A Fast Convex Hull Algorithm for Binary Image

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Convex hull is widely used in computer graphic, image processing, CAD/CAM and pattern recognition. In this work, we derive some new convex hull properties and then propose a fast algorithm based on these new properties to extract convex hull of the object in binary image. It is achieved by computing the extreme points, dividing the binary image into several regions, scanning the regions existing vertices dynamically, calculating the monotone segments, and merging these calculated segments. Theoretical analyses show that the proposed algorithm has low complexities of time and space.

Povzetek: Predstavljen je nov algoritem za obdelavo binarnih slik.

1 Introduction

Convex hull is a central problem in various applications of computational geometry, such as Vornoi diagrams constructing, triangulation computing, etc. It is widely applied to computer graphic [1], image processing [2-3], CAD/CAM and pattern recognition [4-6]. Convex hull of a planar point set S is defined as the intersection of all the half-planes containing S. The shape of convex hull is a convex polygon whose vertices belong to S. For an edge pq, all other points lie on one side of the line running through p and q.

Many research efforts have been devoted to develop algorithms for 2-D convex hull computation. In 1970, Chand et al. [7] initially proposed a convex hull algorithm with $O(n^2)$ time by constructing the borders of convex hull according to the geometric properties of S. Another algorithm with $O(mn)$ time was give by Jarvis [8] ,where m is the number of convex hull vertices. Both of them have a high time complexity. Graham [9] provided a solution to compute the convex hull of a plane. Determine the point with minimal y-coordinate and calculate the angles between the horizontal line and the lines connecting the determined point and other points. According to the sorted angles, vertices are obtained. The divide-and-conquer method [10] was also applied to solve the problem. Point set was divided into two roughly equal-sized subsets. Their convex hulls were recursively computed, respectively. And the entire convex hull was determined by merging the two convex hulls. In another study, Chan [11] used point pairs to calculate the slopes of lines and determine the median values of these slopes, then divided the point set into two parts by median values and recursively computed the convex hull. He gave another algorithm which partitioned the point set and then computed the convex hull of each group, respectively. The entire convex hull was finally obtained by computing the union of the polygons. Exploiting the parallel computational model EREW PRAM, Chen et al. [12] proposed a parallel robust method for constructing convex hull. Brönnimann et al. [13] investigated the storage space of planar convex hull algorithms. As for dynamic planar convex hull, Overmars et al. [14] provided a solution that used $O(\log^2 n)$ time per update operation and maintained a leaf-linked balanced search tree of the vertices on the convex hull in clockwise order. Chan [15] gave a construction for the fully dynamic problem with $O(\log^{1+\varepsilon} n)$ amortized time for updates (for any constant $\varepsilon>0$), and $O(\log n)$ time for extreme point queries. In another work, Brodal and Jacob [16] presented a data structure that maintained a finite set of n points in the plane under insertion and deletion of points in amortized $O(\log n)$ time per operation. In [17], Ye considered the convex hull extraction in binary image and proposed a scheme with two procedures. In the first procedure, the image is scanned and a non-self-intersecting polygon is extracted; in the second procedure, the convex hull is

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extracted from the polygon through checking the convexity of the polygon.

In this work, we firstly investigate the convex hull properties and then derive some new properties, e.g., monotonicity. Finally, we use these new properties to design convex hull algorithm for binary image. The proposed algorithm extracts eight extreme points on the boundary of binary image, and then partitions the image into 5 regions by using the extreme points. During the vertex computation, only these points in 4 regions need to be processed. By orderly scanning, the temporary convex hull is extracted. The entire convex hull is finally obtained by continuously updating the temporary convex hull. As the scanned areas are few and only the vertices of temporary convex hull require storage, the proposed algorithm has low complexities of time and space.

The rest of the paper is organized as follows. In Section 2, new convex hull properties are derived. The convex hull algorithm for binary image and the complexity analysis are then described in Section 3 and Section 4, respectively. Conclusions are drawn in Section 5.

2 Convex hull properties

Convex hull is a convex polygon having the following properties. For an edge pq, all other points lie on one side of the line running through p and q. Any line segment connecting two arbitrary nonadjacent points is in the interior of the polygon. And the interior angle is less than 180 degrees, etc. In this section, we will investigate the convex hull structure and then derive new convex hull properties, which will be applied to improve the efficiency of convex hull algorithm.

