Robustness and Visualization of Decision Models

Andrej Bregar
Informatika d.d.
Vetrinjska ulica 2, SI-2000 Maribor, Slovenia
E-mail: andrej.bregar@informatika.si

József Györkös and Matjaž B. Jurič
University of Maribor, Faculty of Electrical Engineering and Computer Science, Institute of Informatics
Smetanova ulica 17, SI-2000 Maribor, Slovenia
E-mail: jozsef.gyorkos@uni-mb.si, matjaz.juric@uni-mb.si

Keywords: decision support, multi-criteria decision analysis, robustness metrics, mathematical optimization, principal components analysis, utility theory, promethee, electre

Received: June 20, 2008

Robustness analysis and visualization are two of key concepts of multi-criteria decision support. They enable the decision-maker to improve his understanding of both the model and the problem domain. A class of original mathematical optimization based robustness metrics is hence defined in this paper. In addition, several efficient existing techniques that have been successfully used in various ICT projects are presented. They include the stability intervals/regions and the principal components analysis. All approaches are applied to the multi-attribute utility function, and to the PROMETHEE II and ELECTRE TRI methods. Their benefits are discussed and demonstrated on real life cases.

Povzetek: Vpeljane so izvirne, na matematični optimizaciji temelječe metrike robustnosti večkriterijskih odločitvenih modelov ter predstavljeni učinkoviti pristopi k analizi občutljivosti in vizualizaciji, ki so bili uspešno uporabljeni na projektih iz področja informacijsko-komunikacijskih tehnologij.

1 Introduction

The decision model is a formal, simplified representation of the problem domain. It transforms input parameters, which are set by the decision-maker, into numerical or qualitative assessments, also called model assumptions (Power, 2002). These assessments should, however, not directly influence the implemented decision; they should rather be further analysed because they are often derived from data that are subject to uncertainty, imprecision and indetermination (Roy, 1996). These phenomena are the consequence of:

- incomplete domain knowledge or information;
- high domain complexity and high cognitive load of the decision-maker;
- insufficient insight into relations between model parameters;
- nonsystematic subjective assessments of criteria weights and evaluations of alternatives.

It is thus necessary to thoroughly and systematically test the inferred model assumptions. Preference aggregation, which is performed in order to assess alternatives, must represent merely the first phase of the decision-making process since the aim of decision analysis is not only to deal with the common problematics of selecting, ranking or classifying alternatives (Roy, 1996), but primarily to provide the decision-maker with a deep understanding of the problem domain, and to clearly expose the influence of preferential parameters and relations between them on the derived results. For this reason, a technique called the sensitivity analysis is used. It enables the decision-maker to judge in a formal and structured manner (Turban and Aronson, 2001):

- the influence of changes in input data – decision and uncontrollable variables – on the proposed solution that is expressed by the values of output variables;
- the influence of uncertainty on output variables;
- the effects of interactions between variables;
- minimal changes of preferential parameters that are required to obtain (un)desirable results;
- the robustness of both the decision model and the suggested decision in dynamically changing conditions.

Sensitivity/robustness analysis is one of key concepts in the field of multi-criteria decision aiding (Saltelli et al., 1999). It helps the decision-maker to prepare for the uncertain and potentially extreme future, and to improve his understanding of the problem domain by reflecting back on his judgements, synthesising preferences and observing changes. Yet, experiences of researchers and practitioners show that multi-dimensional complexity of the problem domain poses great challenges with regard to the sensitivity analysis as extensive tasks are difficult to communicate (Hodgkin et al., 2005). On the contrary, visual displays are a powerful means of communication for the majority of people. It is therefore recommended to implement and use interactive visual tools, in order to considerably improve the problem solving process.

Several approaches to sensitivity analysis exist that have been defined in conjunction with various decision-making methods (Frey and Patil, 2002; Vincke, 1999b). Because they are designed for specific types of decision models, they do not cover all relevant aspects of problem solving. Especially the following deficiencies should be taken into consideration:

- Existing L_P-metric based optimization methods and algorithms address sensitivity analysis only partially. They eliminate some dilemmas, but to systematically verify robustness it is necessary to simultaneously measure:
    1. the minimal modification of parameters according to which the best alternative loses its priority over any suboptimal alternative;
    2. the smallest modification that suffices for a selected suboptimal aternative to become the best one;
    3. the largest deviation that preserves the preferential relation of two alternatives.
- In the case of outranking methods ELECTRE and PROMETHEE, the robustness is measured only with regard to criteria weights, aggregated credibility degrees or inferred net flows. Other preferential parameters, such as thresholds, are not analysed.

The purpose of this paper is therefore (1.) to introduce a class of original L_P-metric optimization algorithms and programs that can be applied to holistically measure the robustness of decision models in conjunction with both the utility function and the outranking methods, (2.) to extend the concept of robustness analysis in the context of the ELECTRE TRI method to pseudo-criterion related thresholds, (3.) to formally present fundamental existing sensitivity analysis and visualization techniques that the authors have successfully used within the scope of their project work, and (4.) to discuss the benefits of these techniques. It should be noted that the utility function based approaches are adapted solely to determining the influence of criteria weights. This is a common practice because weight derivation is generally more subjective than specification of criterion-wise values of alternatives.

The rest of the paper is organized as follows. Section 2 provides a brief description of three decision methods – utility function, PROMETHEE II and ELECTRE TRI – to which the techniques of robustness measurement are applied. More detailed explanations can be found in the literature (Figueira et al., 2005). Section 3 gives a review of related work. Section 4 formally presents the stability intervals/regions based automatic sensitivity analysis. In Section 5, several new approaches to multi-dimensional robustness analysis are defined, which utilize (non)linear mathematical programming. This Section represents the original contribution of the paper. In Section 6, practical examples are provided. They demonstrate the strengths and benefits of the described techniques, and correspond to the results of projects. Section 7 concludes the paper by giving a resume and directions for further research.

2 Theoretical foundations of decision methods

2.1 Multi-attribute utility function

Since the utility theory was axiomatized by Keeney and Raiffa (1993), it has become the most widespread and probably the most relevant approach to decision analysis. Its foundations lay in the dogma of rational behaviour, so it is based on five axioms that provide a framework for a generic strategy that people should adopt when making reasonable decisions. The central concept of all axioms is the lottery, which is a space of outcomes that occur with certain probabilities. If preferences of the decision-maker satisfy these axioms, a real-valued function exists, which is called the utility function and correlates outcomes with a scale that expresses judgements on the [0, 1] interval.

