Methodology for Interval-Valued Intuitionistic Fuzzy Multiple Attribute Decision Making and Applications to Performance Evaluation of Sustainable Microfinance Groups Lending

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As an important supplement to my country's financial institutions, micro-loan companies serve "agriculture, rural areas and farmers", small and micro enterprises, and individuals, to a certain extent, alleviating the financing difficulties of such groups and regulating private finance. However, micro-loan companies only lend but do not deposit. In the process of lending, due to inadequate risk management, the risk problem has become increasingly prominent. With the continuous growth of the loan amount of rural credit and the continuous increase of microfinance groups lending customers, it faces certain problems in its risk management, which increases the risks of the company in all aspects. The performance evaluation of sustainable microfinance groups lending is a classical multiple attribute decision making (MADM) issues. In this given paper, the interval-valued intuitionistic fuzzy sets (IVIFSs), Hamacher operation sum and Hamacher operation product are introduced. The induced interval-valued intuitionistic fuzzy Hamacher power OWA (I-IVIFHPOWA) operator is built. Then, the I-IVIFHPOWA information operator is employed to solve MADM under IVIFSs. Finally, an example for performance evaluation of sustainable microfinance groups lending is used to test this new defined approach.

Povzetek: Tveganja pri posojanju kmetovalcem so obravnavana z novo metodo večatributnega sistema odločanja s pomočjo mehkih množic.

1 Introduction

Multi-attribute decision-making is a common process of selecting the best alternative from several given alternatives through multiple information decision-making subjects, including invited experts or invited stakeholders in different fields(Ni, Zhao, Xu, & Wang, 2022; Palanikumar, Arulmozhi, & Jana, 2022; Senapati, Chen, & Yager, 2022; S. Tang, Wei, & Chen, 2022; Xing, Cao, Liu, Zhou, & Wu, 2022; Yang & Pang, 2022; M. Zhao, Wei, Wei, Wu, & Guo, 2021). On the one hand, decision-making can effectively overcome some lacks of personal knowledge or common experience, and at the same time, it can fully reflect democracy during common decision-making process, so it has been widely employed in product research and development, credit evaluation, strategic planning program evaluation, investment program evaluation and other fields(Luo, Li, Zhang, Fang, & Chen, 2021; Maisuria, Sonar, & Rathod, 2021; Mousazadeh, Kafaee, & Ashraf, 2021; Shit & Ghorai, 2021; Talafha, Alkouri, Alqaraleh, Zureigat, & Aljarrah, 2021; Verma, 2021). It has become a research hotspot in today's complex decision-making environment. In the actual decision-making process, due to the large amount of uncertain information in the decision-making problem itself, including unquantifiable information, incomplete information or unobtainable information, it is difficult to rely on accurate data to describe it, and the fuzzy set is Representing and manipulating the above data provides a good method, hence the fuzzy multi-attribute decision method(Campbell et al., 2021; Fan, Yan, & Wu, 2021; Zeng, Luo, Zhang, & Li, 2020; S. Zhang et al., 2021; Zhou, Ji, & Xu, 2020). On this basis, with the further complexity of common decision-making and the limitation of personal experience and knowledge level, decision-makers will often show a certain degree of hesitation or uncertainty. The traditional fuzzy set theory (Zadeh, 1965) shows that when dealing with the great limitation, K. T. Atanassov (1986) proposed an intuitionistic fuzzy set (IFSs), which has a stronger performance when dealing with uncertain information. ability. On this basis, K. Atanassov and Gargov (1989) proposed the concepts of interval IFSs (IVIFSs) and interval intuitionistic fuzzy numbers (IVIFNs). The academic community has conducted a lot of research on the MADM problem based on the environment of IVIFNs, such as, attribute weight determination

(Davoudabadi, Mousavi, & Mohagheghi, 2020; W. Z. Wang & Wang, 2008; Z. J. Wang, Li, & Wang, 2009; Z. J. Wang, Wang, Li, & Ieee, 2008; L. S. Zhang, Gao, & Ieee, 2017), determination of decision maker weights(Wan, Xu, & Dong, 2016, 2020; Ye, 2013b; Yue, 2011; Yue & Jia, 2013; S. L. Zhang, Tang, Meng, & Yuan, 2021), similarity measure (Farhadinia & Ban, 2013; Hu & Li, 2013; Ismail & Abdullah, 2012; Mishra et al., 2020; Nguyen & Ieee, 2021; Rani & Jain, 2020; Sonia, Tiwari, & Gupta, 2022; C. P. Wei, Wang, & Zhang, 2011; H. M. Zhang & Yu, 2013; H. Y. Zhang, Dong, Zhang, & Song, 2007), the extension of the classical MADM methods (Bo, He, Li, & Fu, 2020; Dammak, Baccour, & Alimi, 2020; Deveci, Cin, & Kagizman, 2020; Tiwari, Lohani, Muhuri, & Ieee, 2020; Q. Wang, 2021; M. W. Zhao, Wei, Wei, Wu, & Wei, 2021; Zindani, Maity, & Bhowmik, 2020, 2021) and the aggregation methods(Bolturk, Gulbay, & Kahraman, 2020; Kakati & Borkotokey, 2020; Qi, Liang, & Zhang, 2013; Rong, Liu, & Pei, 2021; Senapati, Chen, Mesiar, & Yager, 2022; Senapati, Mesiar, et al., 2022; G. W. Wei & Yi, 2008; Wu, Wei, Wu, & Wei, 2020; X. Xu, 2022; Z. S. Xu & J. Chen, 2007; F. W. Zhang et al., 2019) have all made great progress. The above research provides a good method and idea for solving the MADM based on IVIFNs, but there are also some limitations: First, the similarity between interval intuitionistic fuzzy numbers is mainly measured by calculating distance, but the main distance calculation formulas, including Manhattan, Euclidean, Cosine and Dice (Gupta & Tiwari, 2016; Khalid & Abbas, 2015; D. H. Liu, Chen, & Peng, 2017; Y. Tang, Wen, & Wei, 2017; Verma & Merigo, 2020; Ye, 2012, 2013a, 2018), only consider membership and non-membership degrees. However, in fact, hesitancy is a deep-seated manifestation of decision-makers' awareness and behavior, and is very important to the entire decision-making process. If the influence is ignored, wrong results may occur. From the existing methods, we can’t find the decision methods with combines IOWA operator(Ronald R Yager & Filev, 1999), power average (PA) operator(R. R. Yager, 2001), Hamacher operations(Roychowdhury & Wang, 1998) under IVIFSs. This is the motivation of this study.

