An Integrated Method for Evaluating the Energy-Saving and Emission Reduction of Thermal Power Plants with Interval-Valued Intuitionistic Fuzzy Numbers

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The problems of evaluating the energy-saving and emission reduction of thermal power plants are multiple attribute decision making (MADM) problems. In this paper, the interval-valued intuitionistic fuzzy sets (IVIFSs), Heronian mean (HM) operator, Dombi operations are introduced and the interval-valued intuitionistic fuzzy Dombi power Heronian mean (IVIFDPHM) operator is proposed. Some desirable properties of this operator are established. Then, the IVIFDPHM operator is used to deal with the interval-valued intuitionistic fuzzy (IVIF) multiple attribute decision making (IVIF-MADM) problems. Finally, an illustrative example for evaluating the energy-saving and emission reduction of thermal power plants is given to verify the built approach. In order to show the superiority of IVIFDPHM operator and some comparative studies are also given below. The IVIFDPHM operator is compared with IVIFWA and IVIFWG operators, IVIFZA operator, IVIFZG operator, IVIFCWA operator and I-IIFOWG operator.

Povzetek: Z metodo mehkih množic je obravnavan problem emisij termoelektrarn.

1 Introduction

Since the probability of occurrence of events in the decision environment and the performance characteristics of things under different circumstances are different(Narang, Joshi, & Pal, 2022; Ran, 2022; Seikh & Mandal, 2021; Shahbazova, 2013; Verma & Sharma, 2013), the decision makers(DMs) should consider the uncertainty of things and evaluate them comprehensively after fully analyzing the characteristics of things when analyzing and dealing with multi-attribute decision making (MADM) problems(Ju, Liu, & Ju, 2016; J. D. Qin & Liu, 2016; C. H. Su, Tzeng, & Hu, 2016; Ye, 2016). The Multiple Criteria Decision Making (MCDM) can be divided into two types: multi-attribute decision making (MADM) and multi-objective decision making (MODM), in which the state space of MADM is discrete, and it is mainly used for the state space of MADM is discrete, which is mainly used for the selection and evaluation among given solutions (Choudhary, Nizamuddin, Singh, & Sachan, 2019; Gulistan et al., 2019; Li & Chen, 2018), while the state space of MODM problem is continuous, which is mainly used for the planning and design of unknown solutions. In general, the MADM problem can be regarded as a MODM problem with a finite number of solutions, which can be used to select different solutions according to the different needs of users when the number of attributes is multiple, and the MADM has become an important part of modern decision-making by virtue of its universality and extensiveness (Jan, Zedam, Mahmood, Ullah, & Ali, 2019; Jana, Muhiuddin, & Pal, 2019, 2020). The research on MADM problems in China emerged in the 1970s, and with the continuous development of theoretical research, the research results of MADM research in China have been enriched, such as group decision, mixed multi-attribute decision, time series and intelligent decision, etc. have been gradually applied to various aspects of production life in modern society(Liu & Wang, 2022; Ning, Wei, Lin, & Guo, 2022; S. Wang et al., 2022; W. H. Xu, Shang, & Wang, 2021; Zhao, Gao, Wei, Wei, & Guo, 2022). K. T. Atanassov (1989) built the intuitionistic fuzzy sets (IFSs) on fuzzy set (Zadeh, 1965). K. Atanassov and Gargov (1989) introduced the interval-valued IFSs (IVIFSs). Z.-S. Xu and J. Chen (2007) proposed the IVIF hybrid aggregation (IVIFHA) operator. Z. S. Xu and J. Chen (2007) developed the IVIF hybrid geometric (IIFHG) operator. Z. S. Xu and Yager (2008) developed the uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator. Mu, Zeng, and Liu (2018) defined the IVIF Zhenyuan averaging (IVIFZA) and IVIF Zhenyuan geometric (IVIFZG)operator.Wei(2010)definedtheinduced IVIF ordered weighted geometric (I-IIFOWG) operator.Z. M. Zhang (2016) defined the IVIF Hamacher hybrid arithmetical averaging and IVIF Hamacher hybrid arithmetical geometric operator. S. F. Wang (2017) defined the IVIF Choquet integral operators. Xian, Dong, and Yin (2017) defined the IVIF combined weighted averaging (IVIFCWA) operator. Z. M. Zhang (2017) defined the IVIF Frank weighted averaging and IVIF Frank weighted geometric operator. Tang and Meng (2018) defined the generalized symmetrical IVIF Choquet-Shapley (IG-SIVIFCS) operator. Q. F. Wang and Sun (2018) defined IVIF Einstein geometric Choquet integral operator (IVIFEGC). Wu, Wei, Wu, and Wei (2020) investigated the IVIF Dombi weighted Heronian mean (IVIFDWHM). The existing operators are listed in Table 1.