It is uncomplicated to model the utility function for a single attribute (Zeleny, 1982). However, in practice an alternative is generally chosen by expressing preferences on a set of attributes or criteria $\{x_1, \ldots, x_n\}$. In this case, the alternative a_i is represented with a vector of values $a_i = (x_1(a_i), \ldots, x_n(a_i))$. Its utility is determined by assigning the vector a real value between 0 and 1. It is difficult to directly assess alternatives with the multi-attribute utility function, so this problem is reduced by defining a partial (one-dimensional) utility function for each attribute:

$$u_j(a_i) : x_j(a_i) \to [0, 1] .$$

Partial utilities are aggregated with a decomposition rule. It can have several forms of which the most widely used is the weighted additive decomposition:

$$u(a_i) = \sum\nolimits_{j=1..n} w_j \cdot u_j(a_i) .$$

2.2 PROMETHEE I and II methods

PROMETHEE is a family of methods that are based on the concepts of pseudo-criterion, outranking relation and pairwise comparisons (Brans and Vincke, 1985). For a pair of alternatives a_i and a_j, and for each criterion x_k, the preference function $P_k(a_i,a_j)$ is defined on the interval [0, 1] according to criterion-wise values $g_k(a_i)$ and $g_k(a_j)$, and according to the chosen indifference (q_j), preference (p_j) or Gauss (s_j) thresholds. This function expresses the degree to which a_i outranks (outperforms) a_j. It can have one of six possible shapes of which the linear is the most widely used:

$$P_k(a_i,a_j) = \begin{cases} 0 & , d_k(a_i,a_j) \le q_k , \\ \dfrac{d_k(a_i,a_j) - q_k}{p_k - q_k} & , q_k < d_k(a_i,a_j) < p_k , \\ 1 & , d_k(a_i,a_j) \ge p_k , \end{cases}$$

where $d_k(a_i,a_j) = g_k(a_i) - g_k(a_j)$. The outranking degrees are calculated for both “directions”, so that the $P_k(a_i,a_j)$

and $P_k(a_j,a_i)$ values are obtained. Criterion-wise indices are aggregated by taking criteria weights into account:

$$\pi(a_i,a_j)=\sum\nolimits_{k=1..n} w_k \cdot P_k(a_i,a_j)\ .$$

In the next step, the positive and negative ranking flows $\phi^+(a_i)$ and $\phi^-(a_i)$ are computed for every alternative a_i. They indicate the average degree to which a_i performs better respectively worse than all other alternatives:

$$\phi^+(a_i)=\frac{1}{n-1}\cdot\sum\nolimits_{a_j\in A}\pi(a_i,a_j) \text{ and}$$
$$\phi^-(a_i)=\frac{1}{n-1}\cdot\sum\nolimits_{a_j\in A}\pi(a_j,a_i).$$

The inferred flows can be interpreted in two ways. The PROMETHEE I method considers them simultaneously. A partial rank-order of alternatives is thereby derived, in which the incomparability relation may exist in addition to the preference and indifference relations. More often, a weak rank-order is obtained with the PROMETHEE II method. For this purpose, alternatives are evaluated with the net flow:

$$\phi(a_i)=\phi^+(a_i)-\phi^-(a_i)=$$
$$=\frac{1}{n-1}\cdot\sum\nolimits_{a_j\in A}\sum\nolimits_{k=1..n} w_k\cdot\left(P_k(a_i,a_j)-P_k(a_j,a_i)\right).$$

2.3 Dichotomic ELECTRE TRI method

The above described PROMETHEE I and II methods are designed to rank-order alternatives. Yet, the concepts of pseudo-criterion and outranking relation enable sorting as well. Two variants of PROMETHEE dealing with the sorting problematic have been recently introduced (Araz and Ozkarahan, 2007; Doumpos and Zopounidis, 2004), while the most widespread outranking method for sorting is ELECTRE TRI (Mousseau et al., 2000; Roy, 1991). The latter has been slightly modified within the scope of our research work by following the localization principle and preventing the incomparability relation, in order to allow for group consensus seeking and automated multi-agent negotiation (Bregar et al., 2008).

The dichotomic ELECTRE TRI method compares all alternatives with the profile b. Acceptable choices belong to the positive category C^+, while unsatisfactory ones are members of the negative category C^-. Let $s_j(a_i,b)$ express the degree to which the option a_i outperforms the profile b according to the criterion x_j. Its calculation is based on the indifference and preference thresholds q_j and p_j:

$$s_j(a_i,b)=\max\left(\min\left(\frac{g_j(a_i)-g_j(b)-q_j}{p_j-q_j},1\right),0\right).$$

Analogously, $s_j(b,a_i)$ represents the valued outranking of a_i by b. To express the degree of concordance with the assertion "the alternative a_i belongs to the class C^+", the indices $s_j(a_i,b)$ and $s_j(b,a_i)$ are aggregated with a fuzzy averaging operator:

$$c_j(a_i)=\frac{1}{2}\cdot\left(s_j(a_i,b)+\left(1-s_j(b,a_i)\right)\right).$$

For the sake of compensation of small weaknesses, the indices $c_j(a_i)$ are combined so that each is scaled by the weight w_j which represents the voting power of the j-th criterion and determines its contribution to the decision:

$$c(a_i)=\sum\nolimits_{j=1..n} w_j\cdot c_j(a_i)\ .$$

For each criterion, the discordance index is also defined based on the discordance and veto thresholds u_j and v_j. It reflects the partially noncompensatory degree of veto on the assertion "a_i belongs to C^+":

$$d_j(a_i)=\max\left(\min\left(\frac{g_j(b)-g_j(a_i)-u_j}{v_j-u_j},1\right),0\right).$$

The overall nondiscordance relation is grounded in two ways:

$$\tilde{d}'(a_i)=\prod\nolimits_{j=1..n}\left(1-d_j(a_i)\right) \text{ or}$$
$$\tilde{d}''(a_i)=1-d(a_i), \text{ where } d(a_i)=\max\nolimits_{j=1..n} d_j(a_i)\ .$$

Because of its absolute and noncompensatory nature, the nondiscordance index does not need to be combined with the concordance index. However, the valued outranking relation is usually obtained as a result of the following multiplication:

$$\sigma(a_i)=c(a_i)\cdot\tilde{d}(a_i),$$
$$\text{so that } \tilde{d}(a_i)=\tilde{d}'(a_i) \text{ or } \tilde{d}(a_i)=\tilde{d}''(a_i).$$

As $\sigma(a_i)$ = 0.5 denotes strict equality among the profile and the alternative, an appropriate λ-cut should be used to determine the "crisp" membership of the alternative:

$$a_i\in C^+ \Leftrightarrow \sigma(a_i)\ge\lambda, \text{ where } \lambda\in[0.5,1].$$

3 Existing approaches to sensitivity analysis and visualization

3.1 Techniques and studies

Hites et al. (2006) have explored the applicability of multi-criteria decision-making concepts to the robustness framework by observing the similarities and differences between multi-criteria and robustness problems. In their opinion, a conclusion is called robust if it is true for all or almost all scenarios, where a scenario is a plausible set of parameter values used to solve the problem. In a similar manner, Vincke (1999a) has provided the definition of a robust preference aggregation method. He has analyzed the robustness of eleven methods for the construction of an outranking relation.

Several researchers have investigated the L_P-metric sensitivity analysis of additive multiple attribute value models. Barron and Schmidt (1988) have introduced a procedure for the computation of weights that make the utility of one alternative exceed the utility of a compared alternative by the amount of δ. They have measured the closeness of derived and original weights by the squared deviation. Wolters and Mareschal (1995) have presented a similar method for determining the modification of a given set of weights, which sums up absolute deviations.

