Programming Models-Based Method for Deriving Profits Allocation Scheme with Interval-Valued Cooperative Games

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Owing to the fact that the payoff of grand coalition is limited to players' allocation, what is more, other players will not satisfy with the allocation scheme if one of the possible coalitions allocate too much. Based on this fact, it is better to pursuit the allocation scheme for all players in coalition are as close to the coalition's payoff as possible. In contrast, if the allocation scheme for all players in coalition is far from the coalition's payoff, this will lead to the instability of the coalition. To address this issue, several linear programming models are constructed for taking into account the players' compromise limit constraints. First, the profits allocation model with undominated nonnegative excess vector is extended to interval-valued fuzzy environments. Second, several linear programming models are constructed respectively considering without and with coalitions' compromise limit constraints. Third, an illustrative example in conjunction with comparative analyses is employed to demonstrate the validity and applicability of the proposed models. Finally, the relationship of the models is discussed between without and with coalitions' compromise limit constraints.

Povzetek: Razvitih je več modelov z metodami linearnega programiranja za določanje uspešnih skupin predvsem glede časovne opredelitve.

1 Introduction

Faced with information and globalization of integrative economy, the cooperation between supply chain enterprises becomes more and more important in pursuit of more profits. How to reasonable allocate the profits that obtain from cooperation is a key outstanding issue, which plays a decisive role between the supply chain stability and sustainable development. For this reason, plenty of researchers focusing on the study of profits allocation issues, and making a lot of research results [1-8]. Among these studies, several researchers tried to solve profits allocation issues with game theory [1, 2, 4]. For example, Maafa et al. [1] studied algorithms to compute the Shapley value for cooperative games based on a lattice. Béal et al. [9] studied nonlinear weighted Shapley value for cooperative games with transferable utility. Derks [10] studied the Shapley value of conjunctive-restricted games. Van den Brink et al. [11] introduced the Proper Shapley value and Shapley value, and shared rules for cooperative ventures. Bilbao et al. [6, 7] developed the Shapley value for games on matroids respectively in view of static and dynamic models. Martino developed the probabilistic values and studied the properties of face module, efficiency scenarios and simplicial complexes [12-15]. From these existing researches, it can be observed that most of them studied the cooperation profits with real numbers, that is to say, the profits obtained from the cooperation between supply chain enterprises are determined in priori. Such cooperative is called crisp cooperative games for short.

Due to various unpredictable factors, sometimes the cooperation profits cannot be determined in priori. In this case, it is unsuitable to express cooperation profits with real numbers. To address this issue, much research work used fuzzy numbers to estimate the cooperation profits and established the so-called fuzzy cooperative games [16-20]. Actually, there are two cases of cooperative games with fuzzy payoffs: one is all the players are full take part in cooperative, the coalitions are crisp but the payoffs are determined with fuzzy numbers [21-23]. For example, Monroy et al. [21]

discussed the set valued issues in crisp coalitions and fuzzy payoffs. Zou et al. [22] introduced the generalized Shapley function for cooperative games with fuzzy payoffs. Wu [24] studies the cores and dominance cores of cooperative games endowed with fuzzy payoffs. The other is some of the players are partial take part in cooperative, both the coalitions and the payoffs are uncertain [25-27]. Such as, Wu [25] discussed the cores and convexity of fuzzy games. Liu et al. [26] discussed the average monotonic fuzzy games [30]. Cooperation profits are expressed with triangular fuzzy numbers, that is, triangular fuzzy cooperative games [21]. Review from the cooperative fuzzy games, it can be easily found that there is increasing research on interval-valued cooperative games. Interval-valued cooperative games are the simplest and most natural type of uncertainty. Although sometimes it is difficult to determine the coalition payoffs with real numbers, the lower and upper bounds of the coalition payoffs are easily to be determined in priori. Therefore, it seems to be suitable to use interval-valued cooperative games to solve profits allocation issues in some game's environments.

In order to solve bankruptcy issues, Alparslan-Gök et al. [31] first extended the classical two-person cooperative games to two-person interval-valued cooperative games, and studied their related topics. Afterwards, Alparslan-Gök et al. [32] studied the interval Shapley values and discussed their properties. Mallozzi et al. [30] discussed the core-line and balanced-like conditions of cooperative games with coalition payoffs are represented with interval-valued. Alparslan-Gök et al. [33] discussed the interval-valued stable sets, the interval-valued core and dominance core. Han et al. [34] studied the interval core and interval Shapley values. Meng et al. [35] studied the interval Shapley with interval characteristic functions. Hong et al. [36] based on the proposed satisfactory degree introduced a nonlinear programming method for n-person interval-valued cooperative games. Li et al. [37, 38] developed a direct and effective method for solving a special subclass of interval-valued cooperative games. And Li et al. [39] discussed n-person interval-valued cooperative games with satisfactory degree constraints. Gallardo et al. [40] developed the Shapley value for cooperative games with fuzzy characteristic function.

Previous studies have significantly advanced the research on interval-valued cooperative games. Nevertheless, there are still some shortcomings that needs to be further discussed: (1) Most of the aforementioned works used the Moore's interval subtraction [41] or the partial subtraction operator [42], which may lead to irrational the computation results. This has been pointed out by Li in [43]. (2) In order to overcome the shortcoming of the used of Moore's interval subtraction and partial subtraction operator, Li [43] constructed several nonlinear programming models to obtain interval-valued least square solution. For different coalitions may have different compromise limit values, this should be taken into account in the cooperative games. Yu et al. [28] developed fuzzy Harsanyi solution for a kind of fuzzy coalition games. Since the cooperation profits are expressed with fuzzy numbers, it can be inferred that the cooperation profits are allocated to the players are also fuzzy numbers [29]. With the development of research, some scholars suggest using specific fuzzy numbers to express cooperation profits and establish the corresponding cooperative games. For example, cooperation profits are expressed with interval-valued fuzzy numbers, that is, interval-valued cooperative programming models. However, the models presented in Li [43] have ignored this aspect. (3) The method of ranking interval-valued displays an important role in interval-valued cooperative games. Li [43] defined a distance formula for ranking of interval-valued to solve interval-valued cooperative games. Unfortunately, the distance formula presented in Li [43] has some shortcoming, for that the reasonable of profits allocation scheme needs further discussing. (4) Since the coalitions' payoffs in interval-valued cooperative games are expressed with interval numbers, the allocation sets to the players are also interval numbers. Different ranking of interval numbers reflects differences between the allocation sets, it should be discussed in the games process so as to assist the players to choose most stable coalition. However, there are few studies discussed this aspect.

