Multiple Attribute Decision Making Method Based on the Trapezoid Fuzzy Linguistic Hybrid Harmonic Averaging Operator

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A new method is proposed to solve the multiple attribute decision making (MADM) problems with the trapezoid fuzzy linguistic variables (TFLVs) based on the trapezoid fuzzy linguistic hybrid harmonic averaging ($TFLHHA$) operator. To begin with, this paper reviews the concept and operational rules of the TFLVs, the calculation method of the possibility degree with TFLVs, and the comparison method of TFLVs. Then, some operators are proposed, in order to aggregate the TFLVs, such as the trapezoid fuzzy linguistic weighted harmonic averaging ($TFLWHA$) operator, the trapezoid fuzzy linguistic ordered weighted harmonic averaging ($TFLOWHA$) operator, and the trapezoid fuzzy linguistic hybrid harmonic averaging ($TFLHHA$) operator. Furthermore, based on the $TFLHHA$ operator, a new method solving the MADM problems with the TFLVs is proposed. Finally, an illustrative example is given to show the decision making steps, and it verifies the effectiveness of the developed method.

Povzetek: Članek opisuje metodo za podporo odločanju, ki uporablja mehko logiko.

1 Introduction

In the process of the multiple attribute decision making (MADM), the decision making information, given by the decision makers, often takes the form of the linguistic variables, because of the complexity and uncertainty of the objective things, and the ambiguity of human thinking. Therefore, the MADM under the linguistic context is an interesting research topic which has been receiving more and more attention in recent years [1-4]. Some operators were widely used to aggregate the decision making information in the process of the MADM. Bordogna et al. [5] developed a model within fuzzy set theory by the linguistic ordered weighted average ($LOWA$) operators for the group decision making in the linguistic context. Xu [6] proposed an approach to solve the multiple attribute group decision making problems with the uncertain linguistic information, based on the uncertain linguistic ordered weighted averaging ($ULOWA$) operator and the uncertain linguistic hybrid aggregation ($ULHA$) operator. Wu and Chen [7] introduced the linguistic weighted arithmetic averaging ($LWAA$) operator to aggregate the decision making information which took the form of the linguistic variables. Xu [8] developed some operators for aggregating the triangular fuzzy linguistic variables, such as the fuzzy linguistic averaging (FLA) operator, the fuzzy linguistic weighted averaging ($FLWA$) operator, the fuzzy linguistic ordered weighted averaging ($FLOWA$) operator, and the induced $FLOWA$ ($IFLOWA$) operator.

But in the real situation, the decision-makers sometimes can only provide the decision making information in the form of the trapezoid fuzzy linguistic variables (TFLVs). The trapezoid fuzzy linguistic variable (TFLV) generalizes the linguistic variable, the uncertain linguistic variable and the triangular fuzzy linguistic variable. So the research on the MADM problems with the TFLVs is very significant. But the related decision making methods based on the TFLVs are less. Xu [9] proposed the trapezoid fuzzy linguistic weighted averaging ($TFLWA$) operator to aggregate all the decision making information corresponding to each alternative, and he used the similarity measure to rank the decision alternatives and then the most desirable one is selected. Liang and Chen [10] proposed the trapezoid fuzzy linguistic weighted averaging (TFLWA) operator to aggregate the decision making information, and then all the alternatives were ranked by comparing the possibility degree of the TFLV.

Based on these, this paper extends the $OWHA$ operator [11] and the $UCWHA$ operator [12] to deal with the MADM problems with the trapezoid fuzzy linguistic information, such as the trapezoid fuzzy linguistic weighted harmonic averaging ($TFLWHA$) operator, and the trapezoid fuzzy linguistic ordered weighted harmonic averaging ($TFLOWHA$) operator. The $TFLWHA$ operator only focuses on the attribute weight itself, but it ignores the position weight with respect to the attribute value; and the $TFLOWHA$ operator focuses on the position weight with respect to the attribute value, but it ignores the weight of the attribute value itself. The two operators are one-sided. So in order to avoid the disadvantage of the two operators, the trapezoid fuzzy linguistic hybrid harmonic averaging ($TFLHHA$) operator is proposed to aggregate the attribute values which take the form of the TFLVs. According to the $TFLHHA$ operator, the new method is proposed, which can solve MADM problems with the TFLVs directly.

To do so, the remainder of this paper is structured as follows: In section 2, this paper reviews the concept and the operational rules of the TFLVs, and introduces the comparison method of the TFLVs, in which the calculation method of the possibility degree with the TFLVs is reviewed. In section 3, three operators are proposed in order to aggregate the TFLVs, such as the $TFLWHA$ operator, the $TFLOWHA$ operator, and the $TFLHHA$ operator. In section 4, the decision making steps of the new method is proposed based on the $TFLHHA$ operator. In section 5, an illustrative example is given to show the decision making steps, and it verifies the effectiveness of the developed method. The section 6 concludes this paper.