In addition to the closeness of weights, Ringuest (1997) has developed a second measure of sensitivity: a decision is considered insensitive if the rank order of weights that led to the original best solution must be altered for the optimal solution to change. A method has been defined which applies both criteria simultaneously by searching for solutions that minimize the L_1 and L_∞ distance metrics subject to a set of linear constraints. Jansen et al. (1997) have described the problems that may occur when using standard software for linear programming. Accordingly, they have proposed a framework for performing efficient sensitivity analysis.

Zopounidis and Doumpos (2002) discuss optimality measures for classification and sorting with respect to the assignment of alternatives in the reference set. Two L_1-norm distance metrics determine the classification error and the satisfaction of classification rules, respectively. Mousseau et al. (2001) measure the minimal difference α between the credibilities of alternatives and the cutting level that determines to which classes alternatives should be sorted. The larger is the value of α, the more stable are the assignments. Dias et al. (2002) do not approach the measurement of robustness numerically. Instead, their aim is to identify unrobust alternatives that have a wide range of classes to which they may be sorted, since they are strongly affected by the imprecision of data.

Hodgkin et al. (2005) argue that systematic multi-dimensional sensitivity analysis is not well supported by available facilities. Their review of existing techniques for the display of multi-dimensional data reveals many approaches which may be grouped in three categories:

1. approaches that try to retain all information and display it in some manner;
2. reduction of the dimensionality by applying the multi-variate statistical analysis;
3. displays of sensitivity analysis which focus on the outcomes rather than the input data, such as stability intervals, triangles of the weight space, etc.

Hodgkin et al. describe two softwares for the robustness analysis and visual interactive modelling – the triangle plot and the principal components analysis plot. The first reveals three-dimensional stability regions of the weight space, while the latter reduces dimensionality. Both plots have been evaluated from the perspective of end users. The triangle plot is found to be intuitive and easy to use. It exposes robustness and serves as an analytical device with which users can quickly deduce whether the results are as expected. The principal components analysis plot, on the contrary, is rather a heuristic device that exposes comparisons and directs users to further investigations.

3.2 Variance based methods

It has been established that people have difficulties with interpreting and visualizing information in four or more dimensions. An approach that confronts this problem is the principal components analysis (Jolliffe, 2002), which has already been applied in many fields of science for the purpose of reducing dimensionality and providing a good insight into correlations between variables by preserving a high degree of variance in data. It is often possible to identify a few groups of variables that capture the same key principles, and are hence strongly correlated. Linear combinations of these original variables define a set of principal components forming the unique non-redundant orthogonal basis of a new space of data. Each component corresponds to an axis of the new space. It is selected in such a way that its variance is the highest of all possible choices for this axis. The set of principal components has equal power to the set of original variables, however the sum of variances for only the first two or three principal components generally exceeds 80 percent of variance in original data. For this reason, it is sufficient to consider a small subset of principal components in order to preserve the majority of information. Because of the most simple and understandable interpretation and visualization, the projection on a two-dimensional plane, which is defined by the 1st and the 2nd component, is usually performed.

The principal components analysis may be applied in combination with nearly all multi-criteria decision-aiding methods. Probably the first method that has used it under the name GAIA for almost two decades is PROMETHEE II (Brans and Mareschal, 1994). It takes criteria-wise net ranking flows as the basis for visualization:

$$\phi_k(a_i) = \frac{1}{n-1} \cdot \sum\nolimits_{a_j \in A} P_k(a_i, a_j) - P_k(a_j, a_i)\,.$$

Espinasse et al. (1997) have applied GAIA planes in a multi-agent negotiation framework. They have developed several levels of group planes, which represent decision-makers, coalitions, criteria and weights with the purpose of assisting the mediator during the negotiation process. Radojević and Petrović (1997) have used GAIA within the scope of fuzzy multi-criteria ranking. They have thus extended the applicability of PROMETHEE methods to the cases when criteria values are fuzzy variables.

Saltelli (2001) has studied the properties of variance based methods in the context of importance assessment. He has considered two settings. In the first, the objective has been to identify the most important factor that would lead to the greatest reduction of variance. In the second, the required target variance has been obtained by fixing simultaneously the smallest possible number of factors.

3.3 Integration in decision support systems

In order to make the process of preference assessment interactive, Mustajoki et al. (2005) have developed and described the WINPRE software, which seeks for three-dimensional stability regions in the weight space, ranges of allowed imprecise weights and partial utility intervals. Another decision support system that visualizes utilities of alternatives in the context of group decision-making is RINGS (Kim and Choi, 2001). By observing overlapping of utility ranges for individual decision-makers and the whole group, consensus can be reached. Moreno-Jimenez et al. (2005) have implemented a spreadsheet module for consensus building, which is able to visualize preference structures with radial graphic repesentation maps. Each structure is mapped to a planar polygon whose vertices are placed at the end of rays cast from a central point.

Bana e Costa et al. (1999) have integrated several decision support systems which implement visualization and sensitivity analysis techniques. EQUITY provides graphical cost-benefit efficiency analysis, MACBETH depicts value functions, while V.I.S.A. visualizes partial utilities of alternatives and computes stability intervals. Siskos et al. (1999) have embedded visual components into the MIIDAS system. The decision-maker can shape the value function in terms of its curveness and turning point, graphically perform trade-offs, observe the ordinal regression curve and view the net graph coming from the cluster analysis. Jimenez et al. (2003) have introduced a system that allows for imprecise assignments of weights and utilities, whereby inputs can be subjected to different sensitivity analyses and visualization aids, including:

- pie charts of certainties and probabilities;
- bar charts of weights and utilities;
- graphical representations of utility functions;
- stability intervals of weights;
- several types of simulation techniques designed to randomly modify weights by preserving their rank order or numerical intervals.

4 Stability intervals and regions

4.1 Stability intervals

The inference of stability intervals represents the most basic form of sensitivity analysis, next to the “what-if” analysis which is, in connection with interactive graphic tools, used primarily in the phases of criteria structuring and preference elicitation. It is implemented by many decision support systems that help companies and large corporations make important organizational and business decisions (Forman and Selly, 2001). The purpose of this technique is to determine for what intervals of values of a single parameter (for example, a criterion weight), the rank-order of alternatives is preserved. Its main strength is the ability to identify boundaries of stability intervals automatically, without any manual intervention. It is thus appropriate for robustness checking after the preference aggregation phase completes.