A microfinance institution's expectation of operating costs is the basic logic for determining the choice of loan technology. Since the establishment of microfinance, it is necessary to consider the operating cost corresponding to the specific operating environment to select the loan technology(Sinn, 2013). Generally speaking, the more severe the poverty level in the country, the more poor customers the microfinance institution faces, and the operating costs can only be positioned in low-cost strategies(Allen, 2016). Therefore, microfinance institutions, especially small microfinance institutions, will first consider group lending techniques or village banking techniques. Of course, there are other factors that determine the operating cost of an institution(Haldar & Stiglitz, 2016). For example, a high real loan interest rate will affect the external financing cost or opportunity cost of the institution, while group loan technology or village bank technology has a certain internal financing function and reduces the cost of capital, which promotes Microfinance institutions in poor areas are more inclined to choose group loan technology or village bank technology(Kumar, 2016). In addition, the population density of the host country is also conducive to the development of group loan technology or village banking technology, which helps to further reduce costs(Y. Y. Xu, Cheng, & Zhang, 2020). At the same time, we have also proved that there may be two paths beyond group loans: one is the existing microfinance institutions, which start with group loans and gradually issue personal loans. The proportion of the two technologies can be adjusted. Generally, the proportion of personal loans is increasing; the other is the newly established microfinance institutions, which directly use personal loan technology to carry out business. Basically, there is no possibility to start with a group loan and move to a full-scale personal loan. The performance evaluation of sustainable microfinance groups lending is a classic MAGDM issue. In the face of realistic decision-making problems, the evaluation participants may be hesitant and irrational due to their own knowledge shortcomings and the inherent complex internal mechanism of decision-making. However, the traditional decision-making framework is prone to get results inconsistent with the facts.

Our country is still in a blank state in the field of performance evaluation of sustainable microfinance groups lending. Therefore, it is urgent for researchers in related disciplines to conduct exploratory research in this field to enrich the research content of performance evaluation of sustainable microfinance groups lending in my country. The performance evaluation of sustainable microfinance groups lending has two meanings: (1) for the performance evaluation of sustainable microfinance groups lending, so that designers can improve the structure's sustainable microfinance groups lending resistance ability according to the different sustainable microfinance groups lending; (2) post-performance evaluation of sustainable microfinance groups lending, to provide a basis for microfinance groups lending loss estimation. The problems of performance evaluation of sustainable microfinance groups lending are common MADM issue. In this given paper, the I-IVIFHPOWA operator is proposed with given IOWA operator(Ronald R Yager & Filev, 1999)and power average (PA) operator(R. R. Yager, 2001). Then, the I-IVIFHPOWA information operator is applied to solve the given MADM with IVIFSs. Finally, an example for performance evaluation of sustainable microfinance groups lending is applied to test this new defined operator. Thus, the defined motivation of such new study is: (1) the I-IVIFHPOWA operator is proposed with given IOWA operator; (2) the I-IVIFHPOWA operator is applied to cope with the designed MADM with given IVIFSs; The built I-IVIFHPOWA operator includes the advantages of I-OWA operator, Hamacher information sum and Hamacher information product and power average (PA) operator. Thus, the built I-IVIFHPOWA operator have some main advantages such as, more general formulation in the defined reordering process of the defined arguments through using order-inducing variables, the generalization of give algebraic and Einstein t-conorm and t-norm form, are more general and more flexible and information-aggregation process, whose weighting depend upon the given input arguments and allow values being fused to support and reinforce each other. (4) an empirical example for performance evaluation of sustainable microfinance groups lending has been given. In order to do such work, the reminder sections of such built work proceeds. The IVIFSs is given in Sec. 2. the I-IVIFHPOWA operator is really proposed with given IOWA operator in Sec. 3. The I-IVIFHPOWA operator is applied to cope with MADM with given IVIFSs in Sec. 4. An application for performance evaluation of sustainable microfinance groups lending is applied to depict the mentioned superiority of this new defined operator and some comparative analysis studies are considered in Sec. 5. The final studied conclusion is listed in Sec. 6.

2 Preliminaries

2.1 IVIFSs

K. Atanassov and Gargov (1989) defined the novel IVIFSs.

Definition 1(K. Atanassov & Gargov, 1989). Let L be a given new fixed information set, An IVIFS $\tilde{K}$ on Θ is depicted:

$$\tilde{K} = \left\{ \left\langle \theta, m_k\left(\theta\right), n_k\left(\theta\right) \right\rangle \middle| \theta \in \Theta \right\} \tag{1}$$

where $m_k\left(\theta\right) \subset \left[0,1\right]$ and $n_k\left(\theta\right) \subset \left[0,1\right]$ are interval values numbers, and $0 \le \sup\left(m_k\left(\theta\right)\right) + \sup\left(n_k\left(\theta\right)\right) \le 1, \forall k \in K$.

