A Hesitant Fuzzy Multiplicative Base-criterion Multi-criteria Group Decision Making Method

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Hesitant fuzzy sets have a unique characteristic that its basic element could manifest the assessment values of different decision makers on the same option under a certain criterion. Base-criterion method is a very significant tool for calculating the weights of the criteria in multiple criteria decision-making. In this paper, we developed a novel approach hesitant fuzzy BCM based on hesitant fuzzy multiplicative preference relation for multiple criteria group decision making. The base-comparison of the preferential criterion relative to other criteria is expressed as linguistic terms, which might be converted into hesitant multiplicative elements (HMEs). HMEs are extended along the same length according to the attitude of the decision makers. Then normalized optimal hesitant fuzzy weights are calculated. The normalized optimal hesitant fuzzy weights of criteria may be transferred to crisp values by employing score function. To illustrate the applicability and suitability of hesitant fuzzy BCM, we analyse the optimal transportation mode selection problem and car selection problem under hesitant fuzzy environment. The outcomes of the proposed model indicate that the hesitant fuzzy BCM is highly consistent and can yield appreciable preference ranking of criteria and alternatives.

Povzetek: Predstavljena je metoda odločanja v skupinah s pomočjo obotavljivih mehkih kriterijev.

1 Introduction

Decision making is the process of selecting an optimal alternative from a set of alternatives. Multi-criteria decision-making is an important tool of decision-making process that explicitly evaluates multiple conflicting criteria in decision making (both in daily life and in settings such as business, government and medicine) [1]. MCDM methods are divided into two main categories: Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM). The fundamental difference between MODM and MADM is that MODM have no predetermined alternatives and MADM have limited number of alternatives [2,3]. MODM methods are employed to handle continuous problems, on the other hand, multi-attribute decision making (MADM) methods are used to solve discrete problems. MCDM is commonly used to describe the discrete MADM.

Over the previous years, many MCDM methods have been introduced by researchers such as VIKOR [4], ELECTRE (Elimination and Choice Expressing Reality) [5], TOPSIS (Technique for order preference by similarity to an ideal solution) [6,7], COPRAS (Complex Proportional Assessment) [8,9], SWARA (step-wise weight assessment ratio analysis) [10], ANP (Analytic Network Process) [11,12], AHP (Analytic Hierarchy Process) [13], BWM (Best-Worst Method) [14,15].

In a practical problem, MCDM consists of two parts: (a) obtaining decision information, including criterion weight, (b) ranking the alternatives by a certain approach. The most important part is how we calculate the criterion weights, which has been the foundation for the introduction of many MCDM methods. Recently, Haseli et al [16] developed a novel Base-Criterion Method (BCM) which is a better route to determine the weight of the criteria. First, the decision maker selects the base-criterion (preferential, selective) and then a pairwise comparison is made between the base-criterion and other criteria. This technique is much clearer and more accurate because the execution of secondary comparisons is not necessary. This can achieve the weight of the criterion with less pairwise comparisons than existing MCDM methods. The final weights determined through BCM are very authentic as the comparison is completely consistent while other traditional MCDM techniques such as BWM and AHP have low inconsistency ratio.

It is hard to recognize all the facets of a decision-making problem for a single decision maker. The decisions made by groups are mostly different from those made by individuals. So, it is essential to have opinions from group of experts/decision makers. When more than one expert evaluates an option, it is very possible for them

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to have different opinions. In today's environment, group decision making (GDM) methods pay more attention to the fuzzy information contained in group decision making problems.

Due to the hesitant of the purpose as well as the fuzziness of human mind, human decisions generally collide the characteristics of opacity. Prof. Zadeh in 1965 proposed the fuzzy set theory [17]. Fuzzy mention the things which are obscure. Working system of fuzzy sets is retraced uncertainty and lack of clarity in daily life. After that, several tools have been introduced to handle the fuzzy information such as interval valued fuzzy set [18,19], type-2 fuzzy set [20], intuitionistic fuzzy set [21], and hesitant fuzzy sets [22].

People usually hesitate about one thing or the other while making decisions, which makes it difficult to reach a final agreement. Hesitant Fuzzy Set (HFS) [22,23,24] is very powerful tool for obtaining the optimum alternative in a decision-making process with multi-criteria and multiple individuals. HFS allows the membership having a set of few values. Generally, the grades of preference are not symmetric, but distributed asymmetrically around some value. Saaty's 1/9-9 scale is a useful tool to deal with such a situation, especially in expressing the multiplicative preference relation applied in many areas. A hesitant fuzzy multiplicative preference relation (HFMPR), which is developed based on the fuzzy preference relation [25,26], is the most usual tool for expressing DMs' preferences over decision-making alternatives.