Authors	**The existing operators**
Z.-S. Xu and J. Chen (2007)	IVIFWA operator, IVIFOWA operator, IVIFHA operator
Z. S. Xu and J. Chen (2007)	IIFOWG operator, IIFHG operator
Z. S. Xu and Yager (2008)	UDIFWA operator
Mu et al. (Mu et al., 2018)	IVIFZA operator, IVIFZG operator
Wei (2010)	I-IIFOWG operator
Z. M. Zhang (2016)	IVIF Hamacher hybrid arithmetical averaging operator and IVIF Hamacher hybrid arithmetical geometric operator
S. F. Wang (2017)	IVIF Choquet integral operators
Xian et al. (Xian et al., 2017)	IVIFCWA operator
Z. M. Zhang (2017)	IVIF Frank weighted averaging operator and the IVIF Frank weighted geometric operator
Tang and Meng (2018)	IG-SIVIFCS operator
Q. F. Wang and Sun (2018)	IVIFEGC operator
(Wu et al., 2020)	IVIFDWHM operator

Table 1: The existing operators

The problems of evaluating the energy-saving and emission reduction of thermal power plants (Mu, 2020) are classical MADM problems (Fan, Yan, & Wu, 2021; Huang, Lin, & Chen, 2021; Jana & Pal, 2021; Lu, Zhang, Wu, & Wei, 2021; Rawat & Komal, 2022; Song & Geng, 2021; Tehreem, Hussain, & Alsanad, 2021; Yahya, Abdullah, Chinram, Al-Otaibi, & Naeem, 2021). In this paper, the IVIFSs, Heronian mean (HM) operator, Dombi operations are introduced and the IVIFDPHM operator is proposed. Some desirable properties of this operator are established. Then, the IVIFDPHM operator is used to deal with the IVIF-MADM problem. Finally, an illustrative example for evaluating the energy-saving and emission reduction of thermal power plants is given to verify the built approach.

2 Preliminaries

2.1 IVIFSs

K. Atanassov and Gargov (1989) introduced the IVIFSs.

Definition 1 (K. Atanassov & Gargov, 1989). The IVIFS $\tilde{A}$ over X has the form:

$$\tilde{A}=\left\{\left\langle x,\tilde{\mu}_A(x),\tilde{\nu}_A(x)\right\rangle \middle| x\in X\right\} \tag{1}$$

where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and

$0\le \sup\left(\tilde{\mu}_A(x)\right)+\sup\left(\tilde{\nu}_A(x)\right)\le 1$, $\forall\, x\in X$.

Definition 2 (Z. S. Xu & Yager, 2008). Let $\tilde{a}=\left([a,b],[c,d]\right)$ be the interval-valued intuitionistic fuzzy number (IVIFN), the score function S is:

$$S(\tilde{a})=\frac{a-c+b-d}{2},\quad S(\tilde{a})\in[-1,1]. \tag{2}$$

Definition 3 (Z. S. Xu & Yager, 2008**).** Let $\tilde{a}=\left([a,b],[c,d]\right)$ be the IVIFN, the accuracy function H is:

$$H(\tilde{a})=\frac{a+b+c+d}{2},\quad H(\tilde{a})\in[0,1]. \tag{3}$$

Z. S. Xu and Yager (2008) give order relation for two IVIFNs.

2.2 HM operator

Hara, Uchiyama, and Takahasi (1998) proposed the Heronian mean (HM) operator.

Definition 4 (Hara et al., 1998). The Heronian mean (HM) operator is:

$$\mathrm{HM}^{p,q}\left(a_1,a_2,\cdots,a_n\right)=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}a_i^p a_j^q\right)^{\frac{1}{p+q}} \tag{4}$$

where p, q≥0, then $a_i\ (i=1,2,\cdots,n)$ be a series of crisp numbers.