Definition 2(Z.-S. Xu & J. Chen, 2007). Let $\tilde{k} = \left(\left[ki,kj\right],\left[kp,kq\right]\right)$ be an IVIFN, the built score function S is:

$$S\left(\tilde{k}\right) = \frac{\left(ki+kj\right)-\left(kp+\mathrm{k}q\right)}{2}, \quad S\left(\tilde{k}\right) \in \left[-1,1\right]. \tag{2}$$

Definition 3(Z.-S. Xu & J. Chen, 2007). Let $\tilde{k} = \left(\left[ki,kj\right],\left[kp,kq\right]\right)$ be the IVIFN, the built accuracy function H is:

$$H\left(\tilde{k}\right) = \frac{\left(ki+kj\right)+\left(kp+\mathrm{k}q\right)}{2}, \quad H\left(\tilde{k}\right) \in \left[0,1\right]. \tag{3}$$

Definition 4(H. W. Liu & Wang, 2007). Let $\tilde{k}_1 = \left(\left[ki_1,kj_1\right],\left[kp_1,kq_1\right]\right)$ and $\tilde{k}_2 = \left(\left[ki_2,kj_2\right],\left[kp_2,kq_2\right]\right)$ be IVIFNs, the built score and accuracy information values of $\tilde{k}_1 = \left(\left[ki_1,kj_1\right],\left[kp_1,kq_1\right]\right)$ and $\tilde{k}_2 = \left(\left[ki_2,kj_2\right],\left[kp_2,kq_2\right]\right)$ is defined:

$$S\left(\tilde{k}_1\right) = \frac{ki_1 + ki_1\left(1-ki_1-kp_1\right)+kj_1+kj_1\left(1-kj_1-kq_1\right)}{2}$$

$$S\left(\tilde{k}_2\right) = \frac{ki_2 + ki_2\left(1-ki_2-kp_2\right)+kj_2+kj_2\left(1-kj_2-kq_2\right)}{2} \tag{4}$$

$$H\left(\tilde{k}_1\right) = \frac{ki_1+kp_1+kj_1+kq_1}{2}$$

$$H\left(\tilde{k}_2\right) = \frac{ki_2+kp_2+kj_2+kq_2}{2} \tag{5}$$

For two IVIFNs $\tilde{k}_1 = \left(\left[ki_1,kj_1\right],\left[kp_1,kq_1\right]\right)$ and $\tilde{k}_2 = \left(\left[ki_2,kj_2\right],\left[kp_2,kq_2\right]\right)$, according to Definition 3, then

(1) $if\ S\left(\tilde{k}_1\right) < S\left(\tilde{k}_2\right), then\ \tilde{k}_1 < \tilde{k}_2;$

(2) $if\ S\left(\tilde{k}_1\right) = S\left(\tilde{k}_2\right), H\left(\tilde{k}_1\right) < H\left(\tilde{k}_2\right),$
$\tilde{k}_1 < \tilde{k}_2; if\ H\left(\tilde{k}_1\right) = H\left(\tilde{k}_2\right), then\ \tilde{k}_1 = \tilde{k}_2.$

2.2 Hamacher operations of IVIFSs

P. D. Liu (2014) defined the basic Hamacher operations for IVIFS. The given Hamacher product ($\tilde{k}_1 \otimes_\varepsilon \tilde{k}_2$) and Hamacher sum ($\tilde{k}_1 \oplus_\varepsilon \tilde{k}_2$) on two IVIFSs $\tilde{k}_1$ and $\tilde{k}_2$.

$$\tilde{k}_1 \oplus_\varepsilon \tilde{k}_2 = \left(\begin{array}{l} \left[\frac{ki_1 + ki_2}{1 + ki_1 ki_2}, \frac{kj_1 + kj_2}{1 + kj_1 kj_2} \right], \\ \left[\frac{kp_1 kp_2}{1 + (1 - kp_1)(1 - kp_2)}, \frac{kq_1 kq_2}{1 + (1 - kq_1)(1 - kq_2)} \right] \end{array} \right);$$

$$\tilde{k}_1 \otimes_\varepsilon \tilde{k}_2 = \left(\left[\frac{ki_1 . ki_2}{1 + (1 - ki_1)(1 - ki_2)}, \frac{kj_1 kj_2}{1 + (1 - kj_1)(1 - kj_2)} \right], \left[\frac{kp_1 + kp_2}{1 + kp_1 kp_2}, \frac{kq_1 + kq_2}{1 + kq_1 kq_2} \right] \right);$$

$$\lambda \tilde{k}_1 = \left(\left[\frac{(1 + ki_1)^\lambda - (1 - ki_1)^\lambda}{(1 + ki_1)^\lambda + (1 - ki_1)^\lambda}, \frac{(1 + kj_1)^\lambda - (1 - kj_1)^\lambda}{(1 + kj_1)^\lambda + (1 - kj_1)^\lambda} \right], \left[\frac{2kp_1^\lambda}{(2 - kp_1)^\lambda + kp_1^\lambda}, \frac{2kq_1^\lambda}{(2 - kq_1)^\lambda + kq_1^\lambda} \right] \right), \lambda > 0$$

$$\left(\tilde{k}_1\right)^\lambda = \left(\left[\frac{2ki_1^\lambda}{(2 - ki_1)^\lambda + ki_1^\lambda}, \frac{2kj_1^\lambda}{(2 - kj_1)^\lambda + kj_1^\lambda} \right], \left[\frac{(1 + kp_1)^\lambda - (1 - kp_1)^\lambda}{(1 + kp_1)^\lambda + (1 - kp_1)^\lambda}, \frac{(1 + kq_1)^\lambda - (1 - kq_1)^\lambda}{(1 + kq_1)^\lambda + (1 - kq_1)^\lambda} \right] \right), \lambda > 0.$$

3 I-IVIFHPOWA operator

Then, the IVIFHWA operator (P. D. Liu, 2014) is introduced.