To determine the influence of the criterion $x_i \in X$ on the rank-order of alternatives, its weight w_i continuously increases on the interval from 0 to 1. The weights of all other criteria $x_j \in X \setminus \{x_i\}$ decrease inversely proportioned according to their relative portions d_j that exclude w_i:

$$d_j = \frac{w_j}{s_i}, \text{ where } s_i = \sum_{\substack{k=1..n \\ k \neq i}} w_k \ .$$

If the normalization of weights is required, such that their sum equals to 1, it becomes clear that the weight of the x_j criterion decreases by $\Delta w_j = d_j \cdot \Delta w_i$ when the weight of the observed criterion x_i is increased by Δw_i. Thereby, the theoretical foundations for the graphical represenation of stability intervals are laid. Complementary, the analytical computation of all possible weights w_i at which the rank-order changes is also useful. The utilities of alternatives must be compared in this case for all pairs of a_k and a_l, so that $k, l = 1, \ldots, m$ and $k \neq l$. This requires $(m \cdot (m-1))/2$ pairwise comparisons. Since the weighted additive utility function is applied, the point of indifference between two alternatives can be expressed with a linear equation:

$$w_i \cdot u_i(a_k) + \sum_{j \neq i} d_j \cdot (1 - w_i) \cdot u_j(a_k) =$$
$$w_i \cdot u_i(a_l) + \sum_{j \neq i} d_j \cdot (1 - w_i) \cdot u_j(a_l).$$

The weight w_i is easily derived:

$$\frac{w_i}{1 - w_i} = \frac{\sum_{j \neq i} d_j \cdot (u_j(a_l) - u_j(a_k))}{u_i(a_k) - u_i(a_l)}.$$

Analogously, one-dimensional stability intervals can be found for the PROMETHEE II method, which is based on additive aggregation as well:

$$\frac{w_i}{1 - w_i} = \frac{\sum_{b \in A} \sum_{j \neq i} d_j \cdot (P_j(a_l, b) - P_j(b, a_l) - P_j(a_k, b) + P_j(b, a_k))}{\sum_{b \in A} P_i(a_k, b) - P_i(b, a_k) - P_i(a_l, b) - P_i(b, a_l)}.$$

4.2 Two-dimensional stability regions

It is possible to generalize the stability regions analysis to two or more dimensions. This subsection discusses the interaction of two criteria weights because otherwise the reduction of dimensionality or (non)linear programming must be performed. The latter approach is addressed in the next section. The first is realized by the principal components analysis and is applied by the visual GAIA analysis (Brans and Mareschal, 1994), which projects the multi-dimensional criteria space on a plane, and thereby loses some preferential information.

The two-dimensional sensitivity analysis considers each pair of weights that belong to criteria of the same hierarchical group (let these be the w_i and w_j weights). For a pair of alternatives a_k and a_l, it is determined for which values of w_i and w_j the indifference relation holds. In general, a single point (meaning that alternatives are equivalent for unique weights w_i and w_j), a straight line (implying indifference for an infinite space of weights), or an empty set (meaning that one alternative is preferred to the other for all values of w_i and w_j) is obtained. Lines and points delimit regions within which the rank-order of alternatives remains constant. The stability regions are additionally delimited with borderlines $w_i = 0$, $w_j = 0$ and $w_i + w_j = 1$. It is clear that the new model has one degree of freedom more than the model of stability intervals:

$$w_i \cdot u_i(a_k) + w_j \cdot u_j(a_k) + (1 - w_i - w_j) \cdot \bar{u}(a_k) =$$
$$w_i \cdot u_i(a_l) + w_j \cdot u_j(a_l) + (1 - w_i - w_j) \cdot \bar{u}(a_l),$$

where $\bar{u}(a_k)$ respectively $\bar{u}(a_l)$ is a constant utility of $n-2$ criteria that do not change during analysis:

$$\bar{u}(a_k) = \sum_{h \neq i,j} \frac{w_h}{W} \cdot u_h(a_k), \text{ where } W = \sum_{h \neq i,j} w_h \ .$$

The correlation between weights is now obtained:

$$w_i = \frac{\bar{u}(a_l) - \bar{u}(a_k) - w_j \cdot (u_j(a_k) - u_j(a_l) - \bar{u}(a_k) + \bar{u}(a_l))}{u_i(a_k) - u_i(a_l) - \bar{u}(a_k) + \bar{u}(a_l)}.$$

By setting $w_j = 0$ and $w_j = 1 - w_i$ it can be seen when two alternatives a_k and a_l become equivalent. Analogous two-dimensional sensitivity analysis has been implemented for the PROMETHEE II method as a functionality of the PROMCALC decision support system.

5 Multi-dimensional robustness analysis

Mathematical programming can be applied to judge the influence of arbitrary many simultaneously changing parameters. The motivation for its use lies in the fact that multi-dimensional information is totally preserved, while in the case of visualisation it gets partially lost because of the projection on a plane. For this reason, several original robustness metrics are proposed. They are implemented with optimization algorithms.

5.1 Optimization approaches for the multi-attribute utility function

The goal of the approaches is to test how robust the rank-order of alternatives is with regard to the weights of all criteria that are structured into a common hierarchical group. Thereby, a comprehensive insight into the model and its robustness must be assured with as few metrics as possible. Four mathematical optimization programs are hence defined. The first exposes the minimal change of the weight vector that causes the best ranked alternative to lose its priority over any other, originally less optimal solution, which means that the best ranked alternative changes. This measurement is of essential importance, since a rational decision is to choose an alternative with the highest utility/value. The robustness of such a choice is obtained with the following program:

$$\Delta_w = \min \frac{\left[\sum_{j=1..n} \left(\left|w_j - \widetilde{w}_j\right|\right)^P\right]^{1/P}}{\Delta_w^{\max}}$$

subject to

$$u(a_k) = \sum_{j=1..n} w_j \cdot u_j(a_k) \le$$
$$\le \max_{l \ne k} \left(u(a_l) = \sum_{j=1..n} w_j \cdot u_j(a_l) \right),$$
$$\sum_{j=1..n} w_j = 1,$$
$$0 \le w_j \le 1,\ \forall j = 1,\ldots,n,$$

where $\widetilde{w}_j$ are current and w_j newly derived weights, and where it holds:

$$\widetilde{u}(a_k) = \sum_{j=1..n} \widetilde{w}_j \cdot u_j(a_k) =$$
$$= \max_{l=1..m} \left(\widetilde{u}(a_l) = \sum_{j=1..n} \widetilde{w}_j \cdot u_j(a_l) \right).$$

The parameter P, $1 \le P \le \infty$, determines which one of the L_P distance metrics is used. Usually, the Manhattan norm (L_1), which returns the rectangular distance between two vectors, or the Euclidean norm (L_2), which takes the hypotenuse of a square triangle as the distance, are used because of the simplest interpretation. The distance has to be normalized by division with the largest possible change of the weight vector $\Delta_w^{\max}$. For the case when all criteria weights are allowed to have any value from the [0, 1] interval ($\forall j: dw_j = uw_j - lw_j = 1$), the vector changes maximally when exactly two of its components move from one extreme to the other:

$$w_i = 1,\ \forall k \ne i: w_k = 0 \rightarrow$$
$$\rightarrow w_j = 1,\ i \ne j,\ \forall k \ne j: w_k = 0.$$

In this special situation, $\Delta_w^{\max}$ equals to 2. However, for arbitrary differences dw_j, such that $\neg\forall j: dw_j = 1$ holds, the following mathematical program is solved:

$$\Delta_w^{\max} = \max\left[\sum_{j=1..n} \left(\left|w_j^E - w_j^S\right|\right)^P\right]^{1/P}$$

by deriving

$$w_j^S,\ w_j^E,\ \forall j = 1,\ldots,n$$

subject to

$$\sum_{j=1..n} w_j^S = 1,\ \sum_{j=1..n} w_j^E = 1,\ \forall j = 1,\ldots,n,$$
$$lw_j \le w_j^S \le uw_j,\ lw_j \le w_j^E \le uw_j,\ \forall j = 1,\ldots,n.$$