2 The Trapezoid Fuzzy Linguistic Variables

2.1 The definition of the trapezoid fuzzy linguistic variables

Let $S=\{s_i \mid i=1,2,\cdots,t\}$ be a linguistic term set with odd cardinality, any label s_i represents a possible value of the linguistic variable. Especially, s_1 and s_t represent the lower and the upper values of the linguistic terms, respectively. For example, a linguistic term set S could be given as follows:

S = { s_1 =extremely poor, s_2 =very poor, s_3 =poor, s_4 =slightly poor, s_5 =fair, s_6 =slightly good, s_7 =good, s_8 =very good, s_9 =extremely good}

Usually, in these cases, s_i and s_j must satisfy the following additional characteristics [13]:

(1) The set S is ordered: s_i is worse than s_j, if $i<j$;

(2) Maximum operator: $\max(s_i, s_j)=s_i$, if $s_i \ge s_j$;

(3) Minimum operator: $\min(s_i, s_j)=s_j$, if $s_i \ge s_j$.

Some calculation results, however, may not exactly match any linguistic labels in S in the calculation process. To preserve all the given information, the discrete term set S is extended to a continuous term set $\bar{S}=\{s_i \mid s_0 \le s_i \le s_q, i\in[0,q]\}$, where s_i meets all the characteristics above and $q(q>t)$ is a sufficient large positive integer. If $s_i \in S$, then we call s_i the original term, otherwise, we call s_i the virtual term. In general, the decision makers use the original linguistic terms to evaluate the alternatives, and the virtual linguistic terms can only appear in the process of the operation and ranking [13].

Definition 2.1[8]: Let $s_\alpha, s_\beta \in \bar{S}$, then we defined the distance between s_α and s_β as:

$$d(s_\alpha, s_\beta)=|\alpha-\beta| \tag{1}$$

Definition 2.2[8]: Let $\tilde{s}=[s_\alpha, s_\beta, s_\gamma, s_\eta]\in\tilde{S}$, where $s_\alpha, s_\beta, s_\gamma, s_\eta \in \bar{S}$, and the subscripts $\alpha, \beta, \gamma, \eta$ are non-decreasing numbers, and s_β and s_γ indicate the interval in which the membership value is 1, with s_α and s_η indicating the lower and upper values of $\tilde{s}$, respectively, then $\tilde{s}$ is called the trapezoid fuzzy linguistic variable (TFLV), which is characterized by the following membership function (see Figure 1):

$$\mu_{\tilde{s}}(\theta)=\begin{cases} 0 & s_0 \le s_\theta \le s_\alpha \\ \dfrac{d(s_\theta, s_\alpha)}{d(s_\beta, s_\alpha)} & s_\alpha \le s_\theta \le s_\beta \\ 1 & s_\beta \le s_\theta \le s_\gamma \\ \dfrac{d(s_\theta, s_\eta)}{d(s_\gamma, s_\eta)} & s_\gamma \le s_\theta \le s_\eta \\ 0 & s_\eta \le s_\theta \le s_q \end{cases} \tag{2}$$

where $\tilde{S}$ is the set of all the trapezoid fuzzy linguistic variables. Especially, if any two of $\alpha, \beta, \gamma, \eta$ are equal, then $\tilde{s}$ is reduced to a triangular fuzzy linguistic variable; if any three of $\alpha, \beta, \gamma, \eta$ are equal, then $\tilde{s}$ is reduced to an uncertain linguistic variable [8].

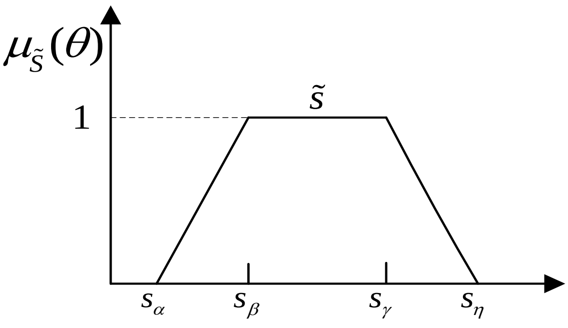

Figure 1 A trapezoid fuzzy linguistic variable $\tilde{s}$

2.2 The operational rules and characteristics of the trapezoid fuzzy linguistic variables

Let $\tilde{s}=[s_\alpha,s_\beta,s_\gamma,s_\eta]$, $\tilde{s}_1=[s_{\alpha_1},s_{\beta_1},s_{\gamma_1},s_{\eta_1}]$ and $\tilde{s}_2=[s_{\alpha_2},s_{\beta_2},s_{\gamma_2},s_{\eta_2}]\in\tilde{S}$ be any three trapezoid fuzzy linguistic variables, and $\lambda\in[0,1]$ and $\lambda_1\in[0,1]$, then their operational rules are defined as follows:

(1) $\tilde{s}_1\oplus\tilde{s}_2=[s_{\alpha_1},s_{\beta_1},s_{\gamma_1},s_{\eta_1}]\oplus[s_{\alpha_2},s_{\beta_2},s_{\gamma_2},s_{\eta_2}]$

$=[s_{\alpha_1+\alpha_2},s_{\beta_1+\beta_2},s_{\gamma_1+\gamma_2},s_{\eta_1+\eta_2}]$;

(2) $\lambda\tilde{s}=\lambda[s_\alpha,s_\beta,s_\gamma,s_\eta]$;