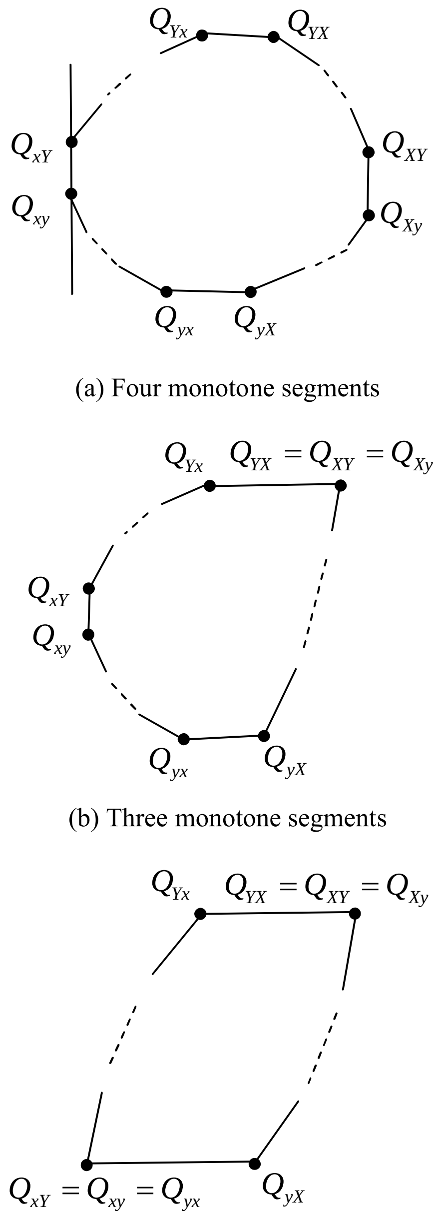
2.1 Extreme points

Let $Q = \{q_1, q_2, \ldots, q_M\}$ be a planar point set. In the subset whose points' x-coordinate are minimal among Q, Q_{xy} and Q_{xY} denote the points with minimal and maximal y-coordinate, respectively. In the subset whose points' x-coordinate are maximal among Q, Q_{Xy} and Q_{XY} represent the points with minimal and maximal y-coordinate, respectively. Likewise, in the subset whose points' y-coordinate are minimal among Q, Q_{yx} and Q_{yX} denote the points with minimal and maximal x-coordinate, respectively. In the subset whose points' y-coordinate are maximal among Q, Q_{Yx} and Q_{YX} represent the points with minimal and maximal x-coordinate, respectively. In the above variables, the first subscript denotes the extremum of coordinate and the second subscript denotes the extremum of the other coordinate under the first coordinate. Subscripts of capitalization and minuscule mean maximum and minimum, respectively, as shown in Fig.1. The definition of these points is given below.

Definition 1. In the planar point set Q, Q_{xy}, Q_{xY}, Q_{Xy}, Q_{XY}, Q_{yx}, Q_{yX} ,Q_{Yx} and Q_{YX} are the extreme points of the convex hull, where Q_{xy} and Q_{xY}, Q_{Xy},and Q_{XY}, Q_{yx} and Q_{yX}, Q_{Yx} and Q_{YX} are the homogeneous extreme points, respectively.

Theorem 1. The extreme points in the planar point set Q are the convex hull vertices.

Proof. Assume that points $q_1, q_2, \ldots, q_M$ are convex hull vertices, and make a line l parallel to y-axis through Q_{xy} and Q_{xY}. Suppose Q_{xy} and Q_{xY} are not the convex hull vertices, as shown in Fig.1 (a). According to their definition, points are all on l or on the right side of l. For any vertex q_i, if q_i is on l, it must locate between Q_{xy} and Q_{xY}. This means that it can't be a convex hull vertex. So these vertices are all on the right side of l. Thus Q_{xy} and Q_{xY} fall in the left side of the convex hull instead of its interior. This contradicts the convex hull definition. Therefore Q_{xy} and Q_{xY} are vertices. Similar proofs can be given to other extreme points.