To overcome the above mention shortcomings, this paper focuses on n-person cooperative games with crisp coalitions and coalitions' payoffs are expressed with interval numbers. The main contributions of this work are highlighted at three aspects.

(1) The proposed models taking into account the compromise limit of players. For players in different coalitions may have different compromise limit values in the games process, the proposed methods are applicable to different cooperative background.

(2) To address the coalition's payoffs are expressed with interval numbers, the profits allocation model with undominated nonnegative excess vector is extended to interval-valued fuzzy environments.

(3) The differences between the allocation sets are discussed in this study, it can assist the players to choose most stable coalition if they obtain several allocation sets at the same time.

The remainder of this study unfolds as follows. In Section 2, the concepts of undominated nonnegative excess vector and n-person interval-valued cooperative games are reviewed. Section 3, several linear programming models are proposed, which respectively without and with compromise limit constraints. In Section 4, an illustrative example in conjunction with comparative analyses is employed to demonstrate the validity and applicability of the proposed methods. And the relationship of the models between without and with coalitions' compromise limit constraints is also discussed. The study ends with conclusions in Section 5.

2 Preliminaries

In this section, the concepts of undominated nonnegative excess vector and n-person interval-valued cooperative games are reviewed.

2.1 Undominated nonnegative excess vector

In this subsection, the concepts of excess vector, nonnegative excess vector and undominated nonnegative excess vector are introduced.

To measure the dissatisfaction of players with an allocation, Schmeidler [44] introduced the concept of excess vector.

Definition 1 [44]. For any coalition $S \subseteq N$ and an allocation x, the excess vector of S with respect to x is defined as follows:

$$e(S,x) = v(S) - \sum_{j=1}^{s} x_j \ , \tag{1}$$

where N is a set of players and $v(S)$ is a payoff function of coalition S, s is the number of players in coalition S. It can be easily found that when the payoff function of coalition $v(S)$ is larger than the sum of the allocation $\sum_{j=1}^{s} x_j$ to all players in coalition S, then the excess vector $e(S,x)$ is positive. In contrast, the excess vector is respectively zero and negative when the payoff function of coalition is equal to and smaller than the sum of the allocation.

For a coalition, it is dissatisfactory if its excess vector is positive. Otherwise, it is satisfactory in the sense that it will be worse if it leaves the grand coalition. That is to say, it may be not reasonable to consider negative excess vector if we want to include all the allocations acceptable by all players. Based on this fact, Chen [45] introduced the notion of nonnegative excess vector.

Definition 2 [45]. For a game (N,v), the nonnegative excess vector of a coalition $S \subseteq N$ with respect to an allocation x is defined as:

$$e^+(S,x) = \max\left\{v(S) - \sum_{j=1}^{s} x_j, 0\right\} \quad .(2)$$

Combining the efficient of the allocation, nonnegative excess vector satisfies the following three constraints, (1) Nonnegative: $\forall\, S \subset N$, $e_S \geq 0$, where e_S is the vector of abbreviation nonnegative excess vector $e^+(S,x)$ of the coalition S; (2) Excess vector of a coalition: $\forall\, S \subset N$, $\sum_{j=1}^{s} x_j \geq v(S) - e_S$; (3) The efficient of the allocation: $\sum_{j=1}^{n} x_j = v(N)$.

On the basis of nonnegative excess vector, Chen [45] developed the concept of undominated nonnegative excess vector.

Definition 3 [45]. Let $X(v,e)$ denote the set of allocations. For a game (N,v), a nonnegative excess vector e is called undominated if there is no other nonnegative excess vector $e' \in X(v,e')$ such that $e' < e$.

It can be concluded that the smaller the vector e_S is, the more stable of the grand coalition N of the game (N,v). To obtain the undominated nonnegative excess vector, Chen [45] developed a linear programming model to derive the maximal-stable games whose total coalitional is minimized:

$$min \quad Z = \sum_{S \subset N} e_S$$

$$s.t. \begin{cases} \sum_{j=1}^{n} x_j = v(N) \\ \sum_{j=1}^{s} x_j \geq v(S) - e_S, \quad S \subset N \\ e_S \geq 0, \quad S \subset N \\ x_j \geq 0, \quad j = 1,2,\cdots,n \end{cases}, \tag{3}$$

where n and s are respectively denote the number of players in coalition N and coalition S, and coalition S is proper subset of N.

2.2 Interval number and interval-valued cooperative games

In this subsection, we recalled some related concepts of interval number and interval-valued cooperative games.

2.2.1 Interval number

Usually, $\bar{a} = [a_L, a_R] = \{a | a_L \leq a \leq a_R, a \in R\}$ is called an interval number, where R is the set of real numbers, $a_R \in R$ and $a_L \in R$ are respectively called the upper bound and lower bound of the interval $\bar{a}$. To develop the used of interval numbers, Moore [46] introduced some interval arithmetic operations.

Definition 4 [46]. Let $\bar{a} = [a_L, a_R]$ and $\bar{b} = [b_L, b_R]$ are any two interval numbers in the interval numbers set $\bar{R}$. Then interval addition, subtraction and scalar multiplication operations are developed as follows:

$$\begin{cases} \bar{a} + \bar{b} = [a_L + b_L, a_R + b_R] \\ \bar{a} - \bar{b} = [a_L - b_R, a_R - b_L] \\ \lambda \bar{a} = [\lambda a_L, \lambda a_R], \quad \lambda \geq 0 \end{cases} . \tag{4}$$

In generally, the relationship $\bar{a} - \bar{b} + \bar{b} = \bar{a}$ does not hold in the subtraction operation presents in Eq. (4), in order to overcome this issue, Banks et al. [47] introduced the concept of Hukuhara difference.