$=[s_{\lambda\alpha},s_{\lambda\beta},s_{\lambda\gamma},s_{\lambda\eta}]$

(3) if $0<\alpha\leq\beta\leq\gamma\leq\eta$, then

$$1/\tilde{s}=(\tilde{s})^{-1}=[\,1/s_\eta,\ 1/s_\gamma,\ 1/s_\beta,\ 1/s_\alpha\,]$$

$$=[s_{1/\eta},s_{1/\gamma},s_{1/\beta},s_{1/\alpha}].$$

In addition, the trapezoid fuzzy linguistic variables have the following characteristics:

(1) $\tilde{s}_1\oplus\tilde{s}_2=\tilde{s}_2\oplus\tilde{s}_1$;

(2) $(\lambda\oplus\lambda_1)\tilde{s}=\lambda\tilde{s}\oplus\lambda_1\tilde{s}$;

(3) $\lambda(\tilde{s}\oplus\tilde{s}_1)=\lambda\tilde{s}\oplus\lambda\tilde{s}_1$.

2.3 The comparison method of the trapezoid fuzzy linguistic variables

Definition 2.3[10]: Let $\tilde{s}_1=[s_{\alpha_1},s_{\beta_1},s_{\gamma_1},s_{\eta_1}]$ and $\tilde{s}_2=[s_{\alpha_2},s_{\beta_2},s_{\gamma_2},s_{\eta_2}]$ be two trapezoid fuzzy linguistic variables, then the possibility degree of $\tilde{s}_1\geq\tilde{s}_2$ is defined as follows:

$$p(\tilde{s}_1\geq\tilde{s}_2)=\min\{\max\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)-(\alpha_2+\beta_2)},0\},1\}$$

(3)

Example 1: Let $\tilde{s}_1=[s_2,s_3,s_5,s_6]$ and $\tilde{s}_2=[s_4,s_5,s_8,s_9]$ be two trapezoid fuzzy linguistic variables, then the possibility degree of $\tilde{s}_1\geq\tilde{s}_2$ is:

$$p(\tilde{s}_1\geq\tilde{s}_2)=\min\{\max\{\frac{(5+6)-(4+5)}{(5+6)-(2+3)+(8+9)-(4+5)},0\},1\}$$

$$=\min\{\max\{0.143,0\},1\}=0.143$$

The characteristics of the possibility degree $p(\tilde{s}_1\geq\tilde{s}_2)$ are shown as follows [10]:

Let $\tilde{s}_1=[s_{\alpha_1},s_{\beta_1},s_{\gamma_1},s_{\eta_1}]$, $\tilde{s}_2=[s_{\alpha_2},s_{\beta_2},s_{\gamma_2},s_{\eta_2}]$, $\tilde{s}_3=[s_{\alpha_3},s_{\beta_3},s_{\gamma_3},s_{\eta_3}]$ be any three trapezoid fuzzy linguistic variables, then

(1) $0\leq p(\tilde{s}_1\geq\tilde{s}_2)\leq1$, $0\leq p(\tilde{s}_2\geq\tilde{s}_1)\leq1$;

(2) $p(\tilde{s}_1\geq\tilde{s}_2)+p(\tilde{s}_2\geq\tilde{s}_1)=1$.

Especially, if $p(\tilde{s}_1\geq\tilde{s}_2)=p(\tilde{s}_2\geq\tilde{s}_1)$, then $p(\tilde{s}_1\geq\tilde{s}_2)=p(\tilde{s}_2\geq\tilde{s}_1)=\frac{1}{2}$;

(3) if $p(\tilde{s}_1\geq\tilde{s}_2)\geq\frac{1}{2}$, and $p(\tilde{s}_2\geq\tilde{s}_3)\geq\frac{1}{2}$, then $p(\tilde{s}_1\geq\tilde{s}_3)\geq\frac{1}{2}$;

(4) if $p(\tilde{s}_1\geq\tilde{s}_2)\geq\frac{1}{2}$, and $p(\tilde{s}_2\geq\tilde{s}_3)\geq\frac{1}{2}$, then $p(\tilde{s}_1\geq\tilde{s}_2)+p(\tilde{s}_2\geq\tilde{s}_3)\geq p(\tilde{s}_1\geq\tilde{s}_3)$

Let $\tilde{s}_i$ and $\tilde{s}_j$ be two trapezoid fuzzy linguistic variables, then the steps of the comparison method are shown as follows:

(1) Utilize the formula (3) to compare the size of $\tilde{s}_i$ and $\tilde{s}_j$, and suppose that $p_{ij}=p(\tilde{s}_i\geq\tilde{s}_j)$, then we can contribute the possibility degree matrix $P=(p_{ij})_{n\times n}$,where $p_{ij}\geq0$, $p_{ij}+p_{ji}=1$, $p_{ii}=\frac{1}{2}$, $i,j=1,2,\cdots,n$. We can easily obtain the result that the matrix $P=(p_{ij})_{n\times n}$ is the complimentary judgment matrix [14].

(2) Sum all the elements of each rows of the possibility degree matrix, and rank the orders of the trapezoid fuzzy linguistic variables based on the values p_i , where

$p_i = \sum_{j=1}^{n} p_{ij}$ ($i = 1, 2, \cdots, n$). The larger the value of p_i is, the larger the trapezoid fuzzy linguistic variable $\tilde{s}_i$ is.