In the present study, using hesitant fuzzy multiplicative numbers the reference comparisons of BCM are executed from group decision making process to have more real and convincing ranking results. So, a novel approach, including BCM under hesitant fuzzy environment has been developed and employed for the first time. We expended the BCM to more authentic HF-BCM.

We systematize the rest of paper as follows: Section 2 demonstrates the basic concepts. Section 3 introduces the proposed methodology. Section 4 describes the application of our proposed method by considering simple example of decision-making problems. Section 5 shows findings and conclusion.

2 Preliminaries

Definition 2.1 ([22],[24]) Let X be a fixed set, HFS on X is in terms of a function that when applied to X returns a subset of [0,1].

Xia and Xu [27] expressed the HFS by a mathematical symbol:

$$A = \{\langle x, h_A(x)\rangle | x \in X\} \quad (1)$$

Where $h_A(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set A and $h = h_A(x)$ is a hesitant fuzzy element (HFE).

Definition 2.2 [27] For a HFE $h, s(h) = \frac{1}{l_h}\sum_{p\in h} p$, is called the score of h, where l_h is the number of the elements in h.

For two HFEs, h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$.

Definition 2.3 [27] Let h_1 and h_2 be two HFEs, then basic operations on HFEs are as follows:

$$h_1 \oplus h_2 = \cup_{p\in h_1, p\in h_2}\{p_1 + p_2\};$$
$$h_1 \ominus h_2 = \cup_{p_1\in h_1, p_2\in h_2}\{p_1 - p_2\};$$
$$h_1 \otimes h_2 = \cup_{p_1\in h_1, p_2\in h_2}\{p_1 . p_2\};$$
$$h_1 \oslash h_2 = \cup_{p_1\in h_1, p_2\in h_2}\left\{\frac{p_1}{p_2}\right\}$$
(2)

Definition 2.4 [13] Let $A = \{A_1, A_2, .., A_n\}$ be a set of n alternatives, then $B = (b_{ij})_{n\times n}$ is called a multiplicative preference relation on $A \times A$, whose element b_{ij} estimates the preference of the alternative A_i over A_j, and is characterized by a ratio scale such as Saaty's ratio scale such that $b_{ij} \in \left[\frac{1}{9}, 9\right]$, and $b_{ij}.b_{ji} = 1, i, j = 1,2, .., n$.

Where b_{ij} unfolds indifference between A_j and A_i; $b_{ij} > 1$ unfolds that A_i is preferred to A_j, especially, $b_{ij} = 9$ unfolds that A_i is absolutely preferred to A_j; $b_{ij} < 1$ unfolds that A_j is preferred to A_i, especially, $b_{ij} = \frac{1}{9}$ unfolds that A_j is absolutely preferred to A_i.

Definition 2.5 [28] Let $X = \{x_1, x_2, \dots . x_n\}$ be a fixed set, then a hesitant fuzzy multiplicative preference relation on the set X is represented by the matrix $H = (h_{ij})_{n\times n} \in X \times X$, where $h_{ij} = \{p_{ij}^l,\ l = 1,2, \dots |h_{ij}|\}$ is a HME which expresses all possible preference degrees of the alternative x_i over x_j given by the DMs. h_{ij} (HME) should satisfies the conditions:

$$p_{ij}^{\sigma(l)} . p_{ji}^{\sigma(|h_{ij}|-l+1)} = 1, \quad p_{ii} = 1, \quad |h_{ij}| = |h_{ji}|, \quad i, j = 1,2, \dots, n. \quad (3)$$

Where all p_{ij} are ranked in ascending order, $p_{ij}^{\sigma(l)}$ represents the l^{th} smallest value in h_{ij} and $|h_{ij}|$ unfolds the number of elements in h_{ij}. In particular, $p_{ii} = 1$ unfolds the indifference between x_i and x_j, $p_{ij} > 1$

indicates that x_i is preferred to x_j, and $p_{ij} < 1$ indicates that x_i is not preferred to x_j or x_j is preferred to x_i.

In fact, if the preference degree of the alternative x_i over x_j is p, then the preference degree of the alternative x_j over x_i should be $1/p$. Thus, in the hesitant multiplicative circumstance, the product of the l^{th} smallest value in h_{ij} and the l^{th} largest value in h_{ji} should be 1. The second condition defines that the preference degree of the alternative x_i over itself should be 1. The third one states that the lengths of h_{ij} and h_{ji} should be the same.

Example 2.1 If a group of decision makers is asked to give the estimation of the degree to which A_i is preferred to A_j $(i \neq j)$, some DMs give p_{ij}^1, some give p_{ij}^2 and others give p_{ij}^3, where $p_{ij}^1, p_{ij}^2, p_{ij}^3 \in \left(\frac{1}{9}, 9\right)$. Then the preference information h_{ij} that A_i is preferred to A_j is given by $h_{ij} = \{p_{ij}^1, p_{ij}^2, p_{ij}^3\}$. For alternatives A_i and A_k $(i \neq j \neq k)$, some DMs in a group may give p_{ik}^1 and the others give p_{ik}^2.