(a) Four monotone segments

(b) Three monotone segments

(c) Two monotone segments

Figure 1: Extreme points and monotone segments of the convex hull

2.2 Convex hull monotonicity and its construction

For segment $Q_{xY}Q_{Yx}$ of convex hull, let its points be numbered in a clockwise order, namely q_m, q_{m+1}, …, q_n ($n > m$), where q_i's coordinate is (x_i, y_i), $q_m = Q_{xY}$ and $q_n = Q_{Yx}$. Then q_m, q_{m+1}, …, q_{n-1} should be on the same side of straight line $\overline{q_m q_n}$ and the x-coordinate and y-coordinate of q_{m+1} should both increase. Suppose that both the x-coordinates and y-coordinates of q_{m+1}, q_{m+2}, …, q_i monotone increase while those of q_{i+1} decrease (either or both of them decrease). Since q_m, q_{i+1}, q_n are on the same side of straight line $\overline{q_{i-1}q_i}$, thus q_{i+1} should lie beneath the line $y = y_i$, as shown in Fig.2 (a). Hence q_{i-1} and q_n are on the different sides of the line $\overline{q_i q_{i+1}}$. This contradicts the fact that q_m, q_{m+1}, …, q_n ($n > m$) are all the convex hull vertices. Therefore both the x-coordinates and y-coordinates of points on segment $Q_{xY}Q_{Yx}$ monotone increase. Similarly, the x-coordinates of points on segment $Q_{YX}Q_{XY}$ monotonically increase while the y-coordinates of them decrease. The x-coordinates of points on segment $Q_{yx}Q_{xy}$ monotonically decrease while the y-coordinates of them increase. Both two coordinates of points on segment $Q_{Xy}Q_{yX}$ monotonically decrease. The monotonicity of these segments is defined as follows.

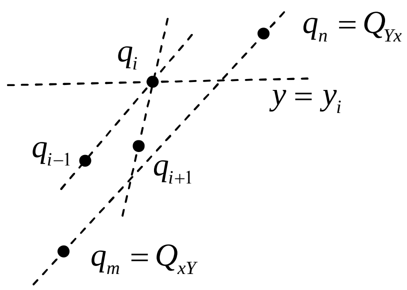


Figure 2: Convex hull monotonicity

Definition 2. If $Q_{xY} \neq Q_{Yx}$, the convex hull segment consisting of vertices from Q_{xY} to Q_{Yx} is called monotone increasing top segment. Likewise, if $Q_{YX} \neq Q_{XY}$, the convex hull segment consisting of vertices from Q_{YX} to Q_{XY} is named monotone decreasing top segment. If $Q_{Xy} \neq Q_{yX}$, the convex hull segment consisting of vertices from Q_{Xy} to Q_{yX} is called monotone decreasing bottom segment. If $Q_{yx} \neq Q_{xy}$, the convex hull segment consisting of vertices from Q_{yx} to Q_{xy} is named monotone increasing bottom segment.

Definition 3. All the monotone (both increasing and decreasing) top and bottom segments are called monotone segment.

If the monotone segments of a given convex hull are already determined, utilizing the definition 2 and the 8 extreme vertices can determine whether a specific monotone segment exists or not. The detailed theorem is as follow.

Theorem 2. The monotone increasing top segment exists if and only if $Q_{xY} \neq Q_{Yx}$. The monotone decreasing top segment exists if and only if $Q_{YX} \neq Q_{XY}$. The monotone increasing bottom segment exists if and only if $Q_{Xy} \neq Q_{yX}$. The monotone decreasing bottom segment exists if and only if $Q_{yx} \neq Q_{xy}$.

Similarly, according to the definition of convex hull monotonicity, the type of monotone segments can be determined by its vertices. Let f(P, A, B) = 0 represent the line equation, where the line runs through points A and B, P is a dynamic point on the line. There is a theorem about the type of monotone segments as follows.

Theorem 3. Let q_m, q_{m+1}, …, q_n ($n - m > 1$) be the vertices on a specific monotone segment of convex hull, and the coordinate of q_i be (x_i, y_i). For arbitrary i, j ($m \le i < j$, $m \le j < n$, $j \ne i$, $j \ne i + 1$), the sufficient and necessary conditions for that this monotone segment is a monotone increasing top segment are that $\begin{cases} \mathrm{f}(q_j, q_i, q_{i+1}) < 0 \\ y_i < y_{i+1} \end{cases}$. Likewise, as for the monotone decreasing top segment, the monotone decreasing bottom segment, monotone increasing bottom segment, their sufficient and necessary conditions are $\begin{cases} \mathrm{f}(q_j, q_i, q_{i+1}) < 0 \\ y_i > y_{i+1} \end{cases}$, $\begin{cases} \mathrm{f}(q_j, q_i, q_{i+1}) > 0 \\ y_i > y_{i+1} \end{cases}$, $\begin{cases} \mathrm{f}(q_j, q_i, q_{i+1}) > 0 \\ y_i < y_{i+1} \end{cases}$, respectively.

According to the convex hull definition, it has 4 monotone segments at most. Since convex hull is a closed shape, it has two monotone segments at least. The number of monotone segments can be determined by the extreme points. According to the number of monotone segments, convex hulls are classified into three types, as shown in Fig.1. Fig.1 (a) shows the convex hull with 4 monotone segments, Fig.1 (b) and Fig.1 (c) show the convex hull with 3 and 2 monotone segments, respectively.