2.3 Dombi Operations of IVIFNs

Definition 5 (Dombi, 1982). Dombi (1982) proposed the Dombi T-norm and T-conorm:

$$\mathrm{D}(t,s)=\frac{1}{1+\left(\left(\frac{1-t}{t}\right)^{\gamma}+\left(\frac{1-s}{s}\right)^{\gamma}\right)^{1/\gamma}} \tag{5}$$

$$\mathrm{D}^c(t,s)=1-\frac{1}{1+\left(\left(\frac{t}{1-t}\right)^{\gamma}+\left(\frac{s}{1-s}\right)^{\gamma}\right)^{1/\gamma}} \tag{6}$$

where $\gamma>0$, $(t,s)\in[0,1]$.

Based on Dombi T-norm and T-conorm, Wu et al. (Wu et al., 2020) defined the operational rules of IVIFNs.

Definition 6 (Wu et al., 2020)**.** For two IVIFNs

$\tilde{a}_1=\left([a_1,b_1],[c_1,d_1]\right)$ and

$\tilde{a}_2=\left([a_2,b_2],[c_2,d_2]\right)$, $\gamma>0$, the

Dombi operational laws are defined:

$$\tilde{a}_1 \oplus \tilde{a}_2 = \left(\begin{array}{l} \left[1 - \dfrac{1}{1 + \left(\left(\dfrac{a_1}{1-a_1} \right)^{\gamma} + \left(\dfrac{a_2}{1-a_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, 1 - \dfrac{1}{1 + \left(\left(\dfrac{b_1}{1-b_1} \right)^{\gamma} + \left(\dfrac{b_2}{1-b_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right], \\ \left[\dfrac{1}{1 + \left(\left(\dfrac{1-c_1}{c_1} \right)^{\gamma} + \left(\dfrac{1-c_2}{c_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, \dfrac{1}{1 + \left(\left(\dfrac{1-d_1}{d_1} \right)^{\gamma} + \left(\dfrac{1-d_2}{d_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right] \end{array} \right); \tag{1}$$

$$\tilde{a}_1 \otimes \tilde{a}_2 = \left(\begin{array}{l} \left[\dfrac{1}{1 + \left(\left(\dfrac{1-a_1}{a_1} \right)^{\gamma} + \left(\dfrac{1-a_2}{a_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, \dfrac{1}{1 + \left(\left(\dfrac{1-b_1}{b_1} \right)^{\gamma} + \left(\dfrac{1-b_2}{b_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right], \\ \left[1 - \dfrac{1}{1 + \left(\left(\dfrac{c_1}{1-c_1} \right)^{\gamma} + \left(\dfrac{c_2}{1-c_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, 1 - \dfrac{1}{1 + \left(\left(\dfrac{d_1}{1-d_1} \right)^{\gamma} + \left(\dfrac{d_2}{1-d_2} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right] \end{array} \right); \tag{2}$$

$$n\tilde{a}_1 = \left(\left[1 - \dfrac{1}{1 + \left(n \left(\dfrac{a_1}{1-a_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, 1 - \dfrac{1}{1 + \left(n \left(\dfrac{b_1}{1-b_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right], \left[\dfrac{1}{1 + \left(n \left(\dfrac{1-c_1}{c_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, \dfrac{1}{1 + \left(n \left(\dfrac{1-d_1}{d_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right] \right); \tag{3}$$

$$\left(\tilde{a}_1 \right)^n = \left(\left[\dfrac{1}{1 + \left(n \left(\dfrac{1-a_1}{a_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, \dfrac{1}{1 + \left(n \left(\dfrac{1-b_1}{b_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right], \left[1 - \dfrac{1}{1 + \left(n \left(\dfrac{c_1}{1-c_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}}, 1 - \dfrac{1}{1 + \left(n \left(\dfrac{d_1}{1-d_1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}} \right] \right). \tag{4}$$

3 Interval-valued intuitionistic fuzzy dombi power heronian mean (IVIFDPHM) operator

In this section, Wu et al. (Wu et al., 2020) investigated the IVIFDHM based on HM (Janous, 2001; Khan, Gwak, Shahzad, & Alam, 2021; Panityakul, Mahmood, Ali, & Aslam, 2021; H. Y. Zhang, Wei, & Chen, 2022) and Dombi operations (Akram, Khan, & Saeid, 2021; Jana, Muhiuddin, Pal, & Al-Kadi, 2021; Khan, Liu, Mahmood, Smarandache, & Ullah, 2018; Qiyas, Abdullah, Chinram, & Muneeza, 2022; Ullah et al., 2021).