Example 2: Let $\tilde{s}_1 = [s_2, s_3, s_5, s_6]$ and $\tilde{s}_2 = [s_4, s_5, s_8, s_9]$ be two trapezoid fuzzy linguistic variables, then we can compare the size of $\tilde{s}_1$ with $\tilde{s}_2$:

(1) The possibility degree of $\tilde{s}_1 \geq \tilde{s}_2$ is:

$$p(\tilde{s}_1 \geq \tilde{s}_2) = \min\{\max\{\frac{(5+6)-(4+5)}{(5+6)-(2+3)+(8+9)-(4+5)}, 0\}, 1\}$$

$$= \min\{\max\{0.143, 0\}, 1\} = 0.143$$

and the possibility degree of $\tilde{s}_2 \geq \tilde{s}_1$ is:

$$p(\tilde{s}_2 \geq \tilde{s}_1) = \min\{\max\{\frac{(8+9)-(2+3)}{(8+9)-(4+5)+(5+6)-(2+3)}, 0\}, 1\}$$

$$= \min\{\max\{0.857, 0\}, 1\} = 0.857$$

Then we can contribute the possibility degree matrix:

$$P = (p_{ij})_{2\times 2} = \begin{bmatrix} 0.5 & 0.143 \\ 0.857 & 0.5 \end{bmatrix}$$

(2) $p_1 = \sum_{j=1}^{2} p_{1j} = 0.5 + 0.143 = 0.643$,

$p_2 = \sum_{j=1}^{2} p_{2j} = 0.875 + 0.5 = 1.375$,

so $p_1 < p_2$.

Then, we can get that: $\tilde{s}_1 < \tilde{s}_2$ ($\tilde{s}_1$ is worse than $\tilde{s}_2$).

3 Some Harmonic Operators with the Trapezoid Fuzzy Linguistic Variables

Definition 3.1: Let $TFLWHA: \tilde{S}^n \to \tilde{S}$, if

$$TFLWHA_w(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = (\sum_{j=1}^{n} \frac{w_j}{\tilde{s}_j})^{-1} \quad (4)$$

where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables, and $\tilde{s}_j \in \tilde{S}$ ($j = 1, 2, \cdots, n$) is the trapezoid fuzzy linguistic variable. $w = (w_1, w_2, \cdots, w_n)$ is the weight vector, and w_i is the weight of $\tilde{s}_i$, where $w_i \geq 0$, $i = 1, 2, \cdots, n$, $\sum_{i=1}^{n} w_i = 1$, then $TFLWHA$ is called the trapezoid fuzzy linguistic weighted harmonic averaging ($TFLWHA$)operator.

Example 3: If $\tilde{s}_1 = [s_2, s_3, s_5, s_6]$ $\tilde{s}_2 = [s_4, s_5, s_8, s_9]$ $\tilde{s}_3 = [s_5, s_6, s_7, s_9]$ and $\tilde{s}_4 = [s_3, s_4, s_5, s_7] \in \tilde{S}$ are four trapezoid fuzzy linguistic variables, and $w = (0.3, 0.2, 0.1, 0.4)$ is the weight vector, then

$$TFLWHA_w(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{s}_4) = (\sum_{j=1}^{4} \frac{w_j}{\tilde{s}_j})^{-1}$$

$$= (\frac{0.3}{[s_2, s_3, s_5, s_6]} \oplus \frac{0.2}{[s_4, s_5, s_8, s_9]} \oplus \frac{0.1}{[s_5, s_6, s_7, s_9]} \oplus \frac{0.4}{[s_3, s_4, s_5, s_7]})^{-1}$$

$$= ([s_{0.05}, s_{0.06}, s_{0.1}, s_{0.15}) \oplus (s_{0.022}, s_{0.025}, s_{0.04}, s_{0.05}] \oplus$$

$$[s_{0.011}, s_{0.014}, s_{0.017}, s_{0.02}] \oplus [s_{0.057}, s_{0.08}, s_{0.1}, s_{0.133}])^{-1}$$

$$= [s_{0.14}, s_{0.179}, s_{0.257}, s_{0.353}]^{-1} = [s_{2.833}, s_{3.891}, s_{5.587}, s_{7.143}]$$

Definition 3.2: Let $TFLOWHA: \tilde{S}^n \to \tilde{S}$, if

$$TFLOWHA_\omega(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = (\sum_{j=1}^{n} \frac{\omega_j}{\tilde{r}_j})^{-1} \quad (5)$$

where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables , and $\tilde{s}_j$, $\tilde{r}_j \in \tilde{S}$ ($j = 1, 2, \cdots, n$) are the trapezoid fuzzy linguistic variables. $\tilde{r}_j$ is the j^{th} largest of $\tilde{s}_i$ ($i = 1, 2, \cdots, n$), and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the position weight vector with $TFLOWHA$, where $\omega_j \geq 0$, $j = 1, 2, \cdots, n$, $\sum_{j=1}^{n} \omega_j = 1$, then $TFLOWHA$ is called the trapezoid fuzzy linguistic ordered weighted harmonic averaging ($TFLOWHA$)operator.