3 Convex hull algorithm for binary image

3.1 Algorithm of monotone segment

Since the extreme points are convex hull vertices, the convex hull can be obtained by determining the vertices on the monotone segments between each pair of extreme points. In this work, a dynamic computation method is applied to determine the convex hull. Calculate the extreme points and determine the monotone segments. By dynamic scanning the boundary of image, the temporal convex hull of the scanned image is obtained. Scan the image boundary pixel by pixel until encountering the last boundary pixel. Thus, the monotone segments are obtained and the convex hull is extracted. The theorem of convex hull computation is as follows.

Theorem 4. Let $Q = \{q_m, q_{m+1}, \ldots, q_n\}$ ($n > m$) be the vertices of a monotone segment of a specific convex hull, the coordinate of q_i and p be (x_i, y_i) and (x, y), respectively, $Q' = \{p\} \cup Q$ and $\min\{y_{n-1}, y_n\} < y < \max\{y_{n-1}, y_n\}$. If p and q_k ($k < n$) are both the points in a specific monotone segment of Q', then $q_m, q_{m+1}, \ldots, q_k, p, q_n$ are all vertices on the monotone segment.

Proof: Suppose that $q_m, q_{m+1}, \ldots, q_n$ ($n > m$) are the vertices of monotone increasing top segment of a specific convex hull. Since $p(x, y)$ belonging to $\{p\} \cup Q$ is a point on the monotone increasing top segment and $y_{n-1} < y < y_n$, then $f(p, q_{n-1}, q_n) > 0$ according to theorem 3. The location of p is shown in Fig.4. If q_k ($k < n$) belonging to $\{p\} \cup Q$ is a vertex with maximum subscript on the monotone increasing top segment, then q_k and p are adjacent vertices on the new monotone increasing top segment. So for all q_i ($i \neq k$), $f(p, q_{n-1}, q_n) < 0$. If q_j ($j < k$) isn't the vertex of new convex hull, then $f(p, q_{j-1}, q_j) > 0$. But $f(q_k, q_{j-1}, q_j) < 0$. It means that q_k and p are on the different side of line $\overline{q_{j-1}q_j}$. So $f(q_k, p, q_j) < 0$. Therefore q_k isn't the vertex of new convex hull. This contradicts the precondition. Hence $q_m, q_{m+1}, \ldots, q_k, p, q_n$ are vertices on the monotone segment. Likewise, similar proofs of other monotone segments can be easily given.

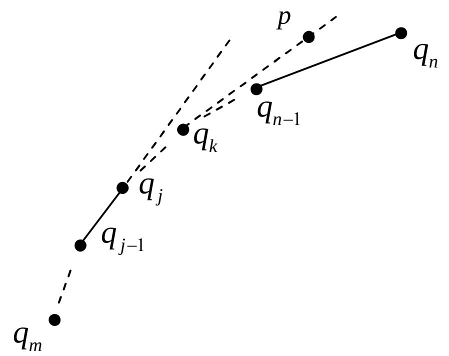


Figure 3: Monotone algorithm

Whether or not a pixel or a boundary point is a convex hull vertex depends on the relation about its position and the line. Take the monotone increasing top segment $q_m, q_{m+1}, \ldots, q_n$ ($n > m$) for example. Theorem 3 shows that $f(q_j, q_i, q_{i+1}) < 0$ for arbitrary i, j ($m \leq i < j$, $m \leq j < n$, $j \neq i$, $j \neq i + 1$). If a point p of image boundary satisfies $f(p, q_{n-1}, q_n) > 0$, then p is outside of the temporary convex hull. Thus p must be a new vertex of convex hull. Start from $k = n-1$ and decrease k by 1 each time. If $f(p, q_k, q_{k-1}) < 0$, then for arbitrary point A ($A \neq q_{k-1}$, $A \neq q_k$), $f(A, q_k, q_{k-1}) < 0$. According to theorem 3, q_k is a new vertex of convex hull. By applying theorem 4, all vertices on this segment can be obtained.

3.2 Convex hull algorithm for binary image

Convex hull of binary image can be determined by its boundary pixel set. In fact, the convex hull of boundary pixel set is equal to the convex hull of binary image. So obtaining the boundary is an important step. Generally, boundary extraction by scanning the whole image requires storing all pixels. However, only few pixels are the convex hull vertices. Reducing the number of scanned pixels can both improve the time and space efficiency of algorithm. In this section, the method scanning from outside to inner is applied to extract the extreme points, as shown in Fig.4. The scanned regions are determined by the extreme points. By dynamic scanning the image boundary, temporary convex hull of the scanned boundary pixel set is computed. Finally, convex hull of binary image is available.