Definition 5 [47]. Let $\bar{a} = [a_L, a_R]$ and $\bar{b} = [b_L, b_R]$ are any two interval numbers, if there exists an interval number $\bar{c}$, such that $\bar{a}=\bar{b}+\bar{c}$, then $\bar{c}$ is called the Hukuhara difference between $\bar{a}$ and $\bar{b}$, denoted as: $\bar{a}-_H\bar{b}=\left[a_L-b_L,a_R-b_R\right]$.

It is noted that, for any two interval numbers $\bar{a}$ and $\bar{b}$, the Hukuhara difference between them does not exist sometimes. For example, let $\bar{a}=\left[1,2\right]$ and $\bar{b}=\left[3,5\right]$, according to definition 5, the Hukuhara difference between them is $\bar{a}-_H\bar{b}=\left[-2,-3\right]$. Since $-2>-3$, then $\left[-2,-3\right]$ is not an interval number, that is to say, the Hukuhara difference between them does not exist. Moreover, it can be concluded that the Hukuhara difference between two interval numbers exist if and only if $a_L-b_L\le a_R-b_R$.

Let $v\left(\bar{a}\right)=\frac{1}{2}\left(a_L+a_R\right)$, $v\left(\bar{b}\right)=\frac{1}{2}\left(b_L+b_R\right)$, $w\left(\bar{a}\right)=a_R-a_L$ and $w\left(\bar{b}\right)=b_R-b_L$ are respectively denoted the middle and wide of interval numbers $\bar{a}$ and $\bar{b}$, Ishibuchi et al. [48] developed the total order between them as follows.

Definition 6 [48]. Let $\bar{a}=\left[a_L,a_R\right]$ and $\bar{b}=\left[b_L,b_R\right]$ are any two interval numbers, $v\left(\bar{a}\right)$ and $v\left(\bar{b}\right)$ are respectively denoted the middle of $\bar{a}$ and $\bar{b}$, $w\left(\bar{a}\right)$ and $w\left(\bar{b}\right)$ are respectively denoted their wide, then the total order between $\bar{a}$ and $\bar{b}$ is developed as, if and only if

$$v\left(\bar{a}\right)>v\left(\bar{b}\right) \text{ and } w\left(\bar{a}\right)<w\left(\bar{b}\right), \tag{5}$$

then $\bar{a}\succ\bar{b}$. Especially, $\bar{a}\sim\bar{b}$ if $v\left(\bar{a}\right)=v\left(\bar{b}\right)$ and $w\left(\bar{a}\right)=w\left(\bar{b}\right)$.

Zhang et al. [49] introduced a new distance formula for measuring interval numbers.

Definition 7 [49]. Let $\bar{a}=\left[a_L,a_R\right]$ and $\bar{b}=\left[b_L,b_R\right]$ are any two interval numbers, $v\left(\bar{a}\right)$ and $v\left(\bar{b}\right)$ are respectively denoted the middle of $\bar{a}$ and $\bar{b}$, $w\left(\bar{a}\right)$ and $w\left(\bar{b}\right)$ are respectively denoted their wide, then the distance between $\bar{a}$ and $\bar{b}$ is developed as follows:

$$D\left(\bar{a},\bar{b}\right)=\left|v\left(\bar{a}\right)-v\left(\bar{b}\right)\right|+\frac{1}{\sqrt{3}}\left|w\left(\bar{a}\right)-w\left(\bar{b}\right)\right|. \tag{6}$$

2.2.2 Interval-valued cooperative games

An n-person interval-valued cooperative game in coalitional form is an ordered pair $\left(N,\bar{v}\right)$, where $N=\{1,2,\cdots,n\}$ is the set of players, the symbol $\bar{v}$ is called the characteristic function of game, and $\bar{v}: 2^n\to R$ is a map, allocating to each coalition $S\in 2^n$ is a closed interval $\bar{v}\left(S\right)\in\bar{R}$, such that $\bar{v}\left(\varnothing\right)=\left[0,0\right]$. For each coalition $S\in 2^n$, the payoff value $\bar{v}\left(S\right)$ related to the coalition S in the n-person interval-valued cooperative game is a closed and bounded interval, which can be denoted as $\bar{v}\left(S\right)=\left[v_L\left(S\right),v_R\left(S\right)\right]$, where $v_L\left(S\right)$ and $v_R\left(S\right)$ are respectively denoted the lower and upper bounds of payoff value $\bar{v}\left(S\right)$. Obvious, the difference between the n-person interval-valued cooperative game and classical cooperative game is that the characteristic function are interval numbers in n-person interval-valued cooperative game, while the characteristic function are real numbers in classical cooperative game.

3 Models for n-person interval-valued cooperative games

For any n-person interval-valued cooperative games, since payoff values are interval numbers, it can be concluded that the allocation vales to each player are also interval numbers. Let $\sum_{j=1}^{s}\bar{x}_j=\left[\sum_{j=1}^{s}x_{jL},x_{jR}\right]$ denote the sum of interval-valued imputation to all players in coalition S, $\bar{v}\left(S\right)=\left[v_L\left(S\right),v_R\left(S\right)\right]$ and $\bar{v}\left(N\right)=\left[v_L\left(N\right),v_R\left(N\right)\right]$ respectively denote the interval-valued payoff function of coalition S and grand coalition N. In this section, the allocation model with undominated nonnegative excess vector presents in section 2 is extended to n-person interval-valued cooperative games. There are two types are concerned, that is, the allocation models respectively without and with compromise limit constraint.