Then the preference information h_{ij} that A_i is preferred to A_k is given by $h_{ij} = \{p_{ik}^1, p_{ik}^2\}$.

Definition 2.6 [29] According to the definition of HME, it can be seen that the number of assessment values in different HMEs may vary and the assessment values in each HME are usually out of order.

To guarantee that the number of values in different HMEs be equal, we can add elements by the linear combination of the maximal and minimal element in h_{ij} with a parameter λ, shown as:

$$\overrightarrow{h_{ij}} = \lambda h_{ij}^{max} + (1-\lambda) h_{ij}^{min} \quad (4)$$

where $\lambda = 1$ and $\lambda = 0$ means the optimistic attitude and pessimistic attitude of DMs, respectively. $\lambda = \frac{1}{2}$ indicates the neutral attitude of the DMs and the adding element h_{ij} is the average value of h_{ij}.

3 Hesitant fuzzy BCM (HF-BCM)

Because of the hesitancy of the purpose as well as the fuzziness of human mind, human decisions generally clasp the characteristics of opacity. To elaborate hesitancy and vagueness involved in decision making, Hesitant fuzzy sets (HFS) have been introduced by Torra and Narukawa [22,24] as an extension of fuzzy sets [17]**.** HFS allows the membership having a set of several possible values to deal with uncertain information. when the assessment values given by experts are different, we could unify them into an HFE. The features of HFE are very compatible with GDM problems. BCM [16] is a novel MCDM method to calculate the weights of the criteria and alternatives. In the BCM, First, the decision maker selects the base-criterion (preferential) and then a pairwise comparison is made between the base-criterion and other criteria. This technique is much clearer and more accurate because the execution of secondary comparisons is not necessary.

In our proposed methodology, hesitant fuzzy BCM method is developed based on hesitant fuzzy multiplicative preference relation for multi-criteria group decision making (MCGDM).

3.1 Hesitant fuzzy BCM with hesitant fuzzy multiplicative preference relations

In a MCGDM, it is too difficult to determine the weights of the criteria because DMs often have different preferences for criteria and also uses natural language. Linguistic terms such as “Equally important”, “Extremely important” and “Strongly important” are used to make the hesitant pairwise comparisons. The decision makers provide the hesitant fuzzy multiplicative preference relations via pairwise comparison on the n criteria by using the Saaty’s 1/9-9 [11] scale. The rules of transformation are listed in Table 1. Suppose an HFMPRs be

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{n1} \end{pmatrix} \quad (5)$$

Where $h_{ij} = \{p_{ij}^l, l = 1,2, \ldots |h_{ij}|\}$ (hesitant multiplicative element (HME)) unfolds the relative hesitant fuzzy preference (HFP) of criterion i to the criterion j given by the decision makers; $p_{ii} = 1$, when $i = j$.

In hesitant fuzzy BCM, Hesitant fuzzy pairwise comparisons are divided into two parts:

Definition 3.1 If i is the base-criterion, then h_{ij} is called hesitant fuzzy base comparison.

Definition 3.2 If i and j are not base-criterion, then h_{ij} is called hesitant fuzzy final comparison.

The basic principle of BCM [16] tells us that not all hesitant fuzzy pairwise comparisons are required to obtain a complete matrix. There is total n^2 HFMNs in the matrix $H = (h_{ij})_{n\times n}$. But we only require $n-1$ hesitant fuzzy pairwise comparisons (hesitant fuzzy base-comparison). The hesitant fuzzy final comparisons are taken from the hesitant fuzzy base comparisons. Without making the hesitant fuzzy final comparisons, optimal hesitant fuzzy weight values are obtained. The HFWs of criteria and alternatives with respect to various criteria could be derived using HF-BCM.

Now, we elaborate the steps of hesitant fuzzy BCM to determine the optimal hesitant fuzzy weights.

Step 1 Determine the decision criteria set and group of experts

Determine a set of decision criteria $\{C_1, C_2, C_3, \ldots, C_n\}$ and group of experts $\{DM_1, DM_2, \ldots, DM_k\}$ on the basis of which decision is taken.

Step 2 Determine the base-criteria (preferential, selective).

Decision makers select one of the criteria as a base-criteria (preferential) from a set of decision criteria $\{C_1, C_2, C_3, \ldots, C_n\}$ but no comparison is performed in this step.

Step 3 Execute the hesitant fuzzy base-comparisons.

Based on Table 1, the relative hesitant fuzzy preference of the base criteria at the other criteria is derived. The linguistic terms are transformed into HMEs. The resulting vector of hesitant fuzzy base-comparisons as follows.

$$H_B = (h_{B1}, h_{B2}, h_{B3}, \ldots, h_{Bn})$$

Table 1: The Saaty’s Scale.