The characteristic of the $TFLOWHA$ operator is: Firstly, The order of the trapezoid fuzzy linguistic variables is ranked, then the position weights are aggregated with them, but there is no relationship between ω_j and $\tilde{s}_j$, and ω_j is only associated with the j^{th} position in the aggregation process, so $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is called the position weight vector.

According to the real situation, the position weight vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is determined. In this paper, the position weight is determined by the method which proposed in literature [15]. The formula is shown as follows:

$$\omega_{i+1} = \frac{C_{n-1}^{i}}{2^{n-1}}, i = 0, 1, \cdots, n-1 \quad (6)$$

Example 4: Let $\tilde{s}_1 = [s_2, s_3, s_5, s_6]$ and $\tilde{s}_2 = [s_4, s_5, s_8, s_9]$ be two trapezoid fuzzy linguistic variables, and we already know that $\tilde{s}_1 < \tilde{s}_2$ (the calculation steps are shown in **Example 1**), then the position weight vector is $\omega = (\frac{C_{2-1}^{0}}{2^{2-1}}, \frac{C_{2-1}^{1}}{2^{2-1}}) = (0.5, 0.5)$,

$$TFLOWHA_{\omega}(\tilde{s}_1,\tilde{s}_2)=(\frac{0.5}{[s_2,s_3,s_5,s_6]}\oplus\frac{0.5}{[s_4,s_5,s_8,s_9]})^{-1}$$
$$=(\frac{0.5}{[s_2,s_3,s_5,s_6]}\oplus\frac{0.5}{[s_4,s_5,s_8,s_9]})^{-1}$$
$$=([s_{0.083},s_{0.1},s_{0.167},s_{0.25}]\oplus[s_{0.056},s_{0.0625},s_{0.1},s_{0.125}])^{-1}$$
$$=[s_{0.139},s_{0.1625},s_{0.267},s_{0.375}]^{-1}$$
$$=[s_{2.667},s_{3.745},s_{6.154},s_{7.217}]$$

The $TFLWHA$ operator only focuses on the weight of the attribute value itself, but it ignores the position weight with respect to the attribute value; and the $TFLOWHA$ operator focuses on the position weight with respect to the attribute value, but it ignores the weight of the attribute value itself. The two operators are one-sided. If the decision makers use these operators to aggregate the decision making information, some information may be lost. So in order to avoid the disadvantage of the two operators, the trapezoid fuzzy linguistic hybrid harmonic averaging ($TFLHHA$) operator is defined as follows:

Definition 3.3: Let $TFLHHA:\tilde{S}^n\rightarrow\tilde{S}$, if

$$TFLHHA_{\omega,w}(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n)=(\sum_{j=1}^{n}\frac{\omega_j}{\tilde{r}_j})^{-1} \tag{7}$$

where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables , and $\tilde{s}_i$, $\tilde{r}_j\in\tilde{S}$ ($i,j=1,2,\cdots,n$) are the trapezoid fuzzy linguistic variables. $\tilde{r}_j$ is the j^{th} largest of $\tilde{s}_i/nw_i$ ($i=1,2,\cdots,n$), where $w=(w_1,w_2,\cdots,w_n)$ is the weight vector, and w_i is the weight of $\tilde{s}_i$, $w_i\geq 0$ ($i=1,2,\cdots,n$), $\sum_{i=1}^{n}w_i=1$, and n is the balancing coefficient. $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ is the position weight vector with $TFLHHA$, where $\omega_j\geq 0$ ($j=1,2,\cdots,n$), $\sum_{j=1}^{n}\omega_j=1$, then $TFLHHA$ is called the trapezoid fuzzy linguistic hybrid harmonic averaging ($TFLHHA$) operator.

Example 5: Let $\tilde{s}_1=[s_2,s_3,s_5,s_6]$ and $\tilde{s}_2=[s_4,s_5,s_8,s_9]$ be two trapezoid fuzzy linguistic variables. We already know that the position weight vector is $\omega=(0.5,0.5)$ (the calculation steps are shown in **Example 4**), and the weight vector is $w=(0.3,0.7)$, given by the decision makers, then based on the method shown in section 2.2, we can calculate that: $\tilde{r}_1=\tilde{s}_1/2w_1=[s_{3.333},s_5,s_{8.333},s_{10}]$, and $\tilde{r}_2=\tilde{s}_2/2w_2=[s_{2.857},s_{3.571},s_{5.714},s_{6.429}]$,

Then,

$$TFLHHA_{\omega,w}(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n)=(\sum_{j=1}^{n}\frac{\omega_j}{\hat{r}_j})^{-1}$$
$$=(\frac{0.5}{[s_{3.333},s_5,s_{8.333},s_{10}]}\oplus\frac{0.5}{[s_{2.857},s_{3.571},s_{5.714},s_{6.429}]})^{-1}$$
$$=([s_{0.05},s_{0.06},s_{0.1},s_{0.15}]\oplus[s_{0.0778},s_{0.0875},s_{0.14},s_{0.175}])^{-1}$$
$$=(s_{0.1278},s_{0.1475},s_{0.24},s_{0.325})^{-1}$$
$$=(s_{3.077},s_{4.167},s_{6.780},s_{7.826})$$

Especially, if $w=(1/n,1/n,\cdots,1/n)$, then $TFLHHA$ operator is reduced to $TFLOWHA$ operator; if $\omega=(1/n,1/n,\cdots,1/n)$, then $TFLHHA$ operator is reduced to the $TFLWHA$ operator. Obviously, $TFLOWHA$ operator and $TFLWHA$ operator are extended from the $TFLHHA$ operator. The $TFLHHA$ operator focuses on not only the importance of the weight of the trapezoid fuzzy linguistic variables itself, but also the importance of the position weight of the trapezoid fuzzy linguistic variables. So this operator is better than the previous ones.