3.1 Model without compromise limit constraints

In this subsection, an allocation model without compromise limit constraints is constructed to obtain the allocation scheme. Owing to the fact that the payoff value of grand coalition is limited to players' allocation, what is more, other players will not satisfy with the allocation scheme if one of the possible coalitions allocates too much. Based on this fact, it is better to pursuit the allocation scheme of all players in coalition S is as close to the coalitional value as possible. Then allocation model presents in Eq. (3) is extended to interval-valued as follows:

$$\min \quad z=\sum_{S\subset N}\bar{e}_S$$
$$s.t.\begin{cases}\sum_{j=1}^{n}\bar{x}_j =_I \bar{v}(N) \\ \sum_{j=1}^{s}\bar{x}_j \geq_I \bar{v}(S) -_I \bar{e}_S, \quad S\subset N \\ \bar{e}_S \geq 0, \quad S\subset N \\ \bar{x}_j \geq 0, \quad j=1,2,\cdots,n \end{cases}, \quad (7)$$

where $\bar{e}_S=[e_{SL},e_{SR}]$ is an interval-valued undominated nonnegative excess vector, $\bar{x}_j=[x_{jL},x_{jR}]$ is the interval-valued allocation to player j and is the decision variable of the model. The symbol "$=_I$", "$\geq_I$" and "$-_I$" are respectively denoted the arithmetic operations of two interval numbers "$=$", "$\geq$" and "$-$". Noting that $\sum_{j=1}^{n}\bar{x}_j =_I \bar{v}(N)$ is equivalent to $\sum_{j=1}^{n}x_{jL}=v_L(N)$ and $\sum_{j=1}^{n}x_{jR}=v_R(N)$, which are described as the efficiency condition and showed the sum of all player's allocation should be equal to the payoff value of grand coalition N. $\bar{e}_S\geq 0$ is equivalent to $e_{SL}\geq 0$ and $e_{SL}\leq e_{SR}$. Similar, $\bar{x}_j\geq 0$ is equivalent to $x_{jL}\geq 0$ and $x_{jL}\leq x_{jR}$. Objective function $\min \quad z=\sum_{S\subset N}\bar{e}_S$ is equivalent to $\min \quad z=\sum_{S\subset N}(e_{SL}+e_{SR})$. Replacing above mention equivalence conditions into Eq. (7), we have:

$$\min \quad z=\sum_{S\subset N}(e_{SL}+e_{SR})$$
$$s.t.\begin{cases}\sum_{j=1}^{n}x_{jL}=v_L(N) \\ \sum_{j=1}^{n}x_{jR}=v_R(N) \\ \sum_{j=1}^{s}\bar{x}_j \geq_I \bar{v}(S) -_I \bar{e}_S, \quad S\subset N \\ e_{SL}\leq e_{SR}, \quad S\subset N \\ x_{jL}\leq x_{jR}, \quad j=1,2,\cdots,n \\ e_{SL}\geq 0, \quad S\subset N \\ x_{jL}\geq 0, \quad j=1,2,\cdots,n \end{cases}. \quad (8)$$

In Eq. (8), according to Definition 5, the constraint condition $\bar{v}(S) -_I \bar{e}_S$ is equal to $[v_L(S)-e_{SL}, v_R(S)-e_{SR}]$ and $v_L(S)-e_{SL}\leq v_R(S)-e_{SR}$. Replacing these equivalence conditions into Eq. (8), we have:

$$\min \quad z=\sum_{S\subset N}(e_{SL}+e_{SR})$$
$$s.t.\begin{cases}\sum_{j=1}^{n}x_{jL}=v_L(N) \\ \sum_{j=1}^{n}x_{jR}=v_R(N) \\ \sum_{j=1}^{s}\bar{x}_j \geq_I [v_L(S)-e_{SL}, v_R(S)-e_{SR}] \\ S\subset N \\ v_L(S)-e_{SL}\leq v_R(S)-e_{SR}, \quad S\subset N \\ e_{SL}\leq e_{SR}, \quad S\subset N \\ x_{jL}\leq x_{jR}, \quad j=1,2,\cdots,n \\ e_{SL}\geq 0, \quad S\subset N \\ x_{jL}\geq 0, \quad j=1,2,\cdots,n \end{cases}. \quad (9)$$

In Eq. (9), according to Eq. (5), the constraint condition $\sum_{j=1}^{s}\bar{x}_j \geq_I [v_L(S)-e_{SL}, v_R(S)-e_{SR}]$ is equal to $\sum_{j=1}^{s}x_{jL}+\sum_{j=1}^{s}x_{jR}\geq v_L(S)+v_R(S)-e_{SL}-e_{SR}$ and $\sum_{j=1}^{s}x_{jR}-\sum_{j=1}^{s}x_{jL}\leq v_R(S)-v_L(S)+e_{SL}-e_{SR}$. Replacing these equivalence conditions into Eq. (9), we have:

$$\min \quad z=\sum_{S\subset N}(e_{SL}+e_{SR})$$
$$s.t.\begin{cases}\sum_{j=1}^{n}x_{jL}=v_L(N) \\ \sum_{j=1}^{n}x_{jR}=v_R(N) \\ \sum_{j=1}^{s}x_{jL}+\sum_{j=1}^{s}x_{jR}\geq \\ v_L(S)+v_R(S)-e_{SL}-e_{SR}, \quad S\subset N \\ \sum_{j=1}^{s}x_{jR}-\sum_{j=1}^{s}x_{jL}\leq \\ v_R(S)-v_L(S)+e_{SL}-e_{SR}, \quad S\subset N \\ v_L(S)-e_{SL}\leq v_R(S)-e_{SR}, \quad S\subset N \\ e_{SL}\leq e_{SR}, \quad S\subset N \\ x_{jL}\leq x_{jR}, \quad j=1,2,\cdots,n \\ e_{SL}\geq 0, \quad S\subset N \\ x_{jL}\geq 0, \quad j=1,2,\cdots,n \end{cases}. \quad (10)$$

Similar to Eq. (10), the optimal solution of Eq. (13) can be easily obtained by utilizing the programming software. The difference between Eq. (13) and Eq. (10) is that, the compromise limit constrains $\bar{e}_S\leq\sigma_S$, $S\subset N$ are taken into account in Eq. (13), while Eq. (10) fail to this. The players choose different allocation models according to different game environments. If the players have very strongly willing to cooperate, they can accept the situation that the payoff values of the coalitions are greater than the allocation values obtained by the players participating in the coalitions. In this case, the optimal solution of Eq. (10) is applicable. In contrast, if the cooperation willing of players is not very strong, they cannot accept the situation that the payoff values of the coalitions are greater than the distribution values obtained by the players participating in the coalitions. In this case, the optimal solution of Eq. (13) is applicable.