1/9-9 scale	0.1-0.9 scale	Linguistic terms
1/9	0.1	Extremely not important
1/7	0.2	Very strongly not important
1/5	0.3	Strongly not important
1/3	0.4	Moderately not important
1	0.5	Equally important
3	0.6	Moderately important
5	0.7	Strongly important
7	0.8	Very strongly important
9	0.9	Extremely important
Other values b/w 1/9-9	Other values b/w 0-1	Intermediate values used to present compromise

where H_B unfolds the hesitant fuzzy base-criteria at the other criteria vector, and h_{Bj} unfolds the hesitant fuzzy preference (HFP) of the base-criteria over the j criteria.

Step 4 Normalization of the hesitant fuzzy base-comparison vector.

According to the definition of HME, it can be seen that the number of assessment values in different HMEs may vary and the values in each HME are usually out of order.

Using Equation 4, the hesitant fuzzy base-comparison vector is normalized according to decision-makers' attitude.

Step 5 Derive the normalized hesitant fuzzy optimal weights.

The normalized optimal hesitant fuzzy weights for each $\frac{h_{wB}}{h_{wj}}$ will be equal to h_{Bj} for all j. The optimal hesitant fuzzy weight values can be determined by absolute differences $\left|\frac{h_{wB}}{h_{wj}} - h_{Bj}\right|$ for all j. Regarding the weight values are non-negative, the normalized optimal hesitant fuzzy weights can be determined by deriving the problem as follows:

$$\text{Min max}\left|\frac{h_{wB}}{h_{wj}} - h_{Bj}\right| \le \xi$$

$$\text{Such that} \quad \begin{cases} \sum_{j=1}^{n} R\left(h_{w_j}\right) = 1 \\ p_j^l \ge 0 \; for \; all \; j \end{cases} \qquad (6)$$

Where $h_{w_B} = \{p_B^l, l = 1,2,..,|h_{ij}|\}$, $h_{w_j} = \{p_j^l, l = 1,2,..,|h_{ij}|\}$, $\xi = \{p^{l\xi}, l = 1,2,..,|h_{ij}|\}$

The Equation 6 can be rewritten as the nonlinearly constrained problem.

min ξ

Such that

$$\begin{cases} \left|\frac{h_{wB}}{h_{wj}} - h_{Bj}\right| \le \xi \\ \sum_{j=1}^{n} R\left(h_{w_j}\right) = 1 \\ p_j^l > 0 \; for \; all \; j; l = 1,2,..,|h_{ij}| \end{cases} \qquad (7)$$

Regarding the HMEs and $\xi = \{k^*\}$, Equation 7 can be rewritten as:

min ξ

Such that

$$\begin{cases} \left|\frac{(p_B^l)}{(p_j^l)} - (p_{Bj}^l)\right| \le (k^*) \\ \sum_{j=1}^{n} R\left(h_{w_j}\right) = 1 \\ p_j^l > 0 \; for \; all \; j; l = 1,2,..,|h_{ij}| \end{cases} \qquad (8)$$

Solution of the Equation 8 gives the normalized optimal hesitant fuzzy weights$\{h_{w_1}, h_{w_2}, \dots, h_{w_n}\}$, then these normalized optimal HFWs can be converted to crisp numbers by employing score function. Figure 1 represents the flowchart of the proposed methodology.

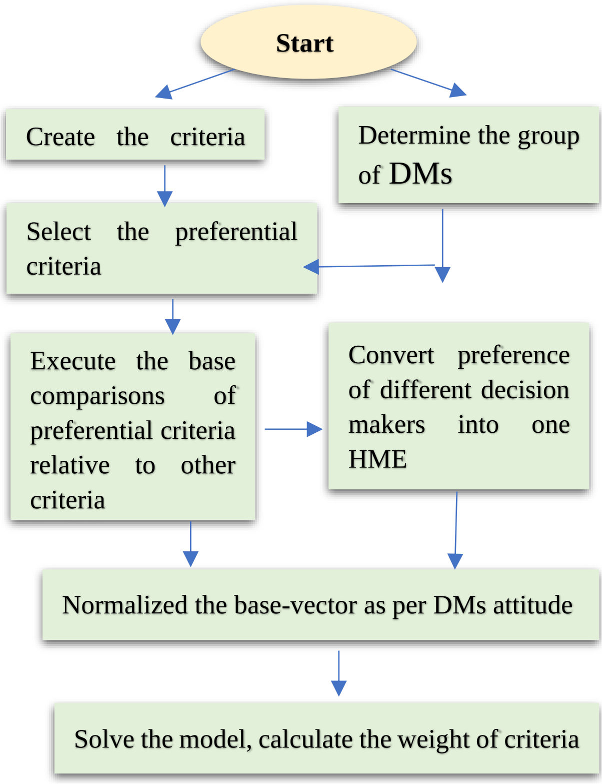

Figure 1: Flowchart of the proposed methodology.