Definition 5(P. D. Liu, 2014). Let $\tilde{k}_a = \left([ki_a, kj_a], [kp_a, kq_a] \right)$ $(a = 1, 2, \cdots, v)$ be the given set of IVIFNs, and let IVIFHWA: $\Theta^n \to \Theta$, if

$$\text{IVIFHWA}_\omega\left(\tilde{k}_1, \tilde{k}_2, \cdots, \tilde{k}_v\right) = \mathop{\oplus}_{a=1}^{v} \left(\omega_a \tilde{k}_a\right)$$

$$= \left(\left[\frac{\prod_{a=1}^{v} \left(1 + (\gamma - 1) ki_a\right)^{\omega_a} - \prod_{a=1}^{v} \left(1 - ki_a\right)^{\omega_a}}{\prod_{a=1}^{v} \left(1 + (\gamma - 1) ki_a\right)^{\omega_a} + (\gamma - 1) \prod_{a=1}^{v} \left(1 - ki_a\right)^{\omega_a}}, \frac{\prod_{a=1}^{v} \left(1 + (\gamma - 1) kj_a\right)^{\omega_a} - \prod_{a=1}^{v} \left(1 - kj_a\right)^{\omega_a}}{\prod_{a=1}^{v} \left(1 + (\gamma - 1) kj_a\right)^{\omega_a} + (\gamma - 1) \prod_{a=1}^{v} \left(1 - kj_a\right)^{\omega_a}} \right], \left[\frac{\gamma \prod_{a=1}^{v} kp_a^{\omega_a}}{\prod_{a=1}^{v} \left(1 + (\gamma - 1)(1 - kp_a)\right)^{\omega_a} + (\gamma - 1) \prod_{a=1}^{v} kp_a^{\omega_a}}, \frac{\gamma \prod_{a=1}^{v} kq_a^{\omega_a}}{\prod_{a=1}^{v} \left(1 + (\gamma - 1)(1 - kq_a)\right)^{\omega_a} + (\gamma - 1) \prod_{a=1}^{v} kq_a^{\omega_a}} \right] \right) \quad (6)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_v)^T$ be the weight and $\omega_a > 0$, $\sum_{a=1}^{v} \omega_a = 1$.

Shi (2017) devised the IVIF Hamacher power weighted average (IVIFHPWA) operator.

Definition 6(Shi, 2017). Let $\tilde{k}_a = ([ki_a, kj_a], [kp_a, kq_a])$ $(a = 1, 2, \cdots, v)$ be a given set of IVIFNs, and let IVIFHPWA: $\Theta^n \to \Theta$, if

$$\mathrm{IVIFHPWA}_\omega\left(\tilde{k}_1, \tilde{k}_2, \cdots, \tilde{k}_v\right) = \overset{v}{\underset{a=1}{\oplus}} \left(\frac{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\tilde{k}_a}{\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} \right)$$

$$= \left(\left[\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} - \prod_{a=1}^{v}\left(1-ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} + (\gamma-1)\prod_{a=1}^{v}\left(1-ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}, \right.\right.$$

$$\left. \frac{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} - \prod_{a=1}^{v}\left(1-kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} + (\gamma-1)\prod_{a=1}^{v}\left(1-kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}} \right]$$

$$\left[\frac{\gamma\prod_{a=1}^{v}kp_a^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)(1-kp_a)\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} + (\gamma-1)\prod_{a=1}^{v}kp_a^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}, \right.$$

$$\left.\left. \frac{\gamma\prod_{a=1}^{v}kq_a^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)(1-kq_a)\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)} + (\gamma-1)\prod_{a=1}^{v}kq_a^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}} \right]\right) \tag{7}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_v\right)^T$ be the weight of $\tilde{k}_a$ and $\omega_a>0$, $\sum_{a=1}^{v}\omega_a=1$, and

$$T\left(\tilde{k}_a\right)=\sum_{\substack{a=1\\a\neq b}}^{v}\omega_a Sup\left(\tilde{k}_b,\tilde{k}_a\right) \quad (8)$$

and $Sup\left(\tilde{k}_b,\tilde{k}_a\right)$ is the information support for $\tilde{k}_b$ from $\tilde{k}_a$, with the given conditions:

(1) $Sup\left(\tilde{k}_a,\tilde{k}_b\right)\in[0,1]$;

(2) $Sup\left(\tilde{k}_b,\tilde{k}_a\right)Sup\left(\tilde{k}_a,\tilde{k}_b\right)$;(

(3) $Sup\left(\tilde{k}_a,\tilde{k}_b\right)\geq Sup\left(\tilde{k}_s,\tilde{k}_t\right)$.

if $d\left(\tilde{k}_a,\tilde{k}_b\right)\geq d\left(\tilde{k}_s,\tilde{k}_t\right)$, where d is a distance information measure.

Ronald R Yager and Filev (1999) built the IOWA operator along with given OWA operator (R. R. Yager, 1988).

Definition 7(Ronald R Yager & Filev, 1999). An IOWA operator is built:

$$IOWA\left(\langle u_1,k_1\rangle,\langle u_2,k_2\rangle,\cdots,\langle u_v,k_v\rangle\right)$$
$$=\sum_{a=1}^{v}w_a k_{\sigma(a)} \quad (9)$$

$k_{\sigma(a)}$ is the value k_i of given OWA pair $\langle u_a,k_a\rangle$ having the i-th largest numbers $u_a\left(u_a\in[0,1]\right)$,and u_i in $\langle u_i,k_i\rangle$ is defined to as order information inducing defined argument value and k_i are given information argument.

Then, the I-IVIFHPOWA operator is newly built along with given IOWA operator(Ronald R Yager & Filev, 1999).

Definition 8. Let $\langle u_a,\tilde{k}_a\rangle(a=1,2,\cdots,v)$ be a given set of 2-tuples information, then the I-IVIFHPOWA operator is:

$$\text{I-IVIFHPOWA}_w\left(\langle u_1,\tilde{k}_1\rangle,\langle u_2,\tilde{k}_2\rangle,\cdots,\langle u_v,\tilde{k}_v\rangle\right)$$
$$=\bigoplus_{a=1}^{v}\left(\frac{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\tilde{k}_{\sigma(a)}}{\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}\right) \quad (10)$$

where $w=\left(w_1,w_2,\cdots,w_v\right)^T$ is a weight information, $w_a>0$, $\sum_{a=1}^{v}w_a=1$, $a=1,2,\cdots,v$, $\tilde{k}_{\sigma(a)}$ is the $\tilde{k}_a$ of IVIFHPOWA pair $\langle u_i,\tilde{k}_i\rangle$ having the j-th largest $u_i\left(u_i\in[0,1]\right)$,and u_i in $\langle u_i,\tilde{k}_i\rangle$ is defined to as order inducing variable and $\tilde{k}_i$ are IVIFNs arguments.