Given that Eq. (10) presents a linear programming model, x_{jL}, $x_{jR} \geq 0$, $j = 1, 2, \cdots, n$ are $2n$ nonnegative decision variables, and $e_{SL} \geq 0$, $S \subset N$ are $2^n - 2$ undominated nonnegative excess variables. It is easily to demonstrate that the set of feasible solutions to Eq. (10) is nonempty. Facilitated by the well-established theory of linear programming, it is convenient to solve Eq. (10) by many available optimization software packages, such as MATLAB, LINGO or CPLEX.

3.2 Model with compromise limit constraints

For different coalitions have different advantages, the allocation to all players in coalition S should be as much as possible. In contrast, if the allocation to all players in coalition S is far from the coalition's payoff value, this will lead to the instability of the coalition. In some cases, the players only accept the allocations within established boundaries, that is, the players in the coalition with compromise limit constraints of the allocations. It is more reasonable that the compromise limits of the coalition should be taken into account in the allocation process. However, the model shows in Eq. (10) may fail to this. In order to overcome this shortcoming, a programming model is constructed for solving interval-valued undominated nonnegative excess vector with compromise limit constraints.

As mention above, it is better to pursuit the allocations to all players in coalition S are as close to the coalition payoff value as possible, and the allocation results are acceptable if the allocation within certain established compromise limit σ_S . Based on this consideration, the allocation model is constructed as follows.

$$\min \quad z = \sum_{S \subset N} \bar{e}_S$$
$$s.t. \begin{cases} \sum_{j=1}^{n} \bar{x}_j =_I \bar{v}(N) \\ \bar{e}_S \leq \sigma_S, \quad S \subset N \\ \sum_{j=1}^{s} \bar{x}_j \geq_I \bar{v}(S) -_I \bar{e}_S, \quad S \subset N \\ \bar{e}_S \geq 0, \quad S \subset N \\ \bar{x}_j \geq 0, \quad j = 1, 2, \cdots, n \end{cases} . \quad (11)$$

Apparently, the difference between Eq. (11) and Eq. (7) is that, the compromise limit constrains $\bar{e}_S \leq \sigma_S$, $S \subset N$ are taken into account in Eq. (11), while Eq. (7) fail to this. Suppose the compromise limit constrain values σ_S provides by the coalition with the equal significance, that is, $\sigma_S = \sigma$, for all $S \subset N$, in this case, the mathematical programming model can be further simplified to:

$$\min \quad z = \sum_{S \subset N} \bar{e}_S$$
$$s.t. \begin{cases} \sum_{j=1}^{n} \bar{x}_j =_I \bar{v}(N) \\ \bar{e}_S \leq \sigma, \quad S \subset N \\ \sum_{j=1}^{s} \bar{x}_j \geq_I \bar{v}(S) -_I \bar{e}_S, \quad S \subset N \\ \bar{e}_S \geq 0, \quad S \subset N \\ \bar{x}_j \geq 0, \quad j = 1, 2, \cdots, n \end{cases} . \quad (12)$$

Moreover, the compromise limit constraints $\bar{e}_S \leq \sigma$, $S \subset N$ are equal to $e_{SL} + e_{SR} \leq \sigma$, $S \subset N$, After a similar simplification presents in section 3.1, we obtain the following linear programming model:

$$\min \quad z=\sum_{S\subset N}\left(e_{SL}+e_{SR}\right)$$
$$s.t.\begin{cases}\sum_{j=1}^{n}x_{jL}=v_L(N)\\ \sum_{j=1}^{n}x_{jR}=v_R(N)\\ e_{SL}+e_{SR}\le\sigma,\quad S\subset N\\ \sum_{j=1}^{s}x_{jL}+\sum_{j=1}^{s}x_{jR}\ge\\ v_L(S)+v_R(S)-e_{SL}-e_{SR},\quad S\subset N\\ \sum_{j=1}^{s}x_{jR}-\sum_{j=1}^{s}x_{jL}\le\\ v_R(S)-v_L(S)+e_{SL}-e_{SR},\quad S\subset N\\ v_L(S)-e_{SL}\le v_R(S)-e_{SR},\quad S\subset N\\ e_{SL}\le e_{SR},\quad S\subset N\\ x_{jL}\le x_{jR},\quad j=1,2,\cdots,n\\ e_{SL}\ge 0,\quad S\subset N\\ x_{jL}\ge 0,\quad j=1,2,\cdots,n\end{cases}. \quad (13)$$

Similar to Eq. (10), the optimal solution of Eq. (13) can be easily obtained by utilizing the programming software. The difference between Eq. (13) and Eq. (10) is that, the compromise limit constrains $\bar{e}_S\le\sigma_S$, $S\subset N$ are taken into account in Eq. (13), while Eq. (10) fail to this. The players choose different allocation models according to different game environments. If the players have very strongly willing to cooperate, they can accept the situation that the payoff values of the coalitions are greater than the allocation values obtained by the players participating in the coalitions. In this case, the optimal solution of Eq. (10) is applicable. In contrast, if the cooperation willing of players is not very strong, they cannot accept the situation that the payoff values of the coalitions are greater than the distribution values obtained by the players participating in the coalitions. In this case, the optimal solution of Eq. (13) is applicable.

4 Models for n-person interval-valued cooperative games

In this section, three new energy factories cooperative problem (adapted from [43]) is used to demonstrate the applicability of the proposed methods.