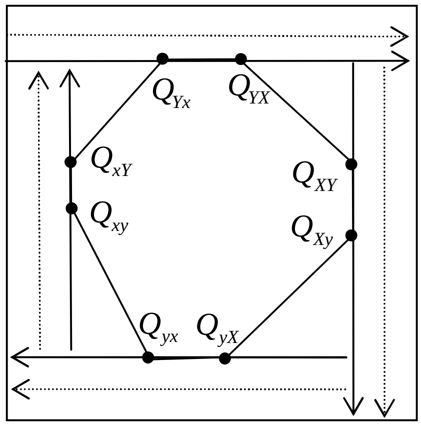


Figure 4: Extreme points of image convex hull

3.2.1 Collect the extreme points

In order to avoid repeated scanning, the method scanning from outside to inner is utilized to collect the extreme points on the image boundary. The detailed steps are as follows.

STEP 1: Begin at the top left of image and scan image from top to bottom until encountering the image boundary. Each row scan starts from left to right. If the scanned row has boundary points, Q_{Yx} and Q_{YX} represent the leftmost and rightmost boundary points, respectively. Thus two extreme points on the image boundary, Q_{Yx} and Q_{YX}, are obtained.

STEP 2: Begin at the line l_1 running through Q_{Yx} and Q_{YX} and scan image from right to left until encountering the image boundary. Each column scan starts from top to bottom. For the column having boundary pixels, let Q_{XY} and Q_{Xy} represent the topmost and bottommost boundary pixels, respectively. Thus two extreme points on the image boundary, Q_{XY} and Q_{Xy}, are obtained.

STEP 3: Begin at the line l_2 running through Q_{XY} and Q_{Xy} and scan image from bottom to top until encountering the image boundary. Each row scan starts from right to left. For the row having boundary pixels, let Q_{yX} and Q_{yx} represent the rightmost and leftmost boundary pixels, respectively. Thus two extreme points on the image boundary, Q_{yX} and Q_{yx}, are obtained.

STEP 4: Start from the line l_3 which is through Q_{yx} and Q_{yX} to the line l_1 and scan image from left to right until encountering the image boundary. Each column scan starts from bottom to top. For the column having boundary points, let Q_{xY} and Q_{xy} represent the topmost and bottommost boundary pixels, respectively. Thus two extreme points on the image boundary, Q_{xY} and Q_{xy}, are obtained.

By the above steps, 8 extreme points of image convex hull are extracted.

3.2.2 Determine the scanned regions

Theorem 1 shows that the 8 extreme points are convex hull vertices. The lines connecting adjacent extreme points divide image into several regions, as shown in Fig.5. There are no boundary pixels outside of the rectangle formed by l1, l2, l3 and l4. If the boundary pixels are on the edges of the rectangle, then they aren't convex hull vertices. In the interior of the rectangle, pixels in region 0 aren't vertices either. Only those in regions 1, 2, 3 and 4 are likely to be vertices. Hence we just need to scan region 1~4 to obtain the boundary pixels and compute the convex hull by applying theorem 4. Since the vertices are numbered in a clockwise order, pixels extracted by scanning the four regions should satisfy theorem 4. The detailed method is as follows.