3.2 Consistency for HF-BCM

The hesitant fuzzy pairwise comparison is fully consistent if

$$h_{Base.i} \times h_{ij} = h_{Base.j} \quad \text{for all } i \text{ and } j.$$

The decision maker should pursuance the following principle in entrusting the hesitant fuzzy multiplicative numbers for hesitant fuzzy base-comparisons.

$$\left(\frac{1}{9},\frac{1}{8},\dots\right) \le \{p_{ij}^l, l = 1,2,\dots|h_{ij}|\} \le (8,9,\dots)$$

$$\left(\frac{1}{9},\frac{1}{8},\dots\right) \le \frac{(p_{Bj}^l, l=1,2,..,|h_{ij}|)}{(p_{Bi}^l, l=1,2...,|h_{ij}|)} \le (8,9,\dots) \qquad (9)$$

4 Case study

In this section, we describe the application of hesitant fuzzy BCM by considering simple examples of decision-making problems.

4.1 Case study 1

A company wants to select the best transportation mode to deliver the product to the market place. As a case study, we adopted the example of mode of transport described in [16] and deal the problem by using our proposed hesitant fuzzy BCM method from group decision making process.

There are three criteria chosen for the optimal transport mode selection issue: (a) load flexibility (b) accessibility (c) cost. The group of decision makers chooses cost criterion as the base-criterion. DMs executes hesitant fuzzy base-comparisons based on HMEs using Table 1 group of decision makers provide the estimation of the degree to which cost is preferred to load flexibility, some DMs provide the preference value 6, and others

Table 2: Hesitant fuzzy pairwise comparisons.

Transportation criteria	Load flexibility	Accessibility	Cost
Base-criterion (Cost)	(6,8)	(1,2,3,4)	(1,1,1,1)

Table 3: Scores of criteria for all the three cases.

Cases	$S(h_{w_1})$	$S(h_{w_2})$	$S(h_{w_3})$	ξ
$\lambda = 1$	0.256	0.725	2.016	0.0000008
$\lambda = 1/2$	0.267	0.717	2.010	0.0000003
$\lambda = 0$	0.290	0.711	1.997	0.0000002

provide the preference value 8. Group of decision makers provide the estimation of the degree to which cost is preferred to accessibility, some DMs provide the preference value 2, some DMs provides the preference value 4, some DMs provides the preference value 1 and other provides 3. On the basis which the hesitant fuzzy base-comparison vector can be obtained as:

$$H_B = [(6,8),(1,2,3,4),(1,1,1,1)]$$

Now, the normalized hesitant fuzzy base-comparison vector can be computed by using Equation 4 as follows:

Case-1 Optimistic attitude of DMs ($\lambda = 1$).

$$H_B = [(6,8,8,8),(1,2,3,4),(1,1,1,1)]$$

Case-2 Neutral attitude of DMs ($\lambda = \frac{1}{2}$).

$$H_B = [(6,7,7,8),(1,2,3,4),(1,1,1,1)]$$

Case-3 Pessimistic attitude of DMs ($\lambda = 0$).

$$H_B = [(6,6,6,8),(1,2,3,4),(1,1,1,1)]$$

The maximum length in the normalized hesitant fuzzy preference is 4. Thus, the number of elements in the normalized hesitant fuzzy preference are 4, and the number of several possible values in the normalized hesitant fuzzy optimal weights is four as well.

Based on Table 2 and Case-1, the normalized optimal hesitant fuzzy weight for each criterion can be obtained by solving the non-linear optimization problem as follows:

$$\min \xi$$

Such that

$$\begin{cases} \left|\frac{(p_3^1,p_3^2,p_3^3,p_3^4)}{(p_1^1,p_1^2,p_1^3,p_1^4)} - (p_{31}^1,p_{31}^2,p_{31}^3,p_{31}^4)\right| \le (k^*,k^*,k^*,k^*) \\ \left|\frac{(p_3^1,p_3^2,p_3^3,p_3^4)}{(p_2^1,p_2^2,p_2^3,p_2^4)} - (p_{32}^1,p_{32}^2,p_{32}^3,p_{32}^4)\right| \le (k^*,k^*,k^*,k^*) \\ \left|\frac{(p_3^1,p_3^2,p_3^3,p_3^4)}{(p_3^1,p_3^2,p_3^3,p_3^4)} - (p_{33}^1,p_{33}^2,p_{33}^3,p_{33}^4)\right| \le (k^*,k^*,k^*,k^*) \\ \sum_{j=1}^{n} R\left(h_{w_j}\right) = 1 \\ p_j^l > 0 \; for \; all \; j. \end{cases} \tag{10}$$