4.1 Problem description

Suppose that there are three new energy factories, denoted as players 1, 2, and 3, who have the ability to produce new energy separately. Three factories denoted the set of players by $N=\{1,2,3\}$. In order to pursuit of more profits, three factories plan to work together for manufacturing a better new energy. Due to the uncertain and incomplete information, they cannot precisely forecast the profits obtain from the new energy. Generally, they only can estimate the ranges of their profits. Namely, the profit of a coalition $S\subseteq N$ of the factories may be expressed with an interval number $\bar{v}(S)=\left[\bar{v}_L(S),\bar{v}_R(S)\right]$. In this case, the optimal allocation problem of profits for the factories may be regarded as an three person interval-valued cooperative game $\bar{v}$ in which the interval-valued characteristic function is equal to $\bar{v}(S)$ for any coalition $S\subseteq N$. Thus, if the factories manufacture the new energy by themselves, according to the information they have, they only can estimate the lower and upper limits of their profits, then the profits are expressed with the interval numbers $\bar{v}(1)=[0,2]$, $\bar{v}(2)=\left[\frac{1}{2},\frac{3}{2}\right]$, and $\bar{v}(3)=[1,2]$, respectively. Similarly, if any two factories cooperatively manufacture the new energy, then their profits are expressed with the interval numbers $\bar{v}(1,2)=[2,3]$, $\bar{v}(2,3)=[4,4]$, and $\bar{v}(1,3)=[3,4]$, respectively. If all three factories, that is, the grand coalition N, cooperatively manufacture the new energy, then the profit is expressed with an interval number $\bar{v}(1,2,3)=[6,7]$.

For this example, the factories try to determine the range of the expected allocation from the grand coalition. In other words, the lower and upper bounds of the interval-valued allocation sets need to be determined.

4.2 Illustration of the proposed models

Let $\bar{x}_1=[x_{1L},x_{1R}]$, $\bar{x}_2=[x_{2L},x_{2R}]$ and $\bar{x}_3=[x_{3L},x_{3R}]$ are respectively denote the allocation sets for three factories. In the following section, two linear programming models respectively without and with compromise limit constraints are constructed to obtain the allocation sets.

Case 1: Model without compromise limit constraints

If the players accept their allocation results without compromise limit constraints, according to Eq. (10), the linear programming model can be constructed as follows.

$$\min \quad z = e_{1L} + e_{1R} + e_{2L} + e_{2R} + e_{3L} + e_{3R} + e_{12L} + e_{12R} + e_{13L} + e_{13R} + e_{23L} + e_{23R}$$

$$s.t.\begin{cases} x_{1L} + x_{2L} + x_{3L} = 6 \\ x_{1R} + x_{2R} + x_{3R} = 7 \\ x_{1L} + x_{1R} \geq 0 + 2 - e_{1L} - e_{1R} \\ x_{2L} + x_{2R} \geq \frac{1}{2} + \frac{3}{2} - e_{2L} - e_{2R} \\ x_{3L} + x_{3R} \geq 1 + 2 - e_{3L} - e_{3R} \\ x_{1L} + x_{2L} + x_{1R} + x_{2R} \geq 2 + 3 - e_{12L} - e_{12R} \\ x_{1L} + x_{3L} + x_{1R} + x_{3R} \geq 3 + 4 - e_{13L} - e_{13R} \\ x_{2L} + x_{3L} + x_{2R} + x_{3R} \geq 0 + 4 - e_{23L} - e_{23R} \\ x_{1R} - x_{1L} \leq 2 - 0 + e_{1L} - e_{1R} \\ x_{2R} - x_{2L} \leq \frac{3}{2} - \frac{1}{2} + e_{2L} - e_{2R} \\ x_{3R} - x_{3L} \leq 2 - 1 + e_{3L} - e_{3R} \\ x_{1R} + x_{2R} - x_{1L} - x_{2L} \leq 3 - 2 + e_{12L} - e_{12R} \\ x_{1R} + x_{3R} - x_{1L} - x_{3L} \leq 4 - 3 + e_{13L} - e_{13R} \\ x_{2R} + x_{3R} - x_{2L} - x_{3L} \leq 4 - 0 + e_{23L} - e_{23R} \\ 2 - e_{1R} \geq 0 - e_{1L} \\ \frac{3}{2} - e_{2R} \geq \frac{1}{2} - e_{2L} \\ 2 - e_{3R} \geq 1 - e_{3L} \\ 3 - e_{12R} \geq 2 - e_{12L} \\ 4 - e_{13R} \geq 3 - e_{13L} \\ 4 - e_{23R} \geq 0 - e_{23L} \\ e_{SL} \leq e_{SR} \\ x_{iL} \leq x_{iR}, \quad i = 1,2,3 \\ e_{SL} \geq 0 \\ x_{iL} \geq 0, \quad i = 1,2,3 \\ S \subseteq \{1,2,3,12,13,23\} \end{cases}$$

Solving above model by utilizing the software LINGO 11.0, we obtain the optimal solution and optimal decision variables as follows: $z^* = 0$; $x_{1L} = 3.5$, $x_{1R} = 4.5$; $x_{2L} = 1$, $x_{2R} = 1$ and $x_{3L} = 1.5$, $x_{3R} = 1.5$. Then the optimal allocation sets for players 1, 2, and 3 as follows: $\bar{x}_1 = [3.5, 4.5]$, $\bar{x}_2 = [1,1]$ and $\bar{x}_3 = [1.5,1.5]$.