S and E denote the starting respectively ending weights, and also the initial respectively final utilities in the next two programs. To find the largest allowed deviation of the weight vector, such that the preferential relation is preserved for a pair of selected alternatives a_1 and a_2, the below optimization problem must be dealt with:

$$\text{maximize } \Delta_l\left(w^S, w^E\right)$$

subject to

$$u^S(a_1) = \sum_{j=1..n} w_j^S \cdot u_j^S(a_1) \ne$$
$$\ne u^S(a_2) = \sum_{j=1..n} w_j^S \cdot u_j^S(a_2),$$
$$u^E(a_1) = \sum_{j=1..n} w_j^E \cdot u_j^E(a_1) =$$
$$= u^E(a_2) = \sum_{j=1..n} w_j^E \cdot u_j^E(a_2),$$
$$\sum_{j=1..n} w_j^S = \sum_{j=1..n} w_j^E = 1,$$
$$w_j^S \in [0,1],\ w_j^E \in [0,1],\ \forall j = 1,\ldots,n.$$

The last addressed problem is to find the smallest change of the weight vector for which any initially suboptimal alternative becomes the best ranked one. As it is similar to the previous optimization problem, the mathematical program is slightly modified:

$$\text{minimize } \Delta_l\left(w^S, w^E\right)$$

subject to

$$u^S(a_1) < u^S(a_i),\ \exists i = 2,\ldots,m,$$
$$u^E(a_1) > u^E(a_i),\ \forall i = 2,\ldots,m,$$
$$u^S(a_i) = \sum_{j=1..n} w_j^S \cdot u_j^S(a_i),\ \forall i = 1,\ldots,m,$$
$$u^E(a_i) = \sum_{j=1..n} w_j^E \cdot u_j^E(a_i),\ \forall i = 1,\ldots,m,$$
$$\sum_{j=1..n} w_j^S = \sum_{j=1..n} w_j^E = 1,$$
$$w_j^S \in [0,1],\ w_j^E \in [0,1],\ \forall j = 1,\ldots,n.$$

It is presupposed that the alternative selected to become optimal for the final inferred distribution of weights is denoted with a_1, and that there exists at least one initially superior alternative.

5.2 Optimization approaches for the ELECTRE TRI method

Three types of distance metrics are defined. They reflect the minimum deviations of weight, veto and preference vectors that cause the reassignment of an alternative to the other category. When, considering the alternative a_i, any of these measures is low, the membership of a_i is not sufficiently robust because only a slight modification of preferences may result in a different decision. The most simple task is to find the smallest change of the weight vector so that the reassignment of a_i to the other class occurs: $a_i \in C^+ \to a_i \in \widetilde{C}^-$ or $a_i \in C^- \to a_i \in \widetilde{C}^+$. The problem is solved with a linear optimization program, for which all used symbols have already been defined:

$$\Delta_w(a_i) = \min \frac{\left[\sum_{j=1..n} \left(\left|w_j - \widetilde{w}_j\right|\right)^P\right]^{1/P}}{\Delta_w^{\max}}$$

by deriving

$$w_j,\ \forall j = 1,\ldots,n$$

subject to

$$\sigma(a_i) = d(a_i) \cdot \left(\sum_{j=1..n} w_j \cdot c_j(a_i)\right) = \lambda,$$

$$\sum_{j=1..n} w_j = 1,\ lw_j \le w_j \le uw_j,\ \forall j = 1,\ldots,n.$$

A harder problem is to measure the robustness of veto and discordance thresholds v_j and u_j. An advanced metric is needed that allows for the aggregation of discordance indices, and indicates the minimal threshold deviations that would cause the observed alternative to reassign:

$$\Delta_v(a_i) = \min\left[\frac{\sum_{j=1..n} \left(\delta_j\right)^P}{\sum_{j=1..n} \left(2 \cdot \left(g_j(b) - p_j - D_j^-\right)\right)^P}\right]^{1/P}$$

by deriving

$$u_j \text{ and } v_j,\ \forall j = 1,\ldots,n$$

subject to

$$\sigma(a_i) = c(a_i) \cdot \prod_{j=1..n} \left(1 - d_j(a_i)\right) = \lambda,$$

$$d_j(a_i) = \max\left(\min\left(\frac{g_j(b) - g_j(a_i) - u_j}{v_j - u_j}, 1\right), 0\right),$$

$$\delta_j = \left|u_j - \widetilde{u}_j\right| + \left|v_j - \widetilde{v}_j\right| + \left|\left(v_j - u_j\right) - \left(\widetilde{v}_j - \widetilde{u}_j\right)\right|,\ \forall j = 1,\ldots,n,$$

$$p_j \le u_j \le v_j \le b_j,\ \forall j = 1,\ldots,n.$$

The program minimizes the distances between previous and new values of discordance and veto thresholds. In addition, it pays regard to the distances between different thresholds ($|v_j - u_j|$), to prevent anomalies that can occur if thresholds converge towards the same value. It clearly demonstrates the problematic of finding the smallest change of u_j and v_j thresholds that causes reclassification. Yet, it has to deal with piecewise linear functions with unknown segments. For this reason, it is substituted with a different optimization program. For each value $g_j(a_i)$, an appropriate partial discordance degree is found so that the product of these degrees equals the required overall discordance $\widetilde{d}(a_i)$ calculated by dividing the fixed cut level λ with the fixed concordance index $c(a_i)$. Then, the criterion-wise coefficient k_j of a linear function is derived according to $g_j(a_i)$ (x-axis) and $\widetilde{d}_j(a_i)$ (y-axis), for each index $j = 1, \ldots, n$. The induced function determines the u_j and v_j thresholds (at $y = 0$ and $y = 1$), and minimizes the distance metric:

$$\Delta_v(a_i) = \min\left[\frac{\sum_{j\in F} \left(\delta_j\right)^P}{\sum_{j\in E} \left(2 \cdot \left(b_j - p_j - D_j^-\right)\right)^P}\right]^{1/P}$$

by deriving

$$\widetilde{d}_j(a_i) \text{ and } k_j,\ \forall j \in F$$

subject to

$$E = \{1,\ldots,n\},\ F \subseteq E,$$

$$\prod_{j\in F} \left(1 - \widetilde{d}_j(a_i)\right) \cdot \prod_{j\in E\setminus F} \left(1 - d_j(a_i)\right) = \widetilde{d}(a_i),$$

$$0 \le \widetilde{d}_j(a_i) \le 1,\ \forall j \in F,$$

$$\delta_j = \delta_j^u + \delta_j^v + \delta_j^{uv},\ \forall j \in F,$$

$$\delta_j^u = u_j - g_j(a_i) + \frac{\widetilde{d}_j(a_i)}{k_j},\ \forall j \in F,$$

$$\delta_j^v = v_j - g_j(a_i) - \frac{1 - \widetilde{d}_j(a_i)}{k_j},\ \forall j \in F,$$

$$\delta_j^{uv} = v_j - u_j - \frac{1}{k_j},\ \forall j \in F,$$

$$\frac{1 - \widetilde{d}_j(a_i)}{D_j^+ - D_j^- - g_j(a_i)} \le k_j \le \frac{\widetilde{d}_j(a_i)}{g_j(a_i) - p_j},\ \forall j \in F.$$

Figure 1 gives the graphical interpretation on how the new u_j and v_j thresholds are inferred by inducing the k_j coefficient. The thresholds may be modified either with a parallel shift of the function or by changing its slope with the increase/decrease of the k_j coefficient. Consequently, their absolute difference or the initial value of u_j must be preserved. The third possibility also exists: by combining the shift and the angle adjustment, all differences Δu, Δv and Δuv become positive.