By putting HMEs of hesitant fuzzy base-comparison vector in the Equation 10, the nonlinearly constrained optimization problem as follows:

$$\min k^*$$

Such that

$$\begin{cases} p_3^1 - 6 * p_1^1 \le k * p_1^1; p_3^1 - 6 * p_1^1 \ge k * p_1^1; \\ p_3^2 - 8 * p_1^2 \le k * p_1^2; p_3^2 - 8 * p_1^2 \ge k * p_1^2; \\ p_3^3 - 8 * p_1^3 \le k * p_1^3; p_3^3 - 8 * p_1^3 \ge k * p_1^3; \\ p_3^4 - 8 * p_1^4 \le k * p_1^4; p_3^4 - 8 * p_1^4 \ge k * p_1^4; \\ p_3^1 - p_2^1 \le k * p_2^1; p_3^1 - p_2^1 \ge k * p_2^1; \\ p_3^2 - 2 * p_2^2 \le k * p_2^2; p_3^2 - 2 * p_2^2 \ge k * p_2^2; \\ p_3^3 - 3 * p_2^3 \le k * p_2^3; p_3^3 - 3 * p_2^3 \ge k * p_2^3; \\ p_3^4 - 4 * p_2^4 \le k * p_2^4; p_3^4 - 4 * p_2^4 \ge k * p_2^4 \\ \left(\frac{1}{12}\right) * \begin{pmatrix} p_1^1 + p_1^2 + p_1^3 + p_1^4 + p_2^1 + p_2^2 + \\ p_2^3 + p_2^4 + p_3^1 + p_3^2 + p_3^3 + p_3^4 \end{pmatrix} = 1; \\ p_1^1 \le p_1^2 \le p_1^3 \le p_1^4; \\ p_2^1 \le p_2^2 \le p_2^3 \le p_2^4; \\ p_3^1 \le p_3^2 \le p_3^3 \le p_3^4; \\ p_1^1, p_2^1, p_3^1, p_4^1 > 0; \\ k^* \ge 0; \end{cases} \tag{11}$$

After solving the Equation 11, the normalized optimal HFWs of criteria are obtained, which are:

$$h_{w_1} = \{0.081,0.162,0.292,0.492\};$$
$$h_{w_2} = \{0.489,0.649,0.779,0.985\};$$
$$h_{w_3} = \{0.489,1.299,2.337,3.941\};$$

By employing score function, the crisp weights of normalized optimal HFWs are obtained.

Load flexibility$S(h_{w_1}) = 0.256$;

Accessibility $S(h_{w_2}) = 0.725$; Cost $S(h_{w_3}) =$ 2.016;

Similarly, the normalized optimal HFWs of criteria are obtained for all the cases. Table 3 shows the calculations.

It can be noted from Table 4 that for BCM [16] and HF-BCM criteria have the same preference order, but there is a slight difference in the criteria weights. Due to $\xi = 0$ (0.00000008), regardless of any values for the consistency index, the consistency ratio is optimal. Also, the proposed method is even better than the HF-BWM [27] in terms of consistency. Since the consistency ratio is the closest value to zero. This example unfolds that the hesitant fuzzy BCM method could consider the ambiguity of DMs in the process of group decision making.

Table 4: Comparison of results.

Methods	**Pairwise comparisons**	**Weights of criterion**	**Consistency**
HF-BCM (Proposed)	HMPRs (1/9-9 scale)	0.267 0.717 2.010	0.0000
BCM [16]	1/9-9 scale	0.076 0.307 0.615	0.0000
BWM [14]	1-9 scale	0.071 0.338 0.589	0.0580
HF-BWM [27]	HMPRs (1-9 scale)	0.413 1.120 1.465	0.0558

4.2 Case study 2

We consider the example of car selection which is handled by using BCM method [16]. In this case study, we solve the same problem by our proposed method HF-BCM from group decision making process. As the number of criteria increases, it becomes difficult to specify values for relative preference in base-comparison. There are six criteria chosen for the car selection problem: (1) convenience (2) fuel consumption (3) safety (4) style (5) acceleration (6) consumer price. The group of decision makers chooses safety criterion as the base-criterion. DMs executes hesitant fuzzy base-comparisons based on HMEs using Table 1. On the basis which the hesitant fuzzy base-comparison vector can be obtained as:

$$H_B=\left[(1,2),\left(\frac{1}{3},\frac{1}{2}\right),(1,1,1),(2,4,6),\left(\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{3}\right)\right]$$

The maximum length in the normalized hesitant fuzzy preference is 3. So, the number of elements in the normalized hesitant fuzzy preference are 3.