4 Multiple Attribute Decision Making Method Based on the Trapezoid Fuzzy Linguistic Variables

A multiple attribute decision making problem under the fuzzy linguistic environment is represented as follows:

Let $X=\{x_1,x_2,\cdots,x_n\}$ be the set of the alternatives, and $U=\{u_1,u_2,\cdots,u_m\}$ be the set of the attributes. Let $w=(w_1,w_2,\cdots,w_m)^T$ be the weight vector of the attributes, and w_j be the weight value of the j^{th} attribute, where $w_j\geq 0$ ($j=1,2,\cdots,m$), $\sum_{j=1}^{m}w_j=1$, given by the decision makers directly. Suppose that $\tilde{A}=(\tilde{a}_{ij})_{n\times m}$ is the fuzzy linguistic decision matrix

$$u_1 \quad u_2 \quad \cdots \quad u_m$$

$$\tilde{A}=\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1m}\\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nm}\end{bmatrix}\begin{matrix}x_1\\ x_2\\ \vdots\\ x_n\end{matrix}$$

where $\tilde{a}_{ij}=[a_{ij}^{(\alpha)},a_{ij}^{(\beta)},a_{ij}^{(\gamma)},a_{ij}^{(\eta)}]\in\tilde{S}$ is the attribute value which takes the form of the trapezoid fuzzy linguistic variables, given by the decision makers, for the alternative $x_i\in X$ ($i=1,2,\cdots,n$) with respect to the attribute $u_j\in U$ ($j=1,2,\cdots,m$). Let $\tilde{a}_i=[\tilde{a}_{i1},\tilde{a}_{i2},\cdots,\tilde{a}_{im}]$ be the vector of the attribute values under the alternative x_i ($i=1,2,\cdots,n$).

Then the decision making steps are shown as follows

Step 1: Construct the weighted linguistic matrix $\tilde{A}'=(\tilde{a}'_{ij})_{n\times m}$

$$\tilde{A}'=\begin{bmatrix}\tilde{a}'_{11} & \tilde{a}'_{12} & \cdots & \tilde{a}'_{1m}\\ \tilde{a}'_{21} & \tilde{a}'_{22} & \cdots & \tilde{a}'_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ \tilde{a}'_{n1} & \tilde{a}'_{n2} & \cdots & \tilde{a}'_{nm}\end{bmatrix}=(\tilde{a}'_{ij})_{n\times m}$$

where $\tilde{a}'_{ij}=\tilde{a}_{ij}/nw_j$, $w=(w_1,w_2,\cdots,w_m)$ is the weight vector of the attributes, $w_j>0$ ($j=1,2,\cdots,m$), $\sum_{j=1}^{m}w_j=1$, n is the balancing coefficient.

Step 2: Utilize the formula (3) to construct the possibility degree matrixes $P_i=(p_{jk}^{(i)})_{m\times m}=(p_{jk}^{(i)}(\tilde{a}'_{ij}\geq\tilde{a}'_{ik}))_{m\times m}$ with respect to the alternative x_i ($i=1,2,\cdots,n$) , and sum all the elements of each rows of the possibility degree matrix P_i , then get the ranking vectors $p^{(i)}=(p_1^{(i)},p_2^{(i)},\cdots,p_j^{(i)})$, ($j=1,2,\cdots,m$), where $p_j^{(i)}=\sum_{k=1}^{m}p_{jk}^{(i)}$. Finally, rank the orders of attribute values $\tilde{a}'_{ij}$ ($j=1,2,\cdots,m$) with respect to the alternative x_i based on the values $p_j^{(i)}$ ($j=1,2,\cdots,m$).

Step 3: Utilize the formula (6) to calculate the position weight vector $\omega=(\omega_1,\omega_2,\cdots,\omega_m)$ of $TFLHHA$ operator.

Step 4: Utilize the formula (7) to calculate the combined attribute values

$$\tilde{z}_i=TFLHHA_{\omega,w}(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_m)=(\sum_{j=1}^{m}\frac{\omega_j}{\tilde{r}_j})^{-1}$$

where $i=1,2,\cdots,n$.

Step 5: Utilize the formula (3) to construct the possibility degree matrix $P=(p_{ij})_{n\times n}$, based on the combined attribute values $\tilde{z}_i$ of each alternative, then sum all the elements of each rows of the possibility degree matrix, where $p_i=\sum_{j=1}^{n}p_{ij}$ ($i=1,2,\cdots,n$). Rank all the combined attribute values of each alternative and select the best alternative based on the values p_i .

5 Illustrative Examples

In this section, a decision making problem of evaluating cars for buying (adapted from literature [1, 9]) is used to illustrate the new method.