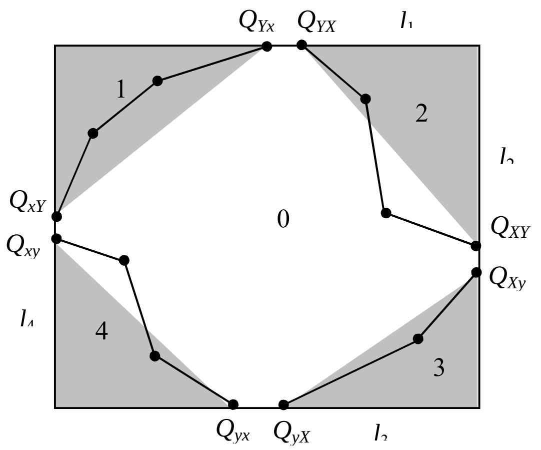


Figure 5: Scanned regions of image

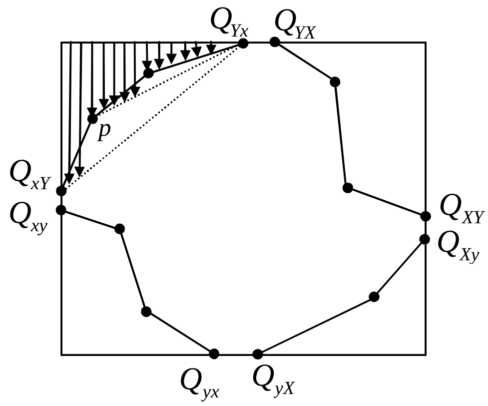
Region 1: Begin at the right side of l_4 and scan the region 1 horizontally from left to right. Each column in region 1 is scanned vertically from top to bottom. If there is no boundary pixel on the scanned line, then scan next column until encountering a boundary point p on the scanned line. Then p is a vertex of temporary convex hull in the scanned image. Compute the monotone increasing top segment of temporary convex hull by theorem 4. To improve the efficiency of algorithm and guarantee that the next scanned boundary pixels must be the vertices of temporary convex hull, the next column scan should stop once reaching the line pQ_{Yx}, as shown in Fig.6 (a). Continue to scan and compute the vertices of temporary convex hull until Q_{Yx} is encountered.

Region 2: Begin at the down side of l_1 and scan region 2 vertically from top to bottom. Each row in region 2 is scanned from right to left. Utilize the similar method introduced in region 1 to determine whether the boundary pixels are vertices or not. Continue to scan until Q_{XY} is encountered, as shown in Fig.6 (b).

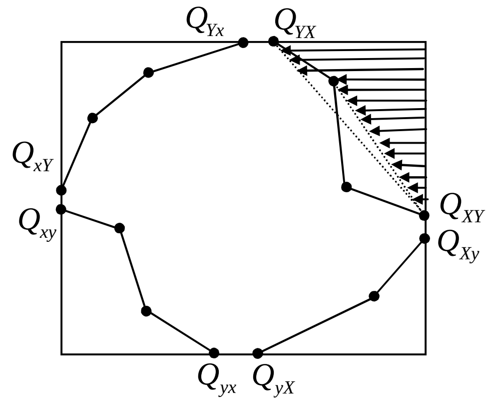
Region 3: Begin at the left side of l_2 and scan region 3 horizontally from right to left. Each column in region 3 is scanned from bottom to top. Utilize the similar method introduced in region 1 to determine whether the boundary pixels are vertices or not. Continue to scan until Q_{yX} is encountered, as shown in Fig.6 (c).

Region 4: Begin at the left side of l_3 and scan region 4 vertically from bottom to top. Each row in region 4 is scanned from left to right. Utilize the similar method introduced in region 1 to determine whether the boundary pixels are vertices or not. Continue to scan until Q_{yx} is encountered, as shown in Fig.6 (d).

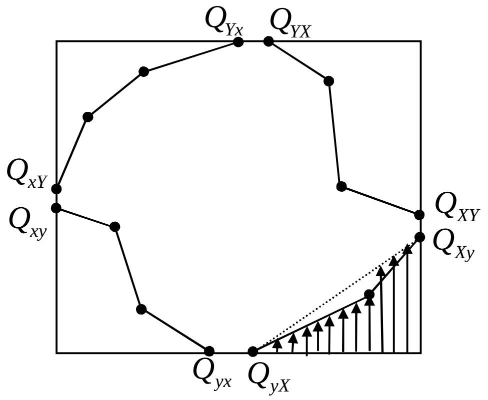
The extracted boundary pixels in the above steps both satisfy the monotone condition and the sequence required by theorem 4. Applying theorem 4 can extract the convex hull of image.