The normalized hesitant fuzzy base-comparison vector

can be computed by using Equation 4 as follows:

Case-1 Optimistic attitude of DMs ($\lambda=1$).

$$H_B=\begin{bmatrix}(1,2,2),\left(\frac{1}{3},\frac{1}{3},\frac{1}{2}\right),(1,1,1),(2,4,6),\\ \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{3},\frac{1}{3}\right)\end{bmatrix}$$

Case-2 Neutral attitude of DMs $\left(\lambda={}^{1}/_{2}\right)$.

$$H_B=\begin{bmatrix}\left(1,\frac{1}{2},2\right),\left(\frac{1}{3},\frac{5}{12},\frac{1}{2}\right),(1,1,1),(2,4,6),\\ \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{7}{24},\frac{1}{3}\right)\end{bmatrix}$$

Case-3 Pessimistic attitude of DMs ($\lambda=0$).

$$H_B=\begin{bmatrix}(1,1,2),\left(\frac{1}{3},\frac{1}{3},\frac{1}{2}\right),(1,1,1),(2,4,6),\\ \left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{4},\frac{1}{4},\frac{1}{3}\right)\end{bmatrix}$$

Based on Table 5 and Case-1, the normalized optimal hesitant fuzzy weight for each criterion can be obtained by solving the non-linear optimization problem as follows: $\min \xi$

Such that

$$\begin{cases}\left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_1^1,p_1^2,p_1^3)}-(p_{31}^1,p_{31}^2,p_{31}^3)\right|\le(k^*,k^*,k^*)\\ \left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_2^1,p_2^2,p_2^3)}-(p_{32}^1,p_{32}^2,p_{32}^3)\right|\le(k^*,k^*,k^*)\\ \left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_3^1,p_3^2,p_3^3)}-(p_{33}^1,p_{33}^2,p_{33}^3)\right|\le(k^*,k^*,k^*)\\ \left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_4^1,p_4^2,p_4^3)}-(p_{34}^1,p_{34}^2,p_{34}^3)\right|\le(k^*,k^*,k^*)\\ \left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_5^1,p_5^2,p_5^3)}-(p_{35}^1,p_{35}^2,p_{35}^3)\right|\le(k^*,k^*,k^*)\\ \left|\frac{(p_3^1,p_3^2,p_3^3)}{(p_6^1,p_6^2,p_6^3)}-(p_{36}^1,p_{36}^2,p_{36}^3)\right|\le(k^*,k^*,k^*)\\ \sum_{j=1}^{n}R\left(h_{w_j}\right)=1\\ p_j^l>0\ for\ all\ j.\end{cases} \quad (12)$$

By putting HMEs of hesitant fuzzy base-comparison vector in the Equation 12, the nonlinearly constrained optimization problem as follows:

$\min k^*$

Such that

$$\begin{cases}p_3^1-1*p_1^1\le k*p_1^1;p_3^1-1*p_1^1\ge k*p_1^1;\\ p_3^2-2*p_1^2\le k*p_1^2;p_3^2-2*p_1^2\ge k*p_1^2;\\ p_3^3-2*p_1^3\le k*p_1^3;p_3^3-2*p_1^3\ge k*p_1^3;\\ p_3^1-\frac{1}{3}*p_2^1\le k*p_2^1;p_3^1-\frac{1}{3}*p_2^1\ge k*p_2^1;\\ p_3^2-\frac{1}{2}*p_2^2\le k*p_2^2;p_3^2-\frac{1}{2}*p_2^2\ge k*p_2^2;\\ p_3^3-\frac{1}{2}*p_2^3\le k*p_2^3;p_3^3-\frac{1}{2}*p_2^3\ge k*p_2^3;\\ p_3^1-2*p_4^1\le k*p_4^1;p_3^1-2*p_4^1\ge k*p_4^1;\\ p_3^2-4*p_4^2\le k*p_4^2;p_3^2-4*p_4^2\ge k*p_4^2;\\ p_3^3-6*p_4^3\le k*p_4^3;p_3^3-6*p_4^3\ge k*p_4^3;\\ p_3^1-\frac{1}{2}*p_5^1\le k*p_5^1;p_3^1-\frac{1}{2}*p_5^1\ge k*p_5^1;\\ p_3^2-\frac{1}{2}*p_5^2\le k*p_5^2;p_3^2-\frac{1}{2}*p_5^2\ge k*p_5^2;\\ p_3^3-\frac{1}{2}*p_5^3\le k*p_5^3;p_3^3-\frac{1}{2}*p_5^3\ge k*p_5^3;\\ p_3^1-\frac{1}{4}*p_6^1\le k*p_6^1;p_3^1-\frac{1}{4}*p_6^1\ge k*p_6^1;\\ p_3^2-\frac{1}{3}*p_6^2\le k*p_6^2;p_3^2-\frac{1}{3}*p_6^2\ge k*p_6^2;\\ p_3^3-\frac{1}{3}*p_6^3\le k*p_6^3;p_3^3-\frac{1}{3}*p_6^3\ge k*p_6^3;\\ \left(\frac{1}{18}\right)*\begin{pmatrix}p_1^1+p_1^2+p_1^3+p_2^1+p_2^2+p_2^3+p_3^1\\ +p_3^2+p_3^3+p_4^1+p_4^2+p_4^3+p_5^1+p_5^2\\ +p_5^3+p_6^1+p_6^2+p_6^3\end{pmatrix}=1;\\ p_1^1\le p_1^2\le p_1^3;\\ p_2^1\le p_2^2\le p_2^3;\\ p_3^1\le p_3^2\le p_3^3;\\ p_4^1\le p_4^2\le p_4^3;\\ p_5^1\le p_5^2\le p_5^3;\\ p_6^1\le p_6^2\le p_6^3;\\ p_1^1,p_2^1,p_3^1>0;\\ k\ge 0\end{cases} \quad (13)$$