Theorem 1. Let $\langle u_a,\tilde{k}_a\rangle(a=1,2,\cdots,v)$ be a set of 2-tuples, then its fused value by I-IVIFHPOWA operator is also an IVIFNs, and

$$
\begin{aligned}
&\text{I-IVIFHPOWA}_w\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)=\mathop{\oplus}_{a=1}^{\mathrm{v}}\left(\frac{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\tilde{k}_{\sigma(a)}}{\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}\right)\\
&=\left(\left[\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.\right.\\
&\qquad\left.\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right]\\
&\left[\frac{\gamma\prod_{a=1}^{v}kp_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kp_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kp_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.\\
&\qquad\left.\left.\frac{\gamma\prod_{a=1}^{v}kq_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kq_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kq_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right]\right)
\end{aligned}
$$

(11)

where $w=\left(w_1,w_2,\cdots,w_v\right)^T$ is a weight values, $w_a>0$, $\sum_{a=1}^{v}w_a=1$, $a=1,2,\cdots,v$, $\tilde{k}_{\sigma(a)}$ is the $\tilde{k}_a$ of IVIFHOWA pair $\left\langle u_i,\tilde{k}_i\right\rangle$ having the j-th largest $u_i\left(u_i\in[0,1]\right)$,and u_i in $\left\langle u_i,\tilde{k}_i\right\rangle$ is designed to as order inducing given variable and $\tilde{k}_i$ are IVIFNs arguments.

Now some special cases of the I-IVIFHPOWA operator are studied:

(1) If $u_j=\tilde{k}_j$ for all j, then I-IVIFHPOWA operator becomes the IVIFHPOWA operator:

$$\text{I-IVIFHPOWA}_w\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)=\bigoplus_{a=1}^{v}\left(\frac{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\tilde{k}_{\sigma(a)}}{\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}\right)$$

$$=\left(\left[\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.\right.$$

$$\left.\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right]$$

$$\left[\frac{\gamma\prod_{a=1}^{v}kp_{\sigma(a)}^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kp_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kp_{\sigma(a)}^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.$$

$$\left.\left.\frac{\gamma\prod_{a=1}^{v}kq_{\sigma(a)}^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kq_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kq_{\sigma(a)}^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right]\right)$$

$$
\begin{aligned}
&= \mathrm{IVIFHPOWA}_{w}\left(\tilde{k}_1,\tilde{k}_2,\cdots,\tilde{k}_v\right)=\overset{v}{\underset{a=1}{\oplus}}\left(\frac{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\tilde{k}_{\sigma(a)}}{\sum_{a=1}^{v} w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}\right)\\
&=\left(\left[\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-ki_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.\right.\\
&\qquad\left.\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}-\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-kj_{\sigma(a)}\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right],\\
&\quad\left[\frac{\gamma\prod_{a=1}^{v}kp_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kp_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kp_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}},\right.\\
&\qquad\left.\left.\frac{\gamma\prod_{a=1}^{v}kq_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)\left(1-kq_{\sigma(a)}\right)\right)^{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{c}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kq_{\sigma(a)}^{\;w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\Big/\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}}\right]\right)
\end{aligned}
$$

(12)

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(v)\right)$ is a permutation of $\left(1,2,\cdots,c\right)$, such that $\tilde{k}_{\sigma(a-1)}\geq\tilde{k}_{\sigma(a)}$ for all $a=2,\cdots,v$.

If $u_a = No.a$ for all a, if a is ordered position of $\left\langle u_a,\tilde{k}_a\right\rangle$, then I-IVIFHPOWA operator becomes the IVIFHPWA operator:

$$
\begin{aligned}
&\mathrm{I\text{-}IVIFHPOWA}_{w}\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)=\overset{v}{\underset{a=1}{\oplus}}\left(\frac{w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)\tilde{k}_{\sigma(a)}}{\sum_{a=1}^{v}w_a\left(1+T\left(\tilde{k}_{\sigma(a)}\right)\right)}\right)\\
&=\mathrm{IVIFHPWA}_{\omega}\left(\tilde{k}_1,\tilde{k}_2,\cdots,\tilde{k}_v\right)=\overset{v}{\underset{a=1}{\oplus}}\left(\frac{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\tilde{k}_a}{\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}\right)
\end{aligned}
$$

$$=\left(\left[\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}-\prod_{a=1}^{v}\left(1-ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-ki_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}},\right.\right.$$

$$\left.\frac{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}-\prod_{a=1}^{v}\left(1-kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}+(\gamma-1)\prod_{a=1}^{v}\left(1-kj_a\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}\right]$$

$$\left[\frac{\gamma\prod_{a=1}^{v}kp_a^{\;\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)(1-kp_a)\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kp_a^{\;\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}},\right.$$

$$\left.\left.\frac{\gamma\prod_{a=1}^{v}kq_a^{\;\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}{\prod_{a=1}^{v}\left(1+(\gamma-1)(1-kq_a)\right)^{\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}+(\gamma-1)\prod_{a=1}^{v}kq_a^{\;\omega_a\left(1+T\left(\tilde{k}_a\right)\right)\Big/\sum_{a=1}^{v}\omega_a\left(1+T\left(\tilde{k}_a\right)\right)}}\right]\right) \tag{13}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_v)^T$ be the weight information of $\tilde{k}_a\,(a=1,2,\cdots,v)$, and $\omega_a>0$, $\sum_{a=1}^{v}\omega_a=1$.

It is easily tested that the I-IVIFHPOWA operator has the given properties.