Case 2: Model with compromise limit constraints

If the players accept their allocation with compromise limit constraints, and suppose the compromise limit constraint values are equal, that is $\sigma_S = \sigma = 2$, $S \subseteq \{1,2,3,12,13,23\}$, according to Eq.(13), the programming model can be constructed as follows:

$$\min \quad z = e_{1L} + e_{1R} + e_{2L} + e_{2R} + e_{3L} + e_{3R} + e_{12L} + e_{12R} + e_{13L} + e_{13R} + e_{23L} + e_{23R}$$

$$s.t.\begin{cases} e_{SL} + e_{SR} \leq 2 \\ x_{1L} + x_{2L} + x_{3L} = 6 \\ x_{1R} + x_{2R} + x_{3R} = 7 \\ x_{1L} + x_{1R} \geq 0 + 2 - e_{1L} - e_{1R} \\ x_{2L} + x_{2R} \geq \frac{1}{2} + \frac{3}{2} - e_{2L} - e_{2R} \\ x_{3L} + x_{3R} \geq 1 + 2 - e_{3L} - e_{3R} \\ x_{1L} + x_{2L} + x_{1R} + x_{2R} \geq 2 + 3 - e_{12L} - e_{12R} \\ x_{1L} + x_{3L} + x_{1R} + x_{3R} \geq 3 + 4 - e_{13L} - e_{13R} \\ x_{2L} + x_{3L} + x_{2R} + x_{3R} \geq 0 + 4 - e_{23L} - e_{23R} \\ x_{1R} - x_{1L} \leq 2 - 0 + e_{1L} - e_{1R} \\ x_{2R} - x_{2L} \leq \frac{3}{2} - \frac{1}{2} + e_{2L} - e_{2R} \\ x_{3R} - x_{3L} \leq 2 - 1 + e_{3L} - e_{3R} \\ x_{1R} + x_{2R} - x_{1L} - x_{2L} \leq 3 - 2 + e_{12L} - e_{12R} \\ x_{1R} + x_{3R} - x_{1L} - x_{3L} \leq 4 - 3 + e_{13L} - e_{13R} \\ x_{2R} + x_{3R} - x_{2L} - x_{3L} \leq 4 - 0 + e_{23L} - e_{23R} \\ 2 - e_{1R} \geq 0 - e_{1L} \\ \frac{3}{2} - e_{2R} \geq \frac{1}{2} - e_{2L} \\ 2 - e_{3R} \geq 1 - e_{3L} \\ 3 - e_{12R} \geq 2 - e_{12L} \\ 4 - e_{13R} \geq 3 - e_{13L} \\ 4 - e_{23R} \geq 0 - e_{23L} \\ e_{SL} \leq e_{SR} \\ x_{iL} \leq x_{iR}, \quad i = 1,2,3 \\ e_{SL} \geq 0 \\ x_{iL} \geq 0, \quad i = 1,2,3 \\ S \subseteq \{1,2,3,12,13,23\} \end{cases}$$

Solving above model by utilizing the software LINGO 11.0, we obtain the optimal solution and optimal decision variables as follows: $z^* = 0$; $x_{1L} = 3.5$, $x_{1R} = 4.5$; $x_{2L} = 1$, $x_{2R} = 1$ and $x_{3L} = 1.5$, $x_{3R} = 1.5$. Then the optimal allocation sets for players 1, 2, and 3 as follows: $\bar{x}_1 = [3.5, 4.5]$, $\bar{x}_2 = [1,1]$ and $\bar{x}_3 = [1.5,1.5]$.

4.3 Computational results and discussion

To validate the feasibility of the proposed models, we conducted a comparative study with other methods based on the same illustrative example.

Li [43] based on the proposed distance formula of interval numbers, developing quadratic programming models respectively consider without and with the efficiency constraint for solving above illustrative example. The interval-valued least square solution is derived by using Lagrange multiplier method. For a better comparison, the optimal solutions and optimal decision variables of Li [43]'s methods and the proposed methods are showed in Table 1.For simplicity, the model without efficiency and compromise limit constrains is denoted Method 1, the model with efficiency but without compromise limit constrains is denoted Method 2, the model with efficiency but without compromise limit constrains is named Method 3 and the model with efficiency and compromise limit constrains is named Method 4.

Table 1: Optimal allocation scheme obtained from different methods.

Methods		Optimal solution	Optimal decision variables		
			$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$
Li [43]'s methods	Method 1	7.06	$\left[\frac{13}{16}, \frac{31}{16}\right]$	$\left[\frac{25}{16}, \frac{27}{16}\right]$	$\left[\frac{37}{16}, \frac{39}{16}\right]$
	Method 2	14	$\left[\frac{5}{4}, \frac{9}{4}\right]$	$\left[2, 2\right]$	$\left[\frac{11}{4}, \frac{11}{4}\right]$
Proposed methods	Method 3	0	$\left[3.5, 4.5\right]$	$\left[2.5, 2.5\right]$	$\left[1.5, 1.5\right]$
	Method 4	0	$\left[3.5, 4.5\right]$	$\left[2.5, 2.5\right]$	$\left[1.5, 1.5\right]$

For the convenience of following discussion, the model without efficiency and compromise limit constrains in Li [43]'s methods is named Method 1. Other entries, that is, Methods 2 to 4, in Table 1 are similarly explained. Review from the Table 1, it can be easily found that, the optimal solutions and optimal decision variables obtained from Li [43]'s methods are different from the proposed methods, and they are also different from Method 1 and Method 2. The optimal solution obtained from Method 2 is the maximum. In contrast, the minimum is obtained from the proposed methods. Among the optimal decision variables, the optimal decision variables obtained from Method 1 does not satisfy the individual rationality, that is

$\sum_{i=1}^{3} x_{iL} = \frac{13}{16} + \frac{25}{16} + \frac{37}{16} = \frac{75}{16} \neq 6$, and

$\sum_{i=1}^{3} x_{iR} = \frac{31}{16} + \frac{27}{16} + \frac{39}{16} = \frac{97}{16} \neq 7$.

Obviously, the optimal decision variables obtained from other methods satisfy the individual rationality.

The reason for the differences among Li [43]'s methods and the proposed methods is explained as follows: the quadratic programming models presented in Li [43]'s methods are based on the distance formula of proposed interval numbers, while the proposed programming model based on undominated nonnegative excess vector whose total coalitional is minimized. The different perspectives for solving the problems lead to different decision-making results. Since the distance formula used in Li [43]'s methods has some shortcoming, that is, the distance formula does not satisfy triangle inequality. But the proposed methods do not use the distance formula of interval numbers. Based on this fact, the optimal solutions and optimal decision variables obtained from the proposed methods seem more convincing. In addition, the models with different limit constrains also lead to different optimal solutions and optimal decision variables, the proposed methods respectively considering without and with compromise limit constrains, these methods are applicable to different cooperative background, while Li [43]'s methods do not consider this aspect.