Table 5: Hesitant fuzzy pairwise comparisons.

Criteria	**Base-criterion (Safety)**
Convenience	(1,2)
Fuel	(1/3,1/2)
Safety	(1,1,1)
Style	(2,4,6)
Acceleration	(1/2)
Price	(1/4,1/3)

Table 6: Scores of criteria for all the three cases.

Cases	$\lambda = 1$	$\lambda = 1/2$	$\lambda = 0$
ξ	0.00000	0.00000	0.00000
$S(h_{w_1})$	0.247	0.343	0.283
$S(h_{w_2})$	0.937	0.921	0.984
$S(h_{w_3})$	0.440	0.428	0.412
$S(h_{w_4})$	0.102	0.088	0.093
$S(h_{w_5})$	0.881	0.856	0.824
$S(h_{w_6})$	1.384	1.360	1.400

After solving the Equation 13, the normalized optimal HFWs of criteria are obtained, which are:

$$h_{w_1} = \{0.161, 0.210, 0.370\};$$
$$h_{w_2} = \{0.487, 0.841, 1.483\};$$
$$h_{w_3} = \{0.160, 0.420, 0.741\};$$
$$h_{w_4} = \{0.080, 0.105, 0.123\};$$
$$h_{w_5} = \{0.321, 0.841, 1.483\};$$
$$h_{w_6} = \{0.643, 1.271, 2.242\};$$

By employing score function, the crisp weights of normalized optimal HFWs are obtained.

Convenience = 0.247, fuel consumption = 0.937, safety = 0.440, style = 0.102, acceleration = 0.881, price = 1.384, ξ = 0.00000007;

Consistency ratio is optimal because ξ = 0 (0.00000007).

Table 6 shows the normalized optimal HFWs of criteria for all the cases.

5 Conclusion

Multi-criteria group decision making problems often have strong uncertainty, which is characterized as fuzziness within the group decision-making. When more than one expert evaluates an option, it is very possible for them to have different opinions. In our proposed methodology, we have developed a unique approach hesitant fuzzy Base-criterion method which is an extension of latest MCDM method BCM in the hesitant situation for multi-criteria group decision making. HFS has its unique advantages in a decision-making problem with multi-criteria and multiple individuals. Employing linguistic variables is more worthy than crisp values, in order to make a base-comparison for criteria and alternatives in the group decision-making process.

The base-comparisons values manifested by linguistic terms could be converted into HMEs that are used in the nonlinear optimization problem and then normalized optimal HFWs of criteria can be obtained. The normalized optimal hesitant fuzzy weights of criteria can be transferred to crisp values by employing score function. The suitability and applicability of the developed HF-BCM is verified by discussing the optimal transportation mode selection problem and car selection problem. In terms of strength and direction, the results of HF-BCM are completely consistent if decision makers have selected HMEs for pairwise-comparisons based on Equation 9. The proposed method HF-BCM has several important features as follows:

- Integration of Base-criterion method with hesitant fuzzy circumstances is novel and provides more reliable and accurate weights of criteria and alternatives for MCGDM.
- Hesitant fuzzy set has its unique advantages over the other methods. In MCGDM, when the assessment values given by DMs vary, we can unify them into one HME. Also, BCM requires fewer comparisons to calculate the weights of criteria and alternative. In this way, the process of calculating lots of group decision making problems is effectively simplified.
- Compared to the BCM method, the hesitant fuzzy BCM method also uses linguistic terms to do base-comparisons. Use of linguistic variables makes pairwise-comparisons more accurate and easier. HF-BCM can gain the more reliable weights.
- The hesitant fuzzy BCM needs less number of pairwise comparisons and highly consistent rather than the hesitant fuzzy AHP and hesitant fuzzy BWM methods.

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