Definition 7 (Wu et al., 2020). Let $\tilde{a}_j = \left(\left[a_j, b_j\right], \left[c_j, d_j\right]\right)$ $\left(i = 1, 2, \cdots, n\right)$ be the IVIFNs with weight $\omega_i = (\omega_1, \omega_2, \ldots, \omega_n)^T$, thereby satisfying $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$, $p, q \geq 0, \gamma > 0$. The fused value by IVIFDWHM operators is also an IVIFN and, and let IVIFDWHM: $Q^n \rightarrow Q$, if

$$\text{IVIFDWHM}_{\omega}^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\frac{2}{n(n+1)} \mathop{\oplus}_{i=1}^{n} \mathop{\oplus}_{j=i}^{n} \left(\left(\omega_i \tilde{a}_i\right)^p \otimes \left(\omega_j \tilde{a}_j\right)^q \right) \right)^{\frac{1}{p+q}}$$

$$= \left(\begin{array}{l} \left[\dfrac{1}{\left(1 + \left(\dfrac{n(n+1)}{2(p+q)} \times \dfrac{1}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\omega_i A_i^{\gamma}\right) + q/\left(\omega_j A_j^{\gamma}\right)} \right)} \right)\right)^{1/\gamma}}, \dfrac{1}{\left(1 + \left(\dfrac{n(n+1)}{2(p+q)} \times 1 \Big/ \sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\omega_i B_i^{\gamma}\right) + q/\left(\omega_j B_j^{\gamma}\right)} \right) \right)\right)^{1/\gamma}} \right], \\ \left[1 - \dfrac{1}{\left(1 + \left(\dfrac{n(n+1)}{2(p+q)} \times 1 \Big/ \sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\omega_i C_i^{\gamma}\right) + q/\left(\omega_j C_j^{\gamma}\right)} \right) \right)\right)^{1/\gamma}}, 1 - \dfrac{1}{\left(1 + \left(\dfrac{n(n+1)}{2(p+q)} \times 1 \Big/ \sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\omega_i D_i^{\gamma}\right) + q/\left(\omega_j D_j^{\gamma}\right)} \right) \right)\right)^{1/\gamma}} \right] \end{array} \right) \quad (7)$$

where

$$A_i = \frac{1-a_i}{a_i}, B_i = \frac{1-b_i}{b_i}, C_i = \frac{c_i}{1-c_i}, D_i = \frac{d_i}{1-d_i}, A_j = \frac{1-a_j}{a_j}, B_j = \frac{1-b_j}{b_j}, C_j = \frac{c_j}{1-c_j}, D_j = \frac{d_j}{1-d_j}$$

Then the properties of IVIFDWHM are listed (Wu et al., 2020).

Property 1. (Idempotency) If $\tilde{a}_j = ([a_j, b_j],[c_j, d_j])(j = 1,2,\ldots,n) = \tilde{a}$ are equal, then

$$\text{IVIFDWHM}_{\omega}^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a} \quad (8)$$

Property 2. (Monotonicity) Let $\tilde{a}_j = ([a_j, b_j],[c_j, d_j])(j = 1,2,\ldots,n)$ and $\tilde{a}'_j = ([a'_j, b'_j],[c'_j, d'_j])(j = 1,2,\ldots,n)$ be IVIFNs. If $a_j \le a'_j, b_j \le b'_j$ *and* $c_j \ge c'_j, d_j \ge d'_j$ hold for all j , then

$$\text{IVIFDWHM}_{\omega}^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \text{IVIFDWHM}_{\omega}^{p,q}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n) \quad (9)$$

Property 3. (Boundedness) Let $\tilde{a}_j = ([a_j, b_j],[c_j, d_j])(j = 1,2,\ldots,n)$ be IVIFNs.
If

$$\tilde{a}^+ = \left(\left(\left[\max_j(a_j), \max_j(b_j)\right],\left[\min_j(c_j), \min_j(d_j)\right]\right)\right)$$

and

$$\tilde{a}^- = \left(\left(\left[\min_j(a_j), \min_j(b_j)\right],\left[\max_j(c_j), \max_j(d_j)\right]\right)\right)$$

then

$$\tilde{a}^- \le \text{IVIFDWHM}_{\omega}^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \tilde{a}^+ \quad (10)$$

In the following, the interval-valued intuitionistic fuzzy Dombi power Heronian mean (IVIFDPHM) operator is built on IVIFDWHM operator (Wu et al., 2020) and power average operator (Yager, 2001).