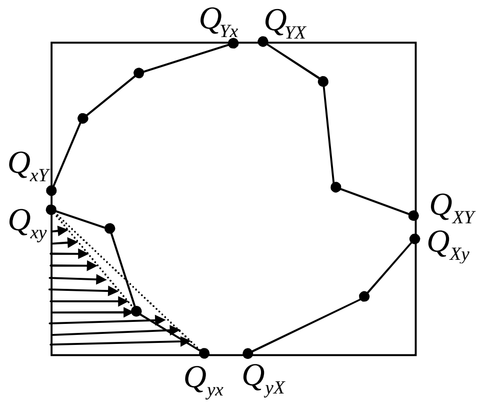
(a) Scan in region 1



(b) Scan in region 2



(c) Scan in region 3



(d) Scan in region 4

Figure 6: Scanned areas in each region

3.2.3 Compute convex hull vertices in scanned areas

Whether or not the pixel in scanned area is a convex hull vertex is just relative to other pixels in this region. By scanning the boundary pixels in each region, convex hull vertices in the corresponding region can be determined, respectively. Take region 1 for example. Let (x_{m1}, y_{m1}) and (x_{m2}, y_{m2}) be the coordinates of Q_{xY} and Q_{Yx}, respectively, f(p, A, B) = 0 be the equation of line running through A and B (p is a dynamic point), $v[i][j]$ and $c[i][j]$ be the pixel value and coordinate of p in the ith row and the jth column of the image, respectively. If $v[i][j] > 0$, p is a boundary pixel, or less a background pixel. Begin at $(x_{m1} + 1, y_{m1} + 1)$ and scan region 1 horizontally from left to right. In region 1, column is scanned from top to bottom. If there is no boundary pixel in the current column, scan next column in its right. If pixel p is a boundary pixel, then it must be a vertex of temporary convex hull. Apply theorem 4 to compute all vertices of temporary convex hull. Scan next column in the right. At this time, the scanned line is above the line pQ_{Yx}, as shown in Fig.6 (a). Stop scanning when the line $x = x_{m2}$ is encountered. Then the monotone segment of convex hull in region 1 is extracted. The detailed algorithm is as follows.

STEP 1: $i=x_{m1}+1$, $j = y_{m2}-1$, $q_1= Q_{xY}$, $A=Q_{xY}$, $n=2$;

STEP 2: IF ($i \geq x_{m2}$) goto **STEP 8**;
//no boundary pixel on the scanned line
IF (f(p, A, Q_{Yx}) ≤ 0) goto **STEP 3**;
//p is a vertex of temporary convex hull
IF ($v[i][j] > 0$) //p is the foreground pixel.
$k = n - 1$, $A = c[i][j]$, goto **STEP** 4;
ELSE goto **STEP** 5; //scan next pixel

STEP 3: $i = i + 1$, $j = y_{m2}-1$, goto **STEP 2**;
//scan next vertical line in the right

STEP 4: IF ($k > 1$) goto **STEP 6**;
ELSE $n = n + 1$， goto **STEP 2**;

STEP 5: $j = j + 1$, goto **STEP 2**;

STEP 6: IF (f(p,q_{k-1},q_k) ≥ 0) // backtrack again.
goto **STEP 7**;
ELSE // finish backtracking
$n = k+1$, $q_n = c[i][j]$, $n = n + 1$, goto **STEP 2**;

STEP 7: IF ($k>2$) //backtrack and process next pixel
$k = k - 1$, goto **STEP 6**;
ELSE //backtrack to the extreme point
$n = 2$, $q_n = c[i][j]$, $n = n + 1$, goto **STEP 2**;

STEP 8: $q_n= Q_{Yx}$,q_1, q_2,…,q_n are convex hull vertices.

3.2.4 Convex hull algorithm for binary image

For the convex hull of binary image, compute the 8 extreme points Q_{xy}, Q_{xY}, Q_{Xy}, Q_{XY}, Q_{yx}, Q_{yX}, Q_{Yx} and Q_{YX}. According to these extreme points, determine the scanned regions of image, as shown in Fig.5. Then, convex hull vertices locate the regions 1~4, which are divided by the lines connecting the adjacent extreme points, as shown in Fig.5. Therefore, only the boundary pixels in these regions require computation. Utilize the monotone properties of convex hull and scan each region dynamically. Then, apply theorem 4 to compute each monotone segment of convex hull. The entire convex hull is obtained by merging these monotone segments. The detailed algorithm is as follows.

STEP 1: Scan the binary image and compute the 8 extreme points, Q_{xy}, Q_{xY}, Q_{Xy}, Q_{XY}, Q_{yx}, Q_{yX}, Q_{Yx} and Q_{YX}.

STEP 2: Utilize the 8 extreme points to determine the four regions where the convex hull vertices may exist.

STEP 3: Scan each region dynamically and obtain convex hull vertices on each monotone segment respectively.

STEP 4: Extract convex hull vertices on each monotone segment according to the following order, $Q_{xY}\rightarrow Q_{Yx}$, $Q_{YX}\rightarrow Q_{XY}$, $Q_{Xy}\rightarrow Q_{yX}$, $Q_{yx}\rightarrow Q_{xy}$. Each extreme point is extracted only one time. Then convex hull is obtained.