On Figure 1, k_0 and k_1 depict the initial respectively the extreme possible induced angle of the linear function. Similarly, y_0 denotes the initial partial discordance degree and y_1 represents the required adjusted degree. Finally, x_0 corresponds to the criterion-wise value of the alternative $g_j(a_i)$. If a_i is the member of the positive category C^+, the discordance degree must increase in order to cause the reassignment, which is a prerequisite to properly measure robustness. Then, $y_1 > y_0$; otherwise $y_1 < y_0$.

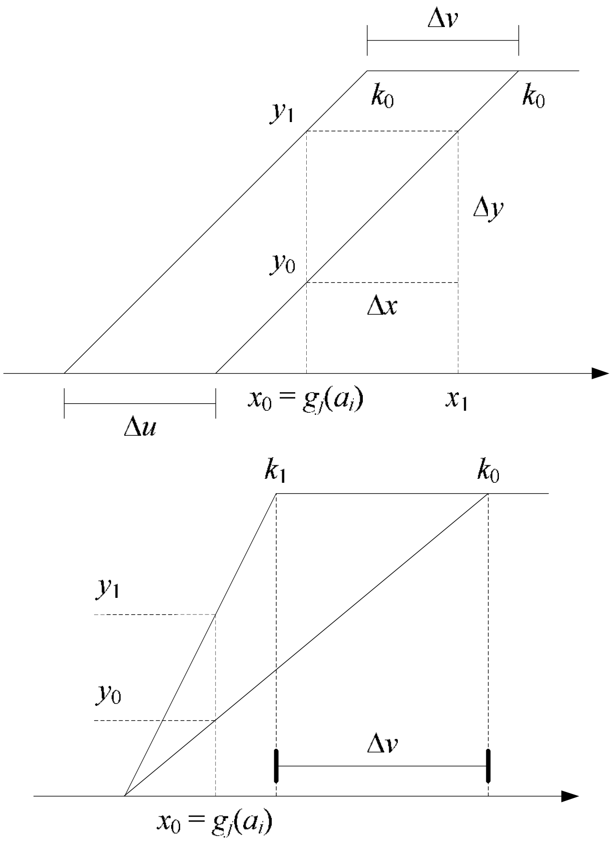

Figure 1: Inference of discordance and veto thresholds with the parallel shift and with the slope adjustment.

The problem of finding the deviations of indifference and preference thresholds that would cause the classification of an alternative into a different category is very similar to the one described above. The optimization is slightly more demanding because it has to deal with symmetry of partial concordance indices. This difficulty is overcome by multiplying each newly derived index with a sign that is determined by comparing the $g_j(a_i)$ and $g_j(b)$ values.

6 Practical examples

All examples described in this Section are based on the utility theory. Partial utility functions are not presented as it is not necessary to be acquainted with them in order to comprehend the discussed use of robustness techniques. Partial utilities are aggregated with the weighted additive decomposition rule, which is defined in Subsection 2.1. Methodological details on the optimization programs and on the computation of stability regions are omitted, since they are thoroughly introduced in Sections 4 and 5. In their original forms, all decision models are extensive. Hence, a subset of the most relevant criteria is treated for the demonstrative purposes. Similarly, the application of robustness algorithms for the ELECTRE TRI method requires a complex example that exceeds the scope of the paper. It can be found in the literature (Bregar, 2009).

Figure 2 shows two examples of stability regions. In the first case, the decision is robust because a substantial modification of the observed weights w_1 and w_2 is needed for the alternative a_1 to gain a higher utility than the best ranked alternative a_2. On the contrary, the decision is not robust in the second example. A small change of current weights suffices for a_1 to be selected as the best available option instead of a_3. In this way, a thorough insight into the decision model is provided in addition to the derived rank-order and assessments of alternatives. The examples are based on the analysis which has been performed for the purpose of toll systems evaluation (Jurič et al., 2005). Since project data are not public, alternatives and criteria are not explicitly named.

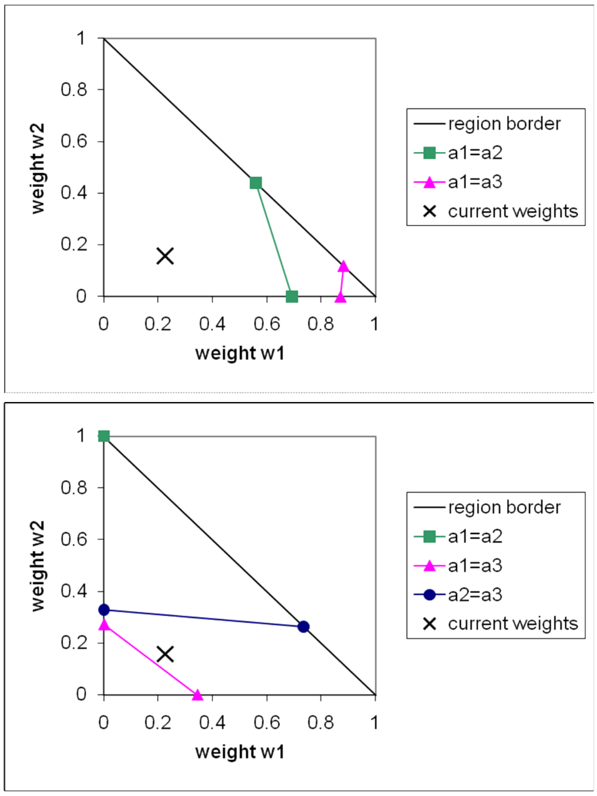

Figure 2: Examples of stability regions.

In order to measure robustness with regard to arbitrary many criteria, mathematical programming has been used for the purpose of above described evaluation, as well as to select the best service-oriented architecture. Because this paper focuses on the formal definition of several new and several established decision analysis techniques, and not on the assessment of service-oriented architectures, any prioritization of the latter is avoided. Hence, the evaluated BEA WebLogic/AquaLogic, IBM SOA, JBoss, Microsoft SOA and Oracle SOA Suite architectures are simply denoted with symbols a_1 to a_5, so that the order is randomly mixed. Although over 100 criteria have been specified, only five are considered here:

- x_1 – service-oriented architecture (global goal),
- x_2 – functionality,
- x_3 – support for business rules,
- x_4 – administrative tools,
- x_5 – business intelligence.

In this example, the criteria x_1 to x_5 are not dealt with in a hierarchically structured manner, yet in practice, x_2 is a subcriterion/descendant of x_1 and x_3 to x_5 are descendants of x_2. To clearly demonstrate the strengths and benefits of the proposed class of robustness analysis techniques, a mathematical optimization program is applied to solve the problem of finding the minimal required modification of the weight vector, such that the best ranked alternative changes. This is the first program from Subsection 5.1. It is operationalized to measure the Euclidean distance and to allow all weights to be between 0 and 1. The obtained results are organized in Table 1.