Definition 8. Let $\tilde{a}_j = ([a_j, b_j],[c_j, d_j])$ be IVIFNs, then the IVIFDPHM is:

$$\text{IVIFDPHM}^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} \left(\left(\frac{(1+T(\tilde{a}_i))}{\sum_{i=1}^{n}(1+T(\tilde{a}_i))}\tilde{a}_i\right)^p \otimes \left(\frac{(1+T(\tilde{a}_j))}{\sum_{j=1}^{n}(1+T(\tilde{a}_j))}\tilde{a}_j\right)^q\right)\right)^{\frac{1}{p+q}} \quad (11)$$

$T(\tilde{a}_a) = \sum_{\substack{j=1 \\ a \ne j}}^{m} Sup(\tilde{a}_a, \tilde{a}_j)$, and $Sup(\tilde{a}_a, \tilde{a}_j)$ is the support for $\tilde{a}_a$ from $\tilde{a}_j$, with the given conditions:

(1) $Sup(\tilde{a}_a, \tilde{a}_j) \in [0,1]$; (1)

(2) $Sup(\tilde{a}_b, \tilde{a}_a) Sup(\tilde{a}_a, \tilde{a}_b)$

(3) $Sup(\tilde{a}_a, \tilde{a}_b) \ge Sup(\tilde{a}_s, \tilde{a}_t)$, if $d(\tilde{a}_a, \tilde{a}_b) \le d(\tilde{a}_s, \tilde{a}_t)$, where d is a distance measure.

Based on the operations of the IVIFSs described, we can drive the Theorem 1.

Theorem 1. Let be $\tilde{a}_j = ([a_j, b_j],[c_j, d_j])$ be IVIFNs, then the aggregated value by IVIFDPHM operator is also an IVIFNs, and

$$
\begin{aligned}
&\mathrm{IVIFDPHM}^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\\
&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\left(\frac{\left(1+T\left(\tilde{a}_i\right)\right)}{\sum_{i=1}^{n}\left(1+T\left(\tilde{a}_i\right)\right)}\tilde{a}_i\right)^{p}\otimes\left(\frac{\left(1+T\left(\tilde{a}_j\right)\right)}{\sum_{j=1}^{n}\left(1+T\left(\tilde{a}_j\right)\right)}\tilde{a}_j\right)^{q}\right)\right)^{\frac{1}{p+q}}
\end{aligned}
$$

$$
=\left(\begin{array}{l}
\left[\dfrac{1}{\left(1+\left(\dfrac{\frac{n(n+1)}{2(p+q)}}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\left(1+T(\tilde{a}_i)\right)A_i^{\gamma}\Big/\sum_{i=1}^{n}\left(1+T(\tilde{a}_i)\right)\right)+q/\left(\left(1+T(\tilde{a}_j)\right)A_j^{\gamma}\Big/\sum_{j=1}^{n}\left(1+T(\tilde{a}_j)\right)\right)}\right)}\right)^{1/\gamma}\right)},\right.\\
\left.\dfrac{1}{\left(1+\left(\dfrac{\frac{n(n+1)}{2(p+q)}}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\left(1+T(\tilde{a}_i)\right)B_i^{\gamma}\Big/\sum_{i=1}^{n}\left(1+T(\tilde{a}_i)\right)\right)+q/\left(\left(1+T(\tilde{a}_j)\right)B_j^{\gamma}\Big/\sum_{j=1}^{n}\left(1+T(\tilde{a}_j)\right)\right)}\right)}\right)^{1/\gamma}\right)}\right],\\
\left[1-\dfrac{1}{\left(1+\left(\dfrac{\frac{n(n+1)}{2(p+q)}}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\left(1+T(\tilde{a}_i)\right)C_i^{\gamma}\Big/\sum_{i=1}^{n}\left(1+T(\tilde{a}_i)\right)\right)+q/\left(1+T(\tilde{a}_j)\right)C_j^{\gamma}\Big/\sum_{j=1}^{n}\left(1+T(\tilde{a}_j)\right)}\right)}\right)^{1/\gamma}\right)},\right.\\
\left.1-\dfrac{1}{\left(1+\left(\dfrac{\frac{n(n+1)}{2(p+q)}}{\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\dfrac{1}{p/\left(\left(1+T(\tilde{a}_i)\right)D_i^{\gamma}\Big/\sum_{i=1}^{n}\left(1+T(\tilde{a}_i)\right)\right)+q/\left(1+T(\tilde{a}_j)\right)D_j^{\gamma}\Big/\sum_{j=1}^{n}\left(1+T(\tilde{a}_j)\right)}\right)}\right)^{1/\gamma}\right)}\right]
\end{array}\right) \tag{12}
$$