4 Complexity analysis

4.1 Time complexity

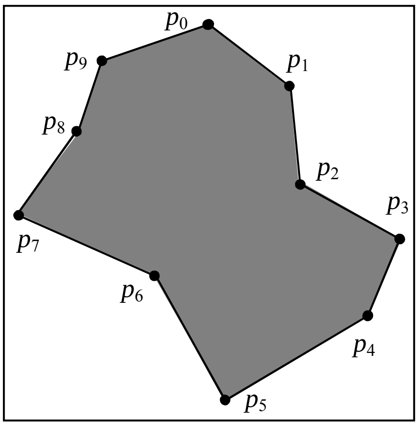
The time complexity is analyzed in the following ways. Suppose that the size of binary image is $N\times N$.

(1) If the image consists of a single pixel, then no convex hull exists. The time complexity is N^2.

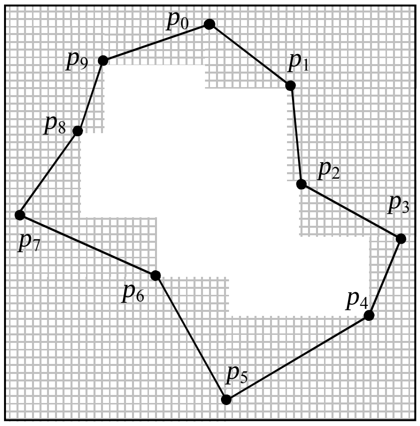
(2) If the image consists of two pixels or all pixels are on a line, then no convex hull exists, either. The time complexity is also N^2.

(3) The binary image has a convex hull if and only if three boundary pixels at least aren't on a line. Suppose that there are S pixels in the polygon whose vertices are the adjacent and inhomogeneous extreme points. The proposed method scans $N^2 - S$ pixels at most. And only $2N$ pixels at most should be computed when it determines whether or not a boundary pixel is a convex vertex. So the time complexity is $O(N^2 - S) + O(N)$.

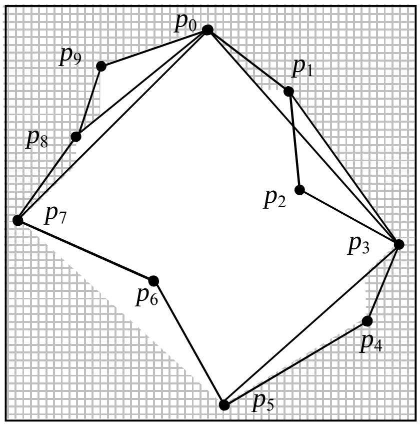
The above analyses show that the bigger the convex hull of binary image, the less the time complexity of the proposed algorithm. The time complexity of convex hull algorithm mainly depends on the size of scanned area. To show the efficiency of time complexity, we compare the proposed algorithm with the algorithm presented in [17]. A typical example is given in Fig.7. Fig.7 (a) is a binary image containing an object whose boundary has 10 vertices. Both algorithms are exploited to extract convex hull of the object in the binary image. Fig.7 (b) and Fig.7 (c) show the scanned areas of the algorithm [17] and the proposed algorithm respectively, where the gray grids denote their scanned areas. It is observed that our algorithm scans less area than the algorithm [17]. In general, if the object's boundary isn't a convex polygon, the scanned areas of the proposed algorithm are less than those of the algorithm [17]. Otherwise, the scanned areas of two algorithms are equivalent. Hence, the proposed algorithm needs less time than the algorithm [17] on average.



(a) Binary image containing an object with 10 vertices



(b) Scanned areas of the algorithm [17]



(c) Scanned areas of the proposed algorithm

Figure 7: A binary image and its scanned areas using different algorithms

4.2 Space complexity

The boundary pixels scanned by the proposed algorithm are the vertices of temporary convex hull. During the convex hull computation, only these vertices require storage. Therefore, the proposed algorithm has a low space complexity. Take Fig.7 for example. The algorithm [17] must store all 10 points from p_0 to p_9. Since p_2 and p_6 aren't scanned, the proposed algorithm doesn't need to compute and store them. So the space complexity of the proposed algorithm is lower than that of the algorithm [17].

5 Conclusions

In this paper, we derive some new convex hull properties, such as monotonicity, and use them to design algorithm for extracting convex hull of object in binary image. The proposed algorithm has a high efficiency by reducing computational cost in the following ways. (1) Divide the binary image into several regions by using the extreme points. Only those boundary pixels in a few regions require computation. (2) To determine a vertex in a given region doesn't need to compute those pixels in other regions. (3) Since the boundary pixels obtained by scanning are computed dynamically, only these vertices of temporary convex hull require storage. Theoretical analyses show that the proposed algorithm has lower complexities of time and space than the algorithm [17] on average.

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