Moreover, from the objective functions presented in Li [43]'s methods and the proposed methods, we find that the smaller the objective function value, the better the imputation result. Based on this fact, the results obtain from the proposed methods are the most optimal. In addition, the optimal decision variables sets not only demonstrate the ranking of the players' expected allocation, but also the differences between the allocation sets. For a better comparison, the ranking of allocation sets and differences between the allocation sets obtained from Li [43]'s methods and the proposed methods are showed in Table 2.

Table 2: Differences between the allocation sets obtained from different methods.

Methods		The ranking of allocation sets	Differences between the allocation sets
Li [43]'s methods	Method 1	$\bar{x}_3 \succ \bar{x}_1 \succ \bar{x}_2$	3.6667
	Method 2	$\bar{x}_3 \succ \bar{x}_1 \succ \bar{x}_2$	5
Proposed methods	Method 3	$\bar{x}_1 \succ \bar{x}_3 \succ \bar{x}_2$	6.58
	Method 4	$\bar{x}_1 \succ \bar{x}_3 \succ \bar{x}_2$	6.58

Take the values of ranking of allocation sets and the differences between the allocation sets in Method 1 for example, according to Eq. (6), we have: $D\left(\bar{x}_1, 0\right) = 1.75$, $D\left(\bar{x}_2, 0\right) = 1.6667$ and $D\left(\bar{x}_3, 0\right) = 2.4167$. Since $D\left(\bar{x}_3, 0\right) > D\left(\bar{x}_1, 0\right) > D\left(\bar{x}_2, 0\right)$, then we have

$\bar{x}_3 \succ \bar{x}_1 \succ \bar{x}_2$. Moreover, since $D\left(\bar{x}_1, \bar{x}_2\right) = 0.5833$, $D\left(\bar{x}_1, \bar{x}_3\right) = 1.3333$ and $D\left(\bar{x}_2, \bar{x}_3\right) = 1.7500$, then the differences between the allocation sets is $D\left(\bar{x}_1, \bar{x}_2\right) + D\left(\bar{x}_1, \bar{x}_3\right) + D\left(\bar{x}_2, \bar{x}_3\right) = 3.6667$. Other values can be obtained in a similar way. Review from the computational results in table 2, we find that, the ranking of the allocation sets and the differences between the allocation sets obtained from different methods are different. The ranking of allocation sets obtain from Li [43]'s methods is $\bar{x}_3 \succ \bar{x}_1 \succ \bar{x}_2$, while obtain from proposed methods is $\bar{x}_1 \succ \bar{x}_3 \succ \bar{x}_2$. And the differences between the allocation sets are also different. The differences between the imputation sets obtain from proposed methods is the greatest, while obtain from Method 1 is the least. It can be easily found that the smaller the differences between the allocation sets, the allocation results are easier to be accepted for all the players, and more stable of the coalition. Based on this fact, the allocation sets presented in Method 1 is the best one.

According to the comparison analysis, the methods presented in this study have the following advantages over the other methods considered.

(1) The proposed models taking into account the compromise limit of coalitions. To ensure the benefits of the coalitions, different coalitions may have different compromise limit constraints. However, the models showed in Li [43]'s methods do not consider this aspect.

(2) The proposed methods based on undominated nonnegative excess vector whose total coalitional is minimized seems more reasonable than Li [43]'s method. Since the distance formula used in Li [43]'s method has some shortcomings. For that, the optimal solutions and optimal decision variables obtained from the proposed methods seem more convincing.

4.4 Sensitivity analysis

According to the previous discussion, the relationships between the proposed models Eq. (10) and Eq. (13) can be concluded as follows: the model presents in Eq. (13) take into accounted the compromise limit constrains of players while Eq. (10) discard this aspect. And the Eq. (13) is reduced to the Eq. (10) if all coalitions' compromise limit constraints tend to infinity such that $\sigma \to \infty$. To better present the conclusion, the numerical analysis results under different values of σ are showed in the table 3.

Table 3: The optimal solutions and optimal decision variables under different values of σ .

Compromise limit values	Optimal solutions	Optimal decision variables $\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$
1.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
1.5	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
2.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
2.5	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
3.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
3.5	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
4.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
4.5	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
5.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
5.5	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$
6.0	0	$[3.5, 4.5]$	$[2.5, 2.5]$	$[1.5,1.5]$

Review from the computational results present in table 3, we find that, the optimal decision variables and optimal solutions keep the same when the compromise limit values vary from 1.0 to 6.0, in this case, Eq. (13) is reduced to the Eq. (10), that is, there is no effect on the allocation results with the compromise limit constrains take into accounted in the model.

5 Conclusion

The aim of the paper is to develop several linear programming models for interval-valued cooperative games in which considering the coalitions' compromise limit constraints. First, the profits allocation model with undominated nonnegative excess vector is extended to interval-valued fuzzy environments. Second, several linear programming models are constructed respectively considering without and with coalitions' compromise limit constraints. Third, an illustrative example in conjunction with comparative analyses is employed to demonstrate the validity and applicability of the proposed models. Finally, the relationship of the models is discussed between without and with coalitions' compromise limit constraints.

The main contributions of this work are highlighted at three aspects: (1) the proposed models taking into account the compromise limit constraints of coalitions. To ensure the benefits of the coalition, different coalitions may have different compromise limit constraints. Based on this fact, the proposed methods are applicable to different cooperative background. (2) To address the coalition's payoffs are interval numbers, the profits allocation model with undominated nonnegative excess vector is extended to interval-valued fuzzy environments. (3) The differences between the allocation sets are discussed in this study, it can assist the players to choose stable coalition if they obtain several allocations sets at the same time. In the further study, the coalitions have different compromise limit constraints will be discussed.

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