where $A_i=\frac{1-a_i}{a_i}, B_i=\frac{1-b_i}{b_i}, C_i=\frac{c_i}{1-c_i}, D_i=\frac{d_i}{1-d_i}$,

$$A_j=\frac{1-a_j}{a_j}, B_j=\frac{1-b_j}{b_j}, C_j=\frac{c_j}{1-c_j}, D_j=\frac{d_j}{1-d_j}.$$

$T(\tilde{a}_a)=\sum_{\substack{j=1\\ a\neq j}}^{m} Sup(\tilde{a}_a,\tilde{a}_j)$, and $Sup(\tilde{a}_a,\tilde{a}_j)$ is the support for $\tilde{a}_a$ from $\tilde{a}_j$, with the given conditions:

(1) $Sup(\tilde{a}_a,\tilde{a}_j)\in[0,1]$;

(2) $Sup(\tilde{a}_b,\tilde{a}_a)Sup(\tilde{a}_a,\tilde{a}_b)$;

(3) $Sup(\tilde{a}_a,\tilde{a}_b)\geq Sup(\tilde{a}_s,\tilde{a}_t)$, if $d(\tilde{a}_a,\tilde{a}_b)\leq d(\tilde{a}_s,\tilde{a}_t)$, where d is a distance measure.

It can be easily proved that the IVIFDPHM operator has the following properties.

Property 4. (Idempotency) If all $\tilde{a}_j\ (j=1,2,\cdots,n)$ are equal, i.e., $\tilde{a}_j=\tilde{a}$ for all j, then

$$\text{IVIFDPHM}^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\tilde{a} \tag{13}$$

Property 5. (Boundedness) Let $\tilde{a}_j\ (j=1,2,\cdots,n)$ be a collection of IVIFNs, and let

$$\tilde{a}^-=\min_j \tilde{a}_j,\quad \tilde{a}^+=\max_j \tilde{a}_j$$

Then

$$\tilde{a}^-\leq \text{IVIFDPHM}^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leq \tilde{a}^+ \tag{14}$$

Property 6. (Monotonicity)

Let $\langle u_j,\tilde{a}_j\rangle (j=1,2,\cdots,n)$ and $\langle u'_j,\tilde{a}'_j\rangle (j=1,2,\cdots,n)$ be two set of IVIFNs, if $\tilde{a}_j\leq\tilde{a}'_j$, for all j, then

$$\begin{aligned}&\text{IVIFDPHM}^{p,q}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\ &\leq \text{IVIFDWHM}^{p,q}(\tilde{a}',\tilde{a}'_2,\cdots,\tilde{a}'_n)\end{aligned} \tag{15}$$

Property 7. (Commutativity)

Let $\tilde{a}_j\ (j=1,2,\cdots,n)$ and $\tilde{a}'_j\ (j=1,2,\cdots,n)$ be two set of IVIFNs, then

$$\begin{aligned}&\text{IVIFDPHM}^{p,q}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\ &=\text{IVIFDWHM}^{p,q}(\tilde{a}',\tilde{a}'_2,\cdots,\tilde{a}'_n)\end{aligned} \tag{16}$$

where $\tilde{a}_j\ (j=1,2,\cdots,n)$ is any permutation of $\tilde{a}'_j\ (j=1,2,\cdots,n)$.