A decision maker intends to buy a car. Four types of cars x_i ($i=1,2,3,4$) are available. He takes into account four attributes to decide which car he should buy: 1) G_1 : economy, 2) G_2 : comfort, 3) G_3 : design, and 4) G_4 : safety. The decision maker evaluates these four types of cars x_i ($i=1,2,3,4$) under the attributes G_j ($j=1,2,3,4$), where the weight vector is $w=(0.3,0.2,0.1,0.4)$ given by the decision makers. He uses the linguistic term set:

S ={ s_1 =extremely poor, s_2 =very poor, s_3 =poor, s_4 =slightly poor, s_5 =fair, s_6 =slightly good, s_7 =good, s_8 =very good, s_9 =extremely good} and provides the linguistic decision making matrix $\tilde{A}=(\tilde{a}_{ij})_{4\times 4}$:

$$\tilde{A}=\begin{bmatrix}[s_2,s_3,s_5,s_6][s_4,s_5,s_8,s_9][s_5,s_6,s_7,s_9][s_3,s_4,s_5,s_7]\\ [s_3,s_5,s_6,s_7][s_5,s_6,s_7,s_8][s_4,s_5,s_8,s_9][s_4,s_5,s_7,s_8]\\ [s_4,s_6,s_8,s_9][s_4,s_5,s_6,s_7][s_6,s_7,s_8,s_9][s_3,s_4,s_5,s_6]\\ [s_5,s_6,s_7,s_9][s_4,s_7,s_8,s_9][s_3,s_5,s_6,s_7][s_6,s_7,s_8,s_9]\end{bmatrix}$$

Step 1: Construct the weighted linguistic matrix $\tilde{A}'=(\tilde{a}'_{ij})_{4\times 4}$, where $\tilde{a}'_{ij}=\tilde{a}_{ij}/nw_j$ ($j=1,2,3,4$).

$$\tilde{A}'=\begin{bmatrix}[s_{1.67},s_{2.5},s_{4.17},s_5][s_5,s_{6.25},s_{10},s_{11.25}]\\ [s_{2.5},s_{4.17},s_5,s_{5.83}][s_{6.25},s_{7.5},s_{8.75},s_{10}]\\ [s_{3.33},s_5,s_{6.67},s_{7.5}][s_{3.33},s_{4.17},s_5,s_{5.83}]\\ [s_{4.17},s_5,s_{5.83},s_{7.5}][s_5,s_{8.75},s_{10},s_{11.25}]\end{bmatrix}$$

$$\begin{bmatrix}[s_{12.5},s_{15},s_{17.5},s_{22.5}][s_{1.875},s_{2.5},s_{3.125},s_{4.375}]\\ [s_{10},s_{12.5},s_{20},s_{22.5}]\ [s_{2.5},s_{3.125},s_{4.375},s_5]\\ [s_{15},s_{17.5},s_{20},s_{22.5}]\ [s_{1.875},s_{2.5},s_{3.125},s_{3.75}]\\ [s_{7.5},s_{12.5},s_{15},s_{17.5}]\ [s_{3.75},s_{4.375},s_5,s_{5.625}]\end{bmatrix}$$

Step 2: Utilize the formula (3) to construct the possibility degree matrixes $P_i = (p_{jk}^{(i)})_{4\times4} = (p_{jk}^{(i)}(\tilde{a}_{ij}^{'} \geq \tilde{a}_{ik}^{'}))_{4\times4}$ with respect to each alternative x_i ($i=1,2,3,4$), and sum all the elements of each rows of the possibility degree matrix P_i , then get the ranking vectors $p^{(i)} = (p_1^{(i)}, p_2^{(i)}, \cdots, p_j^{(i)})$, ($j=1,2,3,4$), where $p_j^{(i)} = \sum_{k=1}^{4} p_{jk}^{(i)}$. Finally, rank the orders of attribute values $\tilde{a}_{ij}^{'}$ ($j=1,2,3,4$) with respect to the alternative x_i based on the values $p_j^{(i)}$ ($j=1,2,3,4$).

$$p_1 = \begin{bmatrix} 0.5 & 0 & 0 & 0.59 \\ 1 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0.41 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(1)} = (p_1^{(1)}, p_2^{(1)}, p_3^{(1)}, p_4^{(1)}) = (1.09, 2.5, 3.5, 0.91)$$

$\tilde{a}_{13}^{'} > \tilde{a}_{12}^{'} > \tilde{a}_{11}^{'} > \tilde{a}_{14}^{'}$.

$$p_2 = \begin{bmatrix} 0.5 & 0 & 0 & 0.66 \\ 1 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0.34 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(2)} = (1.16, 2.5, 3.5, 0.84)$$

$\tilde{a}_{23}^{'} > \tilde{a}_{22}^{'} > \tilde{a}_{21}^{'} > \tilde{a}_{24}^{'}$

$$p_3 = \begin{bmatrix} 0.5 & 0.73 & 0 & 1 \\ 0.27 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(3)} = (2.23, 1.77, 3.5, 0.5)$$

$\tilde{a}_{33}^{'} > \tilde{a}_{31}^{'} > \tilde{a}_{32}^{'} > \tilde{a}_{34}^{'}$.

$$p_4 = \begin{bmatrix} 0.5 & 0 & 0 & 0.78 \\ 1 & 0.5 & 0.0625 & 1 \\ 1 & 0.9375 & 0.5 & 1 \\ 0.22 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(4)} = (1.28, 2.5625, 3.4375, 0.72)$$

$\tilde{a}_{43}^{'} > \tilde{a}_{42}^{'} > \tilde{a}_{41}^{'} > \tilde{a}_{44}^{'}$.