Table 1: Utilities of alternatives and robustness degrees.

Alternative	x_1	x_2	x_3	x_4	x_5
a_1	0.85	0.89	0.55	0.79	0.82
a_2	0.65	0.72	0.60	0.72	0.89
a_3	0.63	0.69	0.30	0.73	0.55
a_4	0.55	0.78	0.42	0.69	0.78
a_5	0.55	0.42	0.61	0.38	0.25
Robustness	0.40	0.62	1.00	0.91	0.15

For each alternative, its criteria-wise utilities are written. The last line contains the measured robustness degrees, which represent the distance between the original and the derived weight vector. The minimal possible robustness degree is 0, while the maximal is 1. It can be observed that these degrees provide far richer information than the computed utilities:

- According to criteria x_2 and x_4, there is almost no difference between the best and the second best alternative. The increase in utility is 0.11 and 0.06, respectively, on the scale from 0 to 1. This does not suffice for the decision-maker to be confident in the proposed decision. However, the degree of robustness is very high (0.62 and 0.91), which means that preferences are firmly stated. Consequently, the reliability of the model drastically improves.
- According to the third criterion, the best and the second best alternative are almost indifferent, as their utilities are 0.61 and 0.60, respectively. It is hence virtually impossible for the decision-maker to rationally choose between them solely on the basis of utilities. However, the robustness index has the highest value of 1, which means that no combination of weights can be found to change the preferential relation a_5 P a_2. In this way, it becomes obvious that a_5 represents the only reasonable solution.
- With regard to x_5, the robustness degree gives a conformation to the fact that the decision-maker should be extremely cautious when choosing a_2 over a_1 or a_4. This should be a clear sign for him to properly revise the decision model.

In the cases when both the difference in utilities of two best ranked alternatives and the degree of robustness are moderate, the proposed technique may be useful as well. Table 2 shows how the weights of subcriteria should be adjusted in order to change the best ranked alternative with respect to the criterion x_1. The weight of the costs subcriterion increases to such an extent (from 0.28 to 0.60) that the derived value is unacceptable.

Table 2: Required adjustments of the weight vector.

Criteria	*Original weights*	*Derived weights*
Functionality	0.32	0.15
Impact on investments	0.40	0.25
Costs	0.28	0.60

Figure 3 depicts the results of the principal components analysis for the fictitious case of selecting an Eastern European country for cooperation on a multilateral ICT project. Criteria are shown as vectors and alternatives as points. It can be clearly seen which alternatives perform well with respect to which criteria. The GAIA analysis additionally includes the so called decision stick on the plane. It is obtained by projecting the weight vector onto the two-dimensional coordinate system, and points in the direction of the best possible alternative.

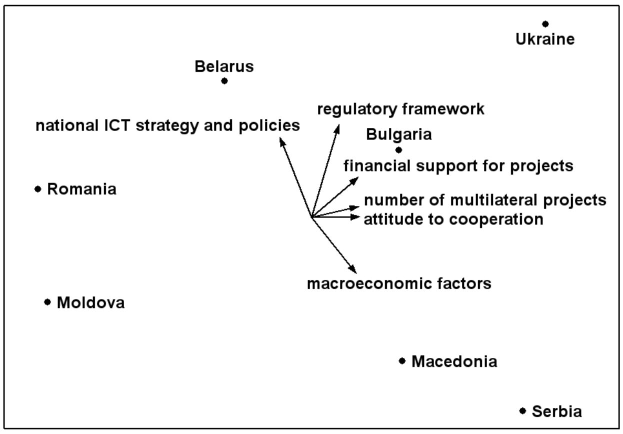

Figure 3: Visualization on the basis of principal components analysis.

Criteria and alternatives (countries) are adopted from the GREAT-IST questionnaire based survey (Györkös et al., 2006), however both the scope of the decision and the data are deliberately as well as significantly modified for the purpose of this example. Randomly generated data in the form of utilities are presented in Table 3. With x_1 to x_6, the following criteria are denoted:

- x_1 – number of multilateral projects,
- x_2 – attitude to international cooperation,
- x_3 – financial support for projects,
- x_4 – national ICT strategy and policies,
- x_5 – regulatory framework,
- x_6 – macroeconomic factors.

Table 3: Fictitious randomly sampled utilities.

Country	x_1	x_2	x_3	x_4	x_5	x_6
Belarus	0.4	0.3	0.6	0.8	0.7	0.3
Bulgaria	0.5	0.5	0.7	0.4	0.9	0.6
Macedonia	0.7	0.6	0.6	0.5	0.5	0.9
Moldova	0.2	0.1	0.2	0.7	0.4	0.2
Romania	0.2	0.1	0.3	1.0	0.4	0.2
Serbia	0.8	0.9	0.8	0.3	0.5	1.0
Ukraine	1.0	1.0	1.0	0.8	0.8	0.7

As is evident from Table 4, most preferential information are preserved on the two-dimensional plane. Nearly 90 percent of cumulative variance is covered by the first two principal components.

Table 4: Variance of principal components.

Principal component	*Percentage of variance*	*Cumulative variance*
1	70.65 %	70.65 %
2	17.65 %	88.30 %
3	10.64 %	98.94 %
4	0.74 %	99.68 %
5	0.27 %	99.95 %
6	0.06 %	100.00 %

7 Conclusion

Robustness analysis and visualization provide for several benefits. They:

1. help the decision-maker in achieving flexibility and adaptability to quickly changing conditions and characteristics of the observed situation or domain;
2. enable better understanding of the problem dealt with and the decision suggested/made;
3. icrease confidence in the decision model, which can be gained through the structured process of subjectively expressing preferential information.

Therefore, several techniques for measuring robustness and for visualizing multiple criteria decision models of various types have been defined. Most of them represent novel approaches to sensitivity analysis, while some are already established, but have been successfully applied on projects. Additional algorithms will be introduced in the scope of future research work, in order to determine:

- for what convex polyhedron of parameter values the observed alternative is selected as the best one, identified as the only acceptable choice, or classified/sorted into the appropriate category;
- for what convex intersections of polyhedrons available alternatives become indifferent or get classified/sorted into the same category.

References

[1] Araz, C., Ozkarahan, I. (2007). Supplier evaluation and management system for strategic sourcing based on new multicriteria sorting procedure. *International Journal of Production Economics*, vol. 106, no. 2, pp. 585–606.

[2] Bana e Costa, C. A., Ensslin, L., Correa, E. C., Vansnick, J.-C. (1999). Decision support systems in action: Integrated application in a multicriteria decision aid process. *European Journal of Operational Research*, vol. 113, no. 2, pp 315–335.

[3] Barron, H., Schmidt, C. P. (1988). Sensitivity analysis of additive multiattribute value models. *Operations Research*, vol. 36, no. 1, pp. 122–127.

[4] Brans, J., Mareschal, B. (1994). The PROMCALC & GAIA decision support system for multicriteria decision aid. *Decision Support Systems*, vol. 12, no. 4–5, pp. 297–310.

