

Enhanced Social Group Optimization algorithm for Solving Optimization Problems

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Keywords: Social group optimization, real-world design problems, computational complexity, metaheuristic algorithms, improved algorithms.

Received: January 12, 2024

In the last decades, the field of global optimization has experienced significant growth, leading to the development of various deterministic and stochastic algorithms designed to tackle a wide range of optimization problems. One notable member of this family is the Social Group Optimization (SGO) algorithm. The Improving Phase and the Acquiring Phase are its two main fundamental phases. The two upgraded versions of SGO with a modified improvement phase are Enhanced Social Group Optimization (ESGO) and Enhanced Modified Social Group Optimization (EMSGO). The key enhancement in these variants focuses on honing, refining skills and abilities to achieve greater effectiveness. To assess the performance of ESGO and EMSGO, an extensive comparative analysis is conducted, involving twelve algorithms, including recently introduced, potent metaheuristic methods. Since both ESGO and EMSGO are modified algorithms, a comparison is conducted between these two algorithms and six recently introduced improved/hybrid algorithms. Subsequently, twenty-six real-world design problems from the mechanical and chemical engineering areas are addressed by applying both modified methods. The simulation results leave no doubt about the capability of ESGO and EMSGO to consistently achieve optimal solutions. Their robust performance, both in comparative evaluations and real-world applications, underscores their potential in solving challenging optimization tasks.

Povzetek: Razširjeni algoritem za optimizacijo socialnih skupin (ESGO) izboljšuje izvorno optimizacijo socialnih skupin z dodajanjem faze posnemanja, kar povečuje raznolikost populacije in globalno iskanje rešitev. ESGO je bil uspešno uporabljen pri reševanju kompleksnih optimizacijskih problemov.

1 Introduction

Numerous optimization difficulties have emerged as a result of the technology's rapid progress and need to be addressed. These problems are common in many industries, such as minerals, machinery, chemicals, electronics, finance, and electronics. complex solution spaces and complex relationships of unknown variables are common features of real-world optimization problems. Large numbers of variables, complex nonlinear constraints, and significant computational effort are frequently present in these situations [1-2]. Because they are unable to balance accuracy and time cost, conventional optimization techniques have difficulties effectively addressing these nonproductivity discontinuity problems [3]. Metaheuristic optimization algorithms have demonstrated superior performance in balancing time cost and solution quality [4]. Because of their straightforward structure and absence of requirement that a problem be continuously derivable, metaheuristic optimization

algorithms have been widely used to handle challenging optimization problems in natural and technical fields [5,6].

Metaheuristic algorithms have advanced significantly in the last few decades in terms of hyperparameter self-adaptation, population structure evolution, and theoretical characterization of the search dynamics [7]. A focus of metaheuristic algorithm research is how to balance algorithm exploration and exploitation for improved performance. To achieve balance, many studies utilize other operators or modify the algorithm settings [8]. To balance the exploration and exploitation of TLBO, Satapathy et al. presented an alternative search pattern technique [9]. Some metaheuristic algorithms' search performance during optimization is significantly influenced by programmable factors like crossover rate, mutation rate, and population size. Adaptive parameter control has been thoroughly investigated by researchers in order to address the problem of parameter value control at various phases of the optimization process [10]. OTLBO, a variation of the teaching learning-based optimization

(TLBO) algorithm that Satapathy et al. presented with orthogonal design and generates an optimal offspring by a statistical optimal method where a new selection strategy is applied to decrease the number of generations and make the algorithm converge faster. [11]. Evolution of the population structure has a significant impact on how well metaheuristic algorithms perform in searches. By include elite factors in the hierarchical population structure, Zhong et al. devised the differential evolution (DE) algorithm variant known as EHDE [12]. Wang et al. presented a four-layered GSA variation with a greater search capability dubbed MLGSA, which was inspired by the two-layered structure GSA [13]. Theoretical examination of the search dynamics has recently drawn a lot of interest from scholars in addition to the aforementioned variables [14].

Metaheuristic optimization algorithms can generally be divided into four groups [15]: algorithms based on physics and chemistry, swarm intelligence, human intelligence, and evolutionary principles. The principles of natural evolution serve as the basis for evolutionary-based algorithms. A common example is genetic algorithms, and its proposal was motivated by Darwinian evolution [16]. Genetic algorithms provide solutions through the concept of crossover and mutation of species in nature. DE [17], evolutionary programming [18], and evolutionary strategies [19] are other evolutionary-based algorithms that have been developed. Swarm-based algorithms, which construct optimization models by replicating animal social behaviour, fall under the second classification. ACO [21] and Particle Swarm Optimization (PSO) [20] are two of the most used swarm-based algorithms. By exchanging data on each person involved in the optimization process, they offer solutions. A few examples of swarm-based algorithms include the Artificial Bee Colony (ABC) [22], Whale Optimization Algorithm (WOA) [23], Butterfly Optimization Algorithm (BOA) [24], Seagull Optimization Algorithm (SOA)[25], Sooty Tern Optimization Algorithm (STOA)[26], Chimp Optimization Algorithm(ChOA)[27], Jelly Fish (JS)[28]. Human-based algorithms, which draw their inspiration from human behaviour, are the third group. Some human-based algorithms are Teaching Learning Based Optimization (TLBO)[29], Social Group Optimization (SGO)[30], Past Present Future (PPF)[31], and Mine Blast Algorithm (MBA)[32]. Physical and chemical-based algorithms, which are motivated by the physical laws and cosmological chemical processes, are the fourth type. Gravitational Search Algorithm (GSA)[34] and Simulated Annealing (SA)[33] are two popular ones. Examples of physical and chemical-based algorithms include the Water Cycle Algorithm (WCA)[35], Ray Optimization(RO)[36], and Artificial Chemical Reaction Optimization Algorithm(ACROA)[37].

The SGO algorithm is a novel human-based algorithm proposed by Satapathy et al., inspired by the social behaviour of human for solving complex problem. It can

be seen from literature that SGO has good performance on solving variety of real-world optimization problems [38-46] like many outstanding algorithms. But the NFL theorem [47] encouraged us to improve the SGO even if their performance is competitive with that of other algorithms. As observed in the literature, an algorithm may perform exceptionally well for a specific set of problems but often struggles with others. This phenomenon is supported by the No Free Lunch (NFL) theorem, which encourages researchers to propose new algorithms or improve existing ones. In this context, the SGO algorithm has been modified to enhance its capability in solving real-world problems.

In this paper, the following key contributions are made:

- The enhanced SGO algorithm is introduced in two forms, ESGO and EMSGO, which incorporate practical problem-solving mechanisms based on rational group dynamics, aligning with the foundational principles of SGO.
- The performance of the proposed ESGO and EMSGO algorithms is evaluated using a comprehensive set of 23 benchmark test functions. When compared to contemporary state-of-the-art algorithms, these solutions demonstrate their competitiveness in providing efficient solutions to these test problems while exhibiting faster convergence.
- Since both ESGO and EMSGO are modified algorithms, a comparison is conducted between these two algorithms and six recently introduced improved/hybrid algorithms.
- To further assess the capabilities of ESGO and EMSGO, they are applied to tackle 26 chemical and mechanical design problems. In most cases, the algorithms yield optimal solutions for these real-world challenges.

The subsequent sections of this paper are organized as follows: Section 2 presents a comprehensive review of the fundamental concepts of SGO and MSGO. Section 3 provides a detailed description of the proposed ESGO and EMSGO algorithms. Section 4 verifies the efficacy of the improved strategies and the superiority of the modified algorithms by conducting experiments using classical test functions and addressing real-world optimization problems. Finally, in Section 5, the conclusions are presented, and avenues for future research are explored.

2 Social group optimization and modified social group optimization

2.1 Social group optimization (SGO)

The SGO algorithm, which is intended to handle complex problems, takes inspiration from human social behaviour. According to this algorithm, members of a social group

are potential solutions since they each have the knowledge and abilities needed to solve the given problem. Individuals' human characteristics match the dimensions of the design factors in the problem. The Enhancing Phase and the Acquiring Phase are the two stages of the optimization process.

Consider a social group denoted as P_i , with i ranging from 1 to pop_size , representing the group's individuals. Each individual, P_i , is characterized by traits ($p_{i1}, p_{i2}, p_{i3}, \dots, p_{iD}$), where D signifies the defining dimensions. Every individual is associated with a fitness value, f_i , reflecting their fitness levels.

Phase 1: Improving Phase

In this phase, the best individual, 'gbest,' shares knowledge with the entire group, enhancing their collective knowledge. Each individual updates their information based on 'gbest' according to the formula:

$$P_{new_i} = c * P_i + r * (g_{best} - P_i) \quad (1)$$

The new solution, P_{new_i} , is accepted only if it improves fitness. Here, 'r' is a random number from $U(0, 1)$, and 'c' is the self-introspection parameter ($c=0.2$).

Phase 2: Acquiring Phase

In this phase, an individual interacts with the best performer ($best_P$) and engages in random interactions with other group members to acquire knowledge. The update is determined by:

Randomly select one person P_r , where $i \neq r$

If $f(P_i) < f(P_r)$

$$P_{new_i} = P_i + r_1 * (P_i - P_r) + r_2 * (best_P - P_i)$$

Else

$$P_{new_i} = P_i + r_1 * (P_r - P_i) + r_2 * (best_P - P_i)$$

End if (2)

The new solution is accepted if it enhances fitness. Here, r_1, r_2 , and r_3 are random numbers from $U(0, 1)$, introducing stochasticity. 'lb' and 'ub' represent lower and upper bounds of design variables. For detailed insights, refer to the paper [30].

2.2 Modified social group optimization (MSGO)

In the MSGO, the Improving Phase remains same as like SGO. Only Acquiring Phase has been modified in the following manner:

Phase 2: Acquiring Phase

A social group member engages in interactions with the best performer ($best_P$) in the same group during the Acquiring Phase. In order to learn, they simultaneously strike up conversations at random with other group members. When the other person knows more than the interacting person does, and the interacting person has a higher Self-Awareness Probability (SAP) of learning that knowledge, new knowledge is acquired. SAP is a measure

of an individual's ability to learn from others. The following is an outline of the Acquiring Phase:

```

For i = 1 to pop_size
  Randomly select one person P_r, where i ≠ r
  If f(P_i) < f(P_r)
    If rand > SAP
      P_new_i = P_i + r_1 * (P_i - P_r) + r_2 * (best_P - P_i)
    Else
      P_new_i = lb + r_3 * (ub - lb)
    end if
  Else
    P_new_i = P_i + r_1 * (P_r - P_i) + r_2 * (best_P - P_i)
  End If
End for
(3)

```

The acceptance of the new solution, P_{new_i} , is contingent upon its ability to yield enhanced fitness relative to the current solution. In this context, r_1, r_2 , and r_3 denote three independent random numbers drawn from a uniform distribution $U(0, 1)$, introducing stochastic elements into the algorithm. The terms 'lb' and 'ub' represent the lower and upper bounds of the corresponding design variable, and the SAP is fixed at 0.7. For a more in-depth understanding of the SGO algorithm, please consult the referenced paper [15].

3 Proposed ESGO (Enhanced SGO) and EMSGO (Enhanced MSGO)

3.1 ESGO, and EMSGO algorithms

To enhance or refine a skill or ability to a higher level of effectiveness in the SGO algorithm, the Improving phase undergoes the following modifications.

Initially, a subset of the best-performing individuals (gbest persons) is selected from the social group. The knowledge level of each individual within this group is then enhanced through interactions with these superior (gbest) persons. As the iterations progress, the number of members in the gbest group diminishes. Eventually, only one dominant (gbest) individual remains within the social group. This modification of improving phase is adapted in the following manner:

- 1) Calculate the number of gbest individuals (NG): $NG = \text{floor}(0.1 * pop_size * (1 - \text{iter} / \text{max_iter})) + 1$
- 2) Sort the fitness values in descending order and store the corresponding best values:
[value_best] = sort(f, 'descend')
- 3) For each of the top NG individuals (indexed as j):
for j = 1:NG
GG_j = P_best_j (4)
GGf_j = value_j
End

- 4) Iterate through the entire population ($i = 1$ to pop_size):
 For $i = 1$ to pop_size
 Randomly select one individual, GG_r .
- If the fitness value of GG_r (GGf_r) < fitness value of the current individual (f_i):
 $\text{Pnew}_i = c * \text{P}_i + r_4 * (\text{GG}_r - \text{P}_i)$ (5)
 Else
 $\text{Pnew}_i = c * \text{P}_i + r_5 * (\text{P}_i - \text{GG}_r)$
 End if

Only accept the new solution, Pnew_i , if it results in a higher level of fitness than the existing solution. Here, r_4 and r_5 are two independent random numbers drawn from a uniform distribution $U(0, 1)$.

The ESGO and EMSGO algorithms are derived by replacing Improving phase by the above modified Improving phase into the SGO and EMSGO algorithms, respectively.

3.2 Phases of ESGO and EMSGO algorithm, Exploration and Exploitation concept

Each of the two phases constituting the ESGO and EMSGO algorithms emphasizes distinct aspects of exploration and exploitation within the optimization framework.

- a) *Improving Phase (Exploration Emphasis)*: Initially, a subset of the top-performing individuals (referred to as "gbest persons") is chosen from the social group. The individuals then undergo a process of knowledge enhancement through interactions with these superior gbest persons. As the iterations progress, the size of the gbest group gradually diminishes, eventually leaving only one dominant gbest individual within the social group. This adjustment in the improving phase facilitates improved knowledge transfer among individuals, thereby enhancing their exploration capabilities more rapidly. Throughout this phase, the primary focus is on exploration.
- b) *Acquiring Phase (Transition to Exploitation)*: A social group member participates in a discussion with the group's top performer during this period. They also strike up conversations at random with other group members in an effort to learn more. When interacting with someone who knows more, an individual absorbs new information because they are more likely to have a higher Self-Awareness Probability (SAP) for learning that information. SAP is a measure of an individual's ability to learn from
- c) others. At this point, people start using the knowledge they have learned for optimization, marking the shift from exploration to exploitation.

3.3 Discussion of computational complexity of ESGO and EMSGO algorithms

BigO ($\text{TSD} + \text{TNC}_f$), where T is the number of iterations, S is the population size or the number of agents, C_f is the cost of function evaluation, and D is the problem dimension, represents the computational complexity of SGO.

In analysing the temporal complexity of most algorithms, three main factors are usually taken into account:

- BigO (SD) computational complexity is usually associated with population initialization, where S denotes population size and D denotes problem dimension.
- BigO (SC_f) is frequently used to limit the computational cost of the initial function evaluation (FE).
- BigO ($\text{TSD} + \text{TNC}_f$) usually sets a limit on the main loop's computational complexity.

Consequently, the overall computational complexity of the ESGO and EMSGO algorithms remains the same as the SGO algorithm, as both the ESGO and EMSGO algorithms are derived by introducing a modified improving phase, where the computational complexity of the modified improving phase is BigO ($\text{TSD} + \text{TNC}_f$).

It follows that SGO, EMSGO, and EMSGO have similar computational complexity, which is represented by the notation BigO ($\text{TSD} + \text{TNC}_f$).

Algorithm 1 gives the pseudo-code for the proposed ESGO and EMSGO algorithm.

Algorithm 1: Pseudo code of ESGO /EMSGO algorithm

- Set up the search agent population (persons).
- Algorithm parameters definition: C, SAP
- While $\text{iter} < \text{Max_iter}$:
 Determine the best current solution by doing fitness calculations
 Select the best solutions and make gbest persons group.
 Within the population (pop_size), for every agent i:
 Update the person using Equation 5.
 End for.
 Perform fitness calculations and update the current position of persons
 Update the current best solution.
 Within the population (pop_size), for every agent i:
 Update the person using Equation 2 for ESGO algorithm/ Update the person using Equation 3 for EMSGO algorithm
 End for
 Perform fitness calculations and update the current position of persons
 Update the current best solution.
- End while.

4 Simulation, experimental result, and discussions

The performance of the ESGO and EMSGO algorithms is demonstrated in this paper through four experiments. In experiment 1, both algorithms are compared with each other and also with their original algorithm SGO and MSGO respectively. In the second experiment, the performances of both algorithms are compared with twelve state-of-the-art algorithms such as African vultures optimization algorithm (AVOA) [48], DE [49], Exponential distribution optimizer (EDO)[50], GWO [51], Kepler optimization algorithm (KOA) [52], Light Spectrum Optimizer (LSO) [53], Mantis Search Algorithm (MSA)[54], Nutcracker optimizer algorithm (NOA) [55], Reptile Search Algorithm (RSA) [56], Slime mould algorithm (SMA) [57], Spider wasp optimizer (SWO)[58], and WOA [23]. In the 3rd experiment, the performance of both algorithms is compared with six improved/hybrid recently introduced algorithms. In the 4th experiment, both algorithms show their performance in solving twenty-six real-world constrained optimization problems of mechanical and chemical design problems.

Every novel optimization algorithm must undergo rigorous evaluation using well-defined benchmark functions to assess and validate its performance. Although there are numerous benchmark functions available, however there is no standardized set of benchmarks that are agreed upon for evaluating new algorithms. In order to validate and benchmark the performance of our proposed ESGO and EMSGO algorithms, the simulations are conducted on a set of twenty-three benchmark functions. These carefully selected benchmark functions serve as a comprehensive testbed for assessing various aspects of the algorithms, including their ability to achieve rapid convergence, escape local optima, and prevent premature convergence. The selection of these benchmark functions is motivated by their widespread use in existing literature [57, 59-63]. Out of the twenty-three functions, seven are unimodal benchmark functions (F1–F7), ideal for benchmarking the exploitation capabilities of algorithms due to their single global optimum. Six are multimodal benchmark functions, while ten are fixed-dimensional multimodal benchmark functions. Each of the multimodal functions, from F8 to F23, contains a multitude of local optima, making them well-suited for evaluating the

exploration capabilities of algorithms. For a comprehensive understanding of these benchmark functions, detailed descriptions can be found in reference [9], and graphical representations are provided in Figure 1. Experiments 1-3 use these benchmark functions to validate the performance comparisons among algorithms. The detailed descriptions of twenty-six real-world constrained optimization problem of mechanical and chemical design problems are given in [64] which is used

in experiment 4 to validate the performance algorithms. Implemented on the Windows 10 operating system, MATLAB 2016a is employed to execute all algorithms. The simulations are conducted on a laptop equipped with an Intel Core i5 processor and 8 GB of memory.

4.1 Algorithm validation

To assess the performance of the ESGO and EMSGO algorithms, a set of 23 benchmark functions is utilized, with results compared against twelve different metaheuristic algorithms, as previously outlined. In Experiment 1, a comparative analysis is conducted between ESGO, SGO, MSGO, and EMSGO, with the results presented in Table 2. In Experiment 2, the modified algorithms are compared with the twelve other algorithms, and the comparative outcomes are showcased in Table 3. Similarly, in Experiment 3, ESGO and EMSGO are compared with six recently introduced improved/hybrid algorithms, with the results imported in Table 5. Throughout these experiments, a fixed parameter, max_FEs, is maintained at 15,000. Consequently, the number of iterations and population size may vary for different algorithms. The parameter configurations for the algorithms align with widely accepted settings utilized by various researchers, as detailed in Table 1.

Experiment 1: The performance comparison of the SGO family of algorithms

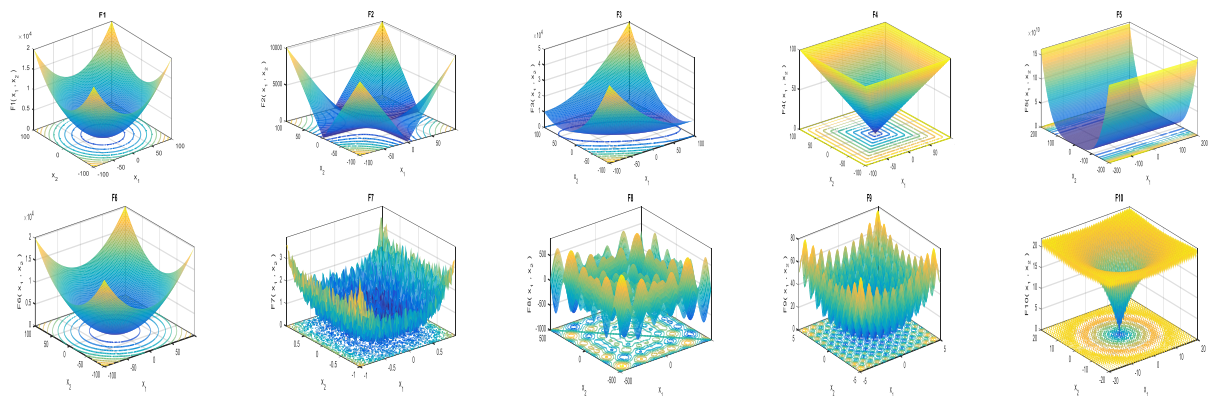
In this specific experiment, the performance of the SGO algorithm family, which includes ESGO, SGO, EMSGO, and MSGO, is assessed through comparative analysis. To ensure the robustness and statistical significance of the findings, the experiment is conducted 30 times. The outcomes are presented in Table 2, detailing key metrics such as the best (BEST), worst (WORST), average (MEAN), and standard deviation (SD) of fitness solutions. Noteworthy results are highlighted in bold within the tables, and the symbol '||' denotes that the value remains consistent with the preceding row.

Discussion

It is seen from Table 2 that the ESGO algorithm reaches the global optimum for twelve functions, only best solution reaches optimal solution in four cases, and in four cases find good solutions in compare to others. The EMSGO algorithm reaches the global optimum for fifteen functions, only best solution reaches optimal solution in one case, and in three cases find good solutions in compare to others. The MSGO algorithm reaches the global optimum for thirteen functions, and only best solution reaches optimal solution in two cases. Similarly, the SGO algorithm reaches the global optimum for eight functions, only best solution reaches optimal solution