Theorem 2(Idempotency). If all $\tilde{k}_a\,(a=1,2,\cdots,v)$ are equal, i.e. $\tilde{k}_a=\tilde{k}$ for all a, then

$$\text{I-IVIFHPOWA}_w\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)=k \tag{14}$$

Theorem 3(Boundedness). Let $\tilde{k}_a\,(a=1,2,\cdots,v)$ be a set of IVIFNs, and let

$$\tilde{a}^-=\min_j\tilde{a}_j,\quad \tilde{k}^+=\max_a\tilde{k}_a$$

Then

$$\tilde{k}^-\le\text{I-IVIFHPOWA}_w\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)\le\tilde{k}^+ \tag{15}$$

Theorem 4 (Monotonicity). Let $\left\langle u_a,\tilde{k}_a\right\rangle(a=1,2,\cdots,v)$ and $\left\langle u'_a,\tilde{k}'_a\right\rangle(a=1,2,\cdots,v)$ be two given set of information IVIFNs, if $\tilde{k}_a\le\tilde{k}'_a$, for all a, then

$$\begin{aligned}&\text{I-IVIFHPOWA}_w\left(\left\langle u_1,\tilde{k}_1\right\rangle,\left\langle u_2,\tilde{k}_2\right\rangle,\cdots,\left\langle u_v,\tilde{k}_v\right\rangle\right)\\&\le\text{I-IVIFHPOWA}_w\left(\left\langle u'_1,\tilde{k}'_1\right\rangle,\left\langle u'_2,\tilde{k}'_2\right\rangle,\cdots,\left\langle u'_v,k'_v\right\rangle\right)\end{aligned} \tag{16}$$

Theorem 5 (Commutativity). Let $\left\langle u_a,\tilde{k}_a\right\rangle(a=1,2,\cdots,v)$ and

$\left\langle u'_a, \tilde{k}'_a \right\rangle (a = 1, 2, \cdots, v)$ be two defined set of information IVIFNs, then

$$\text{I-IVIFHPOWA}_w \left(\left\langle u_1, \tilde{k}_1 \right\rangle, \left\langle u_2, \tilde{k}_2 \right\rangle, \cdots, \left\langle u_v, \tilde{k}_v \right\rangle \right) = \text{I-IVIFHPOWA}_w \left(\left\langle u'_1, \tilde{k}'_1 \right\rangle, \left\langle u'_2, \tilde{k}'_2 \right\rangle, \cdots, \left\langle u'_v, k'_v \right\rangle \right) \tag{17}$$

where $\left\langle u'_a, \tilde{k}'_a \right\rangle (a = 1, 2, \cdots, v)$ is any information permutation of $\left\langle u_a, \tilde{k}_a \right\rangle (a = 1, 2, \cdots, v)$

$$\tilde{k}_u = \left([ki_u, ki_u], [kp_u, kq_u] \right)$$

$$=\text{I-IVIFHPOWA}\left(\left\langle u_{u1}, \tilde{k}_{u1} \right\rangle, \left\langle u_{u2}, \tilde{k}_{u2} \right\rangle, \cdots, \left\langle u_{uy}, \tilde{k}_{uy} \right\rangle \right) = \bigoplus_{v=1}^{y} \left(\frac{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \tilde{k}_{\sigma(uv)}}{\sum_{v=1}^{y} \left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} \right)$$

$$= \left(\left[\frac{\prod_{v=1}^{y} \left(1+(\gamma-1) ki_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} - \prod_{v=1}^{y} \left(1-ki_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}{\prod_{v=1}^{y} \left(1+(\gamma-1) ki_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} + (\gamma-1) \prod_{v=1}^{y} \left(1-ki_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}, \right.\right.$$

$$\left. \frac{\prod_{v=1}^{y} \left(1+(\gamma-1) kj_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} - \prod_{v=1}^{y} \left(1-kj_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}{\prod_{v=1}^{y} \left(1+(\gamma-1) kj_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} + (\gamma-1) \prod_{v=1}^{y} \left(1-kj_{\sigma(uv)}\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}} \right],$$

$$\left[\frac{\gamma \prod_{v=1}^{y} kp_{\sigma(uv)}^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}{\prod_{v=1}^{y} \left(1+(\gamma-1)\left(1-kp_{\sigma(uv)}\right)\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} + (\gamma-1) \prod_{v=1}^{y} kp_{\sigma(uv)}^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}, \right.$$

$$\left.\left. \frac{\gamma \prod_{v=1}^{y} kq_{\sigma(uv)}^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}}{\prod_{v=1}^{y} \left(1+(\gamma-1)\left(1-kq_{\sigma(uv)}\right)\right)^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)} + (\gamma-1) \prod_{v=1}^{y} kq_{\sigma(uv)}^{\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right) \Big/ \sum_{v=1}^{y}\left(1+T\left(\tilde{k}_{\sigma(uv)}\right)\right)}} \right] \right) \tag{18}$$

4 Method for MADM based on defined I-IVIFHPOWA operator with IVIFNs

In such defined section, the MADM are newly investigated with I-IVIFHOWA information operator. Let $A=\{A_1,A_2,\cdots,A_x\}$ be a group of possible solutions, and $G=\{G_1,G_2,\cdots,G_y\}$ be the group of criterions. Suppose that $\tilde{K}=\left(\tilde{k}_{uv}\right)_{x\times y}=\left(\left[ki_{uv},kj_{uv}\right],\left[kp_{uv},kq_{uv}\right]\right)_{x\times y}$ is the IVIF-matrix, $\left[ki_{uv},kj_{uv}\right]\subset[0,1]$, $\left[kp_{uv},kq_{uv}\right]\subset[0,1]$, $j_{uv}+q_{uv}\le 1$, $u=1,2,\cdots,x$, $v=1,2,\cdots,y$.

Then, the I-IVIFHPOWA operator is given to deal with defined MADM with information IVIFNs. The method involves the following information steps:

Step 1. Utilize the defined $\tilde{K}=\left(\tilde{k}_{uv}\right)_{x\times y}=\left(\left[ki_{uv},kj_{uv}\right],\left[kp_{uv},kq_{uv}\right]\right)_{x\times y}$ and defined I-IVIFHPOWA operator to obtain the information overall values $\tilde{k}_u\,(u=1,2,\cdots,x)$ of the information alternative A_u.

Step 2. Obtain the information scores $S\left(\tilde{k}_u\right)(u=1,2,\cdots,x)$ of $\tilde{k}_u\,(u=1,2,\cdots,x)$ to rank the defined alternatives $A_u\,(u=1,2,\cdots,x)$ and then to obtain the best choice.