4 Model for MADM based on the I-IVIFDWHM operator with IVIFNs

In this section, we shall investigate the MADM based on IVIFDPHM operator with IVIFNs. Let $A=\{A_1,A_2,\cdots,A_m\}$ be alternatives, and $G=\{G_1,G_2,\cdots,G_n\}$ be the of attributes. Suppose that $\tilde{R}=(\tilde{r}_{ij})_{m\times n}=([a_{ij},b_{ij}],[c_{ij},d_{ij}])_{m\times n}$ is the IVIF decision matrix, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

In the following, we apply the IVIFDPHM operator to MADM with IVIFNs. The method involves the following steps:

Step 1. Utilize the decision information $\tilde{R}$, and IVIFDPHM operator

$$\begin{aligned}\tilde{r}_i&=([a_i,b_i],[c_i,d_i])\\ &=\text{IVIFDPHM}^{p,q}(\tilde{a}_{i1},\tilde{a}_{i2},\ldots,\tilde{a}_{in})\\ &=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{s=1}^{n}\mathop{\oplus}_{j=s}^{n}\left(\left(\frac{(1+T(\tilde{a}_{is}))}{\sum_{s=1}^{n}(1+T(\tilde{a}_{is}))}\tilde{a}_{is}\right)^{p}\otimes\left(\frac{(1+T(\tilde{a}_{ij}))}{\sum_{j=1}^{n}(1+T(\tilde{a}_{ij}))}\tilde{a}_{ij}\right)^{q}\right)\right)^{\frac{1}{p+q}}\end{aligned} \tag{17}$$

to derive the overall values $\tilde{r}_i\ (i=1,2,\cdots,m)$ for A_i.

Step 2. Calculate the scores $S(\tilde{r}_i), H(\tilde{r}_i)$ of overall values $\tilde{r}_i \ (i=1,2,\cdots,m)$.

Step 3. Rank all the alternatives $A_i \ (i=1,2,\cdots,m)$ and select the best one(s) in accordance with $S(\tilde{r}_i)$ and $H(\tilde{r}_i) \ (i=1,2,\cdots,m)$.

Step 4. End.

5 Numerical example

This section presents a numerical example to evaluate the energy-saving and emission reduction of thermal power plants (Adapted from Mu, 2020). There is a panel with five possible thermal power plants $P_i \ (i=1,2,3,4,5)$ to evaluate. The company selects four attributes to evaluate the five possible thermal power plants (Adapted from Mu, 2020): ①G_1 is the energy-saving system management; ②G_2 is resource and energy consumption; ③G_3 is the comprehensive utilization of resources; ④G_4 is environmental impact. The five possible thermal power plants $P_i \ (i=1,2,3,4,5)$ are to be evaluated using the IVIFNs, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix} ([0.23,0.46],[0.34,0.45]) & ([0.52,0.65],[0.32,0.35]) & ([0.45,0.48],[0.51,0.52]) & ([0.16,0.27],[0.42,0.48]) \\ ([0.36,0.42],[0.37,0.58]) & ([0.43,0.57],[0.42,0.43]) & ([0.36,0.39],[0.27,0.33]) & ([0.29,0.32],[0.42,0.47]) \\ ([0.32,0.41],[0.45,0.54]) & ([0.32,0.61],[0.36,0.39]) & ([0.22,0.26],[0.46,0.49]) & ([0.39,0.42],[0.29,0.32]) \\ ([0.34,0.45],[0.49,0.54]) & ([0.35,0.47],[0.39,0.41]) & ([0.39,0.42],[0.56,0.58]) & ([0.28,0.31],[0.56,0.59]) \\ ([0.41,0.43],[0.37,0.41]) & ([0.31,0.38],[0.41,0.47]) & ([0.27,0.31],[0.34,0.38]) & ([0.49,0.51],[0.47,0.49]) \end{bmatrix}$$

In the following, we apply the IVIFDPHM operator to MADM for evaluating the energy-saving and emission reduction of thermal power plants with IVIFNs. The method involves the following steps:

Step 1. Utilize the matrix $\tilde{R}$, and IVIFDPHM operator, the values $\tilde{r}_i$ of thermal power plants $P_i \ (i=1,2,3,4,5)$ are obtain, $p=2, q=3$.