Step 3: utilize the formula (6) to calculate the position vector of $TFLHHA$ operator: $\omega = (0.125, 0.375, 0.375, 0.125)$.

Step 4: utilize the formula (7) to calculate the combined attribute values:

$\tilde{z}_1 = (s_{2.65}, s_{3.73}, s_{5.73}, s_{7.02})$,

$\tilde{z}_2 = (s_{3.67}, s_{5.27}, s_{6.55}, s_{7.55})$,

$\tilde{z}_3 = (s_{3.33}, s_{4.5}, s_{5.63}, s_{6.53})$,

$\tilde{z}_4 = (s_{4.65}, s_{6.39}, s_{7.39}, s_{8.87})$

Step 5: utilize the formula (3) to construct the possibility degree matrix, based on $\tilde{z}_i$:

$$P = \begin{bmatrix} 0.5 & 0.33 & 0.46 & 0.15 \\ 0.67 & 0.5 & 0.66 & 0.29 \\ 0.54 & 0.34 & 0.5 & 0.12 \\ 0.85 & 0.71 & 0.88 & 0.5 \end{bmatrix}$$

then sum all the elements of each row of the possibility degree matrix, we can get $p_1 = 1.44$ $p_2 = 2.12$ $p_3 = 1.5$ $p_4 = 2.94$. Based on the values p_i , rank all combined attribute values of each alternative and select the best alternative, then we can get $x_4 > x_2 > x_3 > x_1$, so the best alternative is x_4.

In order to verify the effective of this method, we utilized the method shown in literature [9] to solve this illustrate example.

Step1: From the linguistic decision making matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$, we can get the vector of the ideal point of the attribute values corresponding to the alternative x_i ($i=1,2,3,4$): $\tilde{I} = (\tilde{I}_1, \tilde{I}_2, \tilde{I}_3, \tilde{I}_4)$, and

$\tilde{I}_1 = (s_5, s_6, s_8, s_9), \tilde{I}_2 = (s_5, s_7, s_8, s_9),$

$\tilde{I}_3 = (s_6, s_7, s_8, s_9), \tilde{I}_4 = (s_6, s_7, s_8, s_9)$

Step2: Utilize the $TFLWA$ operator to derive the overall values $\tilde{z}_i$ ($i=1,2,3,4$)of the alternative x_i ($i=1,2,3,4$) and $\tilde{z}$ of the ideal point $\tilde{I}$

$\tilde{z}_1 = (s_{3.1}, s_{4.1}, s_{5.8}, s_{7.3}), \tilde{z}_2 = (s_{3.9}, s_{5.2}, s_{6.8}, s_{7.8}),$

$\tilde{z}_3 = (s_{3.8}, s_{5.1}, s_{6.4}, s_{7.4}), \tilde{z}_4 = (s_5, s_{6.5}, s_{7.5}, s_{8.8})$

$\tilde{z} = (s_{5.5}, s_{6.7}, s_8, s_9)$

Step3: We get the similarity degree $s(\tilde{z}, \tilde{z}_i)$ between $\tilde{z}$ and $\tilde{z}_i$ ($i=1,2,3,4$) based on the similarity degree formula

$s(\tilde{z}, \tilde{z}_1) = 0.876$, $s(\tilde{z}, \tilde{z}_2) = 0.924$,

$s(\tilde{z}, \tilde{z}_3) = 0.910$, $s(\tilde{z}, \tilde{z}_4) = 0.981$

Step 4: Rank the order of $s(\tilde{z}, \tilde{z}_i)$ ($i=1,2,3,4$), then we can get: $x_4 > x_2 > x_3 > x_1$.

Analysis:

The order calculated by this method is the same as the order calculated by the method proposed in literature [9], so it is demonstrated that the method proposed in this paper is feasible and effective, and it is also verified that the $TFLHHA$ operator is effective. It provided the new idea to solve the MADM problems under the linguistic context, and it provided the new idea of aggregating the trapezoid fuzzy linguistic variables in the MADM problems.

6 Conclusions

This paper proposed a new method of the MADM problems based on the $TFLHHA$ operator. The new method can deal with the MADM problems where the decision making information takes the form of the $TFLV$s directly, and makes the computation process of the $TFLV$s easily without the loss of the information. This method is easy to use and understand, and it enriched and developed the theory and method of the MADM, This method can solve these MADM problems where the attribute values take the form of the fuzzy linguistic variables, such as fuzzy linguistic variables, the uncertain fuzzy linguistic variables, the triangular fuzzy linguistic variables, the trapezoid fuzzy linguistic variables, and the mixed fuzzy linguistic variables, if we can transform these fuzzy linguistic variables into the trapezoid fuzzy linguistic variables. But this method can only solve the MADM problem under the linguistic context. So it is the limitation of this paper. In the future, we will apply this method to solve the real-life MADM problems in the linguistic context, and we will continued working in the decision making method of the MADM problems with the $TFLV$.

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