[5] Brans, J., Vincke, Ph. (1985). A preference ranking organisation method: The PROMETHEE method for multi-criteria decision-making. *Management Science*, vol. 31, no. 6, pp. 647–656.

[6] Bregar, A. (2009). *An Aggregation/Disaggregation Approach to Integrative Negotiations*. PhD thesis. University of Maribor, Faculty of Electrical Engineering and Computer Science. URLs: http://dkum.uni-mb.si/Dokument.php?id=7309 and http://slais.ijs.si/theses/2009-02-02-Bregar.pdf.

[7] Bregar, A., Györkös, J., Jurič, M. B. (2008). Interactive aggregation/disaggregation dichotomic sorting procedure for group decision analysis based on the threshold model. *Informatica (Vilnius)*, vol. 19, no. 2, pp. 161–190.

[8] Dias, L., Mousseau, V., Figueira, J., Climaco, J. (2002). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI. *European Journal of Operational Research*, vol. 138, no. 2, pp. 332–348.

[9] Doumpos, M., Zopounidis, C. (2004). Multicriteria classification approach based on pairwise comparisons. *European Journal of Operational Research*, vol. 158, no. 2, pp. 378–389.

[10] Espinasse, B., Picolet, G., Chouraqui, E. (1997). Negotiation support sytems: A multi-criteria and multi-agent approach. *European Journal of Operational Research*, vol. 103, no. 2, pp. 389–409.

[11] Figueira, J., Greco, S., Ehrgott, M. (2005). *Multiple Criteria Decision Analysis: State of the Art Surveys*. Springer.

[12] Forman, E. H., Selly, M. A. (2001). *Decision by Objectives*. World Scientific.

[13] Frey, H. C., Patil, S. M. (2002). Identification and review of sensitivity analysis methods. *Risk Analysis*, vol. 22, no. 3, pp. 553–578.

[14] Györkös, J., Bregar, A., Cizelj, B. (2006). *Report on GREAT-IST WG3: Questionnaire analysis*. University of Maribor, Faculty of Electrical Engineering and Computer Science.

[15] Hites, R., De Smet, Y., Risse, N., Salazar-Neumann, M., Vincke, Ph. (2006). About the applicability of MCDA to some robustness problems. *European Journal of Operational Research*, vol. 174, no. 1, pp. 322–332.

[16] Hodgkin, J., Belton, V., Koulouri, A. (2005). Supporting the intelligent MCDA user: A case study in multi-person multi-criteria decision support. *European Journal of Operational Research*, vol. 160, no. 1, pp. 172–189.

[17] Jansen, B., De Jong, J. J., Roos, C., Terlaky, T. (1997). Sensitivity analysis in linear programming: just be careful! *European Journal of Operational Research*, vol. 101, no. 1, pp. 15–28.

[18] Jimenez, A., Rios-Insua, S., Mateos, A. (2003). A decision support system for multiattribute utility evaluation based on imprecise assignments. *Decision Support Systems*, vol. 36, no. 1, pp. 65–79.

[19] Jolliffe, I. T. (2002). *Principal Component Analysis*. Springer.

[20] Jurič, M. B., Bregar, A., Rozman, I. (2005). *Evalvacija sistemov cestninjenja: Odločitveni model cestninjenja z vrednotenjem*. University of Maribor, Faculty of Electrical Engineering and Computer Science.

[21] Keeney, R. L., Raiffa, H. (1993). *Decisions with Multiple Objectives: Preferences and Value Trade-Offs*. Cambridge University Press.

[22] Kim, J. K., Choi, S. H. (2001). A utility range-based interactive group support system for multiattribute decision making. *Computers & Operations Research*, vol. 28, no. 5, pp. 485–503.

[23] Moreno-Jimenez, J. M., Joven, J. A., Pirla, A. R., Lanuza, A. T. (2005). A spreadsheet module for consistent consensus building in AHP-group decision making. *Group Decision and Negotiation*, vol. 14, no. 2, pp. 89–108.

[24] Mousseau, V., Figueira, J., Naux, J.-Ph. (2001). Using assignment examples to infer weights for ELECTRE TRI method: Some experimental results. *European Journal of Operational Research*, vol. 130, no. 2, pp. 263–275.

[25] Mousseau, V., Slowinski, R., Zielniewicz, P. (2000). A user-oriented implementation of the ELECTRE TRI method integrating preference elicitation support. *Computers & Operations Research*, vol. 27, no. 7, pp. 757–777.

[26] Mustajoki, J., Hämäläinen, R. P., Salo, A. (2005). Decision support by interval SMART/SWING – Incorporating imprecision in the SMART and SWING methods. *Decision Sciences*, vol. 36, no. 2, pp. 317–339.

[27] Power D. J. (2002). *Decision Support Systems: Concepts and Resources for Managers*. Greenwood Publishing.

[28] Radojević, D., Petrović, S. (1997). A fuzzy approach to preference structure in multicriteria ranking. *International Transactions in Operational Research*, vol. 4, no. 5–6, pp. 419–430.

[29] Ringuest, J. L. (1997). L_P-metric sensitivity analysis for single and multi-attribute decision analysis. *European Journal of Operational Research*, vol. 98, no. 3, pp. 563–570.

[30] Roy, B. (1991). The outranking approach and the foundation of ELECTRE methods. *Theory and Decision*, vol. 31, no. 1, pp. 49–73.

[31] Roy, B. (1996). *Multicriteria Methodology for Decision Aiding*. Kluwer Academic Publishers.

[32] Saltelli, A. (2001). Sensitivity analysis for importance assessment. *Proceedings of the 3rd International Symposium on Sensitivity Analysis of Model Output*, pp. 3–18.

[33] Saltelli, A., Tarantola, S., Chan, K. (1999). A role for sensitivity analysis in presenting the results from MCDA studies to decision makers. *Journal of Multi-Criteria Decision Analysis*, vol. 8, no. 3, pp. 139–145.

[34] Siskos, Y., Spyridakos, A., Yannacopoulos, D. (1999). Using artificial intelligence and visual techniques into preference disaggregation analysis: The MIIDAS system. *European Journal of Operational Research*, vol. 113, no. 2, pp. 281–299.

[35] Turban, E., Aronson, J. E. (2001). *Decision Support Systems and Intelligent Systems*. Prentice-Hall.

[36] Vincke, Ph. (1999). Robust and neutral methods for aggregating preferences into an outranking relation. *European Journal of Operational Research*, vol. 112, no. 2, pp. 405–412.

[37] Vincke, Ph. (1999). Robust solutions and methods in decision aid. *Journal of Multi-Criteria Decision Anaysis*, vol. 8, no. 3, pp. 181–187.

[38] Wolters, W. T. M., Mareschal, B. (1995). Novel types of sensitivity analysis for additive MCDM methods. *European Journal of Operational Research*, vol. 81, no. 2, pp. 281–290.

[39] Zeleny, M. (1982). *Multiple Criteria Decision Making*. McGraw-Hill.

[40] Zopounidis, C., Doumpos, M. (2002). Multicriteria classification and sorting methods: A literature review. *European Journal of Operational Research*, vol. 138, no. 2, pp. 229–246.