$$\tilde{r}_1=([0.18,0.25],[0.42,0.47])$$
$$\tilde{r}_2=([0.49,0.56],[0.19,0.24])$$
$$\tilde{r}_3=([0.34,0.37],[0.23,0.29])$$
$$\tilde{r}_4=([0.42,0.45],[0.23,0.27])$$
$$\tilde{r}_5=([0.26,0.29],[0.32,0.39])$$

Step 2. Calculate the scores $S(\tilde{r}_i) \ (i=1,2,\cdots,5)$ of the overall IVIFNs $\tilde{r}_i \ (i=1,2,\cdots,5)$

$$S(\tilde{r}_1)=-0.19, S(\tilde{r}_2)=0.58, S(\tilde{r}_3)=0.36$$
$$S(\tilde{r}_4)=0.43, S(\tilde{r}_5)=0.15$$

Step 3. Rank all the thermal power plants $P_i \ (i=1,2,3,4,5)$ in accordance with $S(\tilde{r}_i)$: $P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$, and thus the most desirable thermal power plant is P_2.

In order to show the effects on the ranking results by changing parameters of p,q in the IVIFDPHM operator, all the results are shown in Tables 2.

In order to show the superiority of IVIFDPHM operator and some comparative studies are also given below. The IVIFDPHM operator is compared with IVIFWA and IVIFWG operators (Z. X. Su, Xia, & Chen, 2011), IVIFZA operator, IVIFZG operator (Mu et al., 2018), IVIFCWA operator (Xian et al., 2017) and I-IIFOWG operator (Wei, 2010). Eventually, the obtained results of these selected methods are obtained in Table 3. From Table 3, the best choice is P_2, while the worst choice is P_1. In other words, these given methods' order is same. Different given methods may tackle MADM from given different angles.

Table 2: Ranking results for different parameters of IVIFDPHM operator.

(p,q)	$S(\tilde{r}_1)$	$S(\tilde{r}_2)$	$S(\tilde{r}_3)$	$S(\tilde{r}_4)$	$S(\tilde{r}_5)$	Order
(1,1)	0.2153	0.4661	0.3431	0.4116	0.305	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(3,3)	0.2796	0.5238	0.4312	0.4892	0.3902	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(5,5)	0.3031	0.5464	0.467	0.5247	0.4241	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(7,7)	0.3163	0.5601	0.4877	0.5457	0.4436	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(9,9)	0.3251	0.5696	0.5014	0.5594	0.4566	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(3,4)	0.2874	0.5312	0.4433	0.4995	0.4017	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(3,5)	0.294	0.5377	0.4536	0.5088	0.4114	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(3,6)	0.2998	0.5434	0.4625	0.517	0.4198	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(4,3)	0.2871	0.5308	0.4422	0.5014	0.4006	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(5,3)	0.2935	0.537	0.4517	0.512	0.4097	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$
(6,3)	0.2988	0.5424	0.46	0.5211	0.4176	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$

Table 3: The obtained results

Methods	order	The best choice	The worst choice
IVIFWA operator (Z. X. Su et al., 2011)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
IVIFWG operator (Z. X. Su et al., 2011)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
IVIFZA operator (Mu et al., 2018)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
IVIFZG operator (Mu et al., 2018)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
IVIFCWA operator (Xian et al., 2017)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
I-IIFOWG operator (Wei, 2010)	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1
IVIFDPHM operator	$P_2 \succ P_4 \succ P_3 \succ P_5 \succ P_1$	P_2	P_1

5 Conclusion

The problems of evaluating the energy-saving and emission reduction of thermal power plants are classical MADM problems. In this paper, the definition of IVIFSs, HM operator, Dombi operations are introduced and the IVIFDPHM operator is proposed. Some desirable properties of this operator are established, such as commutativity, idempotency and monotonicity. Then, the IVIFDPHM operator is used to deal with the IVIF-MADM problems. Finally, an illustrative example for evaluating the energy-saving and emission reduction of thermal power plants is given to verify the built approach. In the future, we shall continue working in the extension and application of the developed operators to other domains (Jana, Pal, & Liu, 2022; Kumar & Chen, 2022; Palanikumar, Arulmozhi, & Jana, 2022; Q. D. Qin, Liang, Li, Chen, & Yu, 2017; Senapati, Chen, & Yager, 2022; Yang & Pang, 2022).

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