Step 3. Rank the information choices $A_u\,(u=1,2,\cdots,x)$ and obtain the best information one(s) by given $S\left(\tilde{k}_u\right)$ and $H\left(\tilde{k}_u\right)\,(u=1,2,\cdots,x)$.

Step 4. End.

5 Numerical Example

In recent years, with the goal of achieving targeted poverty alleviation, the central government has encouraged and guided various financial institutions to increase their support for poverty alleviation and development. Strengthening financial poverty alleviation efforts, providing poverty alleviation micro-credit for poor households and low-income farmers, establishing poverty alleviation re-loans, and financial bonds for poverty alleviation and relocation in other places are important policy measures for poverty alleviation. The "Strategic Plan for Rural Revitalization (2018-2022)" pointed out that it is necessary to innovate financial agricultural products and services, explore the pilot work of "two rights" mortgages, and develop new types of credit-based financial agricultural products and services. Therefore, in the process of granting credit, what kind of loan technology is adopted by financial institutions to ensure the timely repayment of principal and interest of credit funds, and to realize the financial sustainability of financial support for agriculture, that is, the practical problems that financial institutions have to face are also the financial problems of developing countries. The old problem in poverty alleviation. Looking back at poverty alleviation practices around the world, the success of microfinance is striking, and people attribute the success of microfinance to a unique mechanism—the Grameen Bank of Bangladesh's group lending technology is the most typical example. Typical elements of this technique include: Borrowers form a team of five with joint responsibility for the loan; two members of the team pay off the loan before the other members can get the loan; weekly meetings, weekly/biweekly repayments money; make weekly deposits to form group savings. Group loans have worked well at Grameen Bank in Bangladesh, with a repayment rate of 97%, breaking the stereotype that the poor cannot be trusted. Microfinance institutions issued syndicated loans to the poor, successfully resolving the high default rate of policy loans in developing countries after World War II. Under the vigorous promotion of international institutions such as the World Bank, Grameen Bank's group lending technology has been rapidly promoted around the world, and has even become synonymous with microfinance. Consistent with international practice, in the development of microfinance in my country for more than 20 years, group loan technology has always been valued and achieved remarkable results. In the 1980s, in the poverty alleviation projects of the International Agricultural Development Fund in China, some of the credit projects adopted the group loan technology, and achieved remarkable results. In 1994, the Rural Development Research Institute of the Chinese Academy of Social Sciences promoted the establishment of the Hebei Yixian Poverty Alleviation Economic Cooperative, which completely borrowed the group loan technology of the Village Bank, and the repayment was guaranteed. Subsequently, the Agricultural Bank of China and the Rural Credit Cooperatives successively took over the microfinance business. The technology is still used, but it has been partially revised according to the actual situation. In the new round of financial reform started in 2006, the group loan technology was written into the pilot charter by the village-level mutual aid fund, and was vigorously promoted in practice. However, in the process of group loan technology promotion, it has been facing various difficulties, some elements have been modified or even replaced by other loan technologies.

For example, rural credit cooperatives in some areas abandoned the practice of weekly meetings in group loans, and changed the repayment method to one-time payment upon maturity. In some places, the joint guarantee loan is only a mere formality, which is used to cope with the inspection of the superior. Some microfinance institutions conduct their own credit ratings for customers, carry out credit loans, or issue loans in the form of guarantees by village cadres, civil servants, and husband and wife guarantees. In recent years, my country's agricultural land management rights, forest rights, and housing property mortgage loans in some areas also reflect this trend. As a result, microfinance has formed a diversified loan technology pattern, and the proportion of group loan technology has gradually declined, forming the current basic trend. The performance evaluation of sustainable microfinance groups lending is a classical MADM issue. This section gives a numerical example for performance evaluation of sustainable microfinance groups lending. There is the information panel with five potential microfinance groups $MG_u\ (u=1,2,3,4,5)$ to be chosen and four information attributes to assess the performance evaluation of sustainable microfinance groups lending: ①G_1 is the loan amount; ②G_2 is the pay back ability; ③G_3 is the loan utilization rate; ④G_4 is the loan repayment ability. The five possible microfinance groups $MG_u\ (u=1,2,3,4,5)$ are to be always evaluated through IVIFNs by DMs under four given information attributes, as depicted in matrix $\tilde{K}$.

$$\tilde{K}=\begin{bmatrix} ([0.26, 0.51],[0.44, 0.41]) & ([0.23, 0.42],[0.45, 0.57]) & ([0.24, 0.56],[0.32,0.46]) & ([0.44,0.52],[0.18, 0.31]) \\ ([0.31, 0.56],[0.26, 0.36]) & ([0.42, 0.56],[0.32, 0.41]) & ([0.28, 0.64],[0.27, 0.33]) & ([0.38, 0.53],[0.26, 0.47]) \\ ([0.25, 0.37],[0.51, 0.63]) & ([0.36, 0.48],[0.52, 0.54]) & ([0.19, 0.61],[0.35, 0.38]) & ([0.19, 0.43],[0.34, 0.56]) \\ ([0.42, 0.59],[0.32, 0.44]) & ([0.33, 0.41],[0.39, 0.59]) & ([0.35, 0.52],[0.28, 0.49]) & ([0.46, 0.51],[0.31, 0.39]) \\ ([0.43,0.52],[0.36,0.45]) & ([0.41,0.56],[0.38,0.46]) & ([0.41, 0.66],[0.16, 0.35]) & ([0.45, 0.52],[0.36, 0.48]) \end{bmatrix}$$

Then, the I-IVIFHPOWA operator is usefully applied to defined MADM for solving the performance evaluation of sustainable microfinance groups lending with information IVIFNs. The built method involves the following information steps:

Step 1. If the I-IVIFHPOWA operator is applied to defined MADM with information IVIFNs.

Step 2. Invited given famous experts use given induced information variables to depict the given information complex personal attitude characteristics of the group opinions of different board information members. The results are often shown in information Table 1.

Table 1: Inducing variables

	G_1	G_2	G_3	G_4
MG_1	15	13	12	9
MG_2	11	20	19	22
MG_3	19	12	17	13
MG_4	20	13	23	24
MG_5	21	15	18	13

Step 3. The decision newly information obtained through information matrix $\tilde{R}$ is combined along with I-IVIFHPOWA, the values $\tilde{k}_u$ of the microfinance groups are obtained as $MG_u\ (u=1,2,\cdots,5)$.

$$\mathrm{MG}_1=([0.3414, 0.4205],[0.3608, 0.5432])$$
$$\mathrm{MG}_2=([0.5112, 0.4937],[0.1797, 0.3256])$$
$$\mathrm{MG}_3=([0.4633, 0.5354],[0.2878, 0.469])$$
$$\mathrm{MG}_4=([0.4318, 0.5295],[0.1636, 0.2785])$$
$$\mathrm{MG}_5=([0.3616, 0.5288],[0.3367, 0.4754])$$

Step 4. Calculate the given scores $S(MG_u)$ of IVIFNs $MG_u\ (u=1,2,\cdots,5)$

$$S(MG_1)=-0.0913, S(MG_2)=0.4709$$
$$S(MG_3)=0.2617, S(MG_4)=0.3874$$
$$S(MG_5)=-0.0616$$

Step 5. Rank all given information choices $MG_u\ (u=1,2,3,4,5)$ with along information scores: $MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$, and thus the most desirable information microfinance groups is MG_2.

6 Compare analysis

Firstly, compared with defined IVIFWA & IVIFWG information operators (Su, Xia, & Chen, 2011). For IVIFWA information operator, the designed calculating results are $S(MG_1)=0.0787, S(MG_2)=0.1512,$ $S(MG_3)=0.3451, S(MG_4)=0.0476, S(MG_5)=0.0431$. Thus, the ranking order is $MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$ .For information IVIFWG operator, the information calculating values are: $S\left(MG_1\right) = \text{-}0.0121, S\left(MG_2\right) = 0.1243,$ $S\left(MG_3\right) = 0.3226, S\left(MG_4\right) = 0.0372,$ $S\left(MG_5\right) = 0.0096.$ So the ranking order is $MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$.

In the end, compared with information IVIF-CODAS method(Yeni & Ozcelik, 2019). The information assessment scores are: $AS_1 = -0.8018$, $AS_2 = 0.1750$, $AS_3 = 1.4769$, $AS_4 = -0.3658$, $AS_5 = -0.4659$. Therefore, the order is $MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$.

Eventually, the obtained information results of these methods are obtained in information Table 2. From information Table 2, the best choice information is MG_2, while the worst information choice is MG_1. In other words, these information methods' order is slightly different. Different given methods may cope with MADM from different information angles. These five information models have their given information advantages: (1) IVIFWA operator emphasis group decision influences; (2) IVIFWG operator emphasis individual decision influences; (3) The advantage of the IVIF-CODAS model lies in the use of two distance measures. (4) the I-IVIFHPOWA operator is proposed with IOWA information operator; (2) the I-IVIFHPOWA operator is defined to study the MADM under information IVIFSs; (3) The built I-IVIFHPOWA operator includes the advantages of I-OWA operator, Hamacher information sum and Hamacher information product and power average (PA) operator. Thus, the built I-IVIFHPOWA operator have some main advantages such as, more general formulation in the defined reordering process of the defined arguments through using order-inducing variables, the generalization of give algebraic and Einstein t-conorm and t-norm form, are more general and more flexible and information-aggregation process, whose weighting depend upon the given input arguments and allow values being fused to support and reinforce each other.

Table 2: Evaluation results of these selected information methods

Methods	order	The best choice	The worst choice
IVIFWA operator (Su et al., 2011)	$MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$	MG_2	MG_1
IVIFWG operator (Su et al., 2011)	$MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$	MG_2	MG_1
IVIF-CODAS model(Yeni & Ozcelik, 2019)	$MG_2 \succ MG_4 \succ MG_5 \succ MG_3 \succ MG_1$	MG_2	MG_1
I-IVIFHPOWA operator	$MG_2 \succ MG_4 \succ MG_3 \succ MG_5 \succ MG_1$	MG_2	MG_1

7 Conclusion

The existing literature suggests that the operating cost expectation determined by the microfinance business environment determines the choice of credit technology, and the development and changes of the business environment also prompt the evolution of low-cost models to higher costs. Group loan and village bank technology are good loan technologies suitable for poor groups (especially poor groups). With the optimization of the business environment, customers have begun to expand, and some of them have gradually adopted personal loan technology, and even small loans that fully use personal loan technology have emerged. credit institution. Through the empirical research of micro data, it is verified that the loan operating cost has a significant impact on the choice of credit technology, and the macro environment such as population density, poverty level, and real loan interest rate also has a significant impact on the choice of credit technology. Institutional age and credit operating cost reflect the evolutionary path of credit technology from group loan-group loan/individual loan, group loan-individual loan. Therefore, combining the characteristics of the business environment, selecting appropriate loan technology and carrying out innovation in a timely manner is the proper meaning of microfinance technology selection. The performance evaluation of sustainable microfinance groups lending is a classical MAGDM issue. Thus, in this given paper, the IVIFSs, Hamacher sum information, Hamacher product information is introduced and the I-IVIFHPOWA information operator is proposed. Then, the I-IVIFHPOWA information operator is defined to study MADM under information IVIFSs. Finally, an information example for performance evaluation of sustainable microfinance groups lending is used to test this new defined operator. Thus, the main contribution of this information study is: (1) the I-IVIFHPOWA operator is proposed with IOWA information operator; (2) the I-IVIFHPOWA operator is defined to study the MADM under information IVIFSs; (3) The built I-IVIFHPOWA operator includes the advantages of I-OWA operator, Hamacher information sum and Hamacher information product and power average (PA) operator. (4) an empirical example for performance evaluation of sustainable microfinance groups lending has been given. In the future, we shall continue working into the development and wide application of the built operators to other defined uncertain information and fuzzy information domains.

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