3: Compute the distances:

$$B_i = \rho_i \det(F_i)^{1/n} F_i^{-1}, 1 \leq i \leq C$$

$$D_{ikBi}^2 = (z_k - c_i)^T B_i (z_k - c_i), 1 \leq i \leq C, 1 \leq k \leq N$$

Step 4: Update the partition matrix:

$$\mu_{ik}^{(l)} = \frac{1}{\sum_{j=1}^C (D_{ikBi} / D_{jkBi})^{2/(m-1)}}$$

iterate until $\|U^{(l)} - U^{(l-1)}\| < \varepsilon$.

Appendix B

The training of the SOM network is as following:

Step 1: Give initial values for neighborhood radius $h_{ci}(t)$ and learning rate $\alpha(t)$

Step 2: Choose the steps K

Step 3: Choose one vector z from the learning data Z

Step 4: Find c , BMU (best matching unit) from the initialized network, which distance is closest to the input vector z . Euclidian distance is used.

$$\|z - c_c\| = \min \{\|z - c_i\|\}$$

Step 5: The updating of the weight vectors. Only the weight vectors which are inside the neighborhood radius are updated.

$$c_i(t+1) = c_i(t) + \alpha(t)h_{ci}(t)[z(t) - c_i(t)]$$

Step 6: Set $t = t + 1$. If $t = K$, stop. Otherwise go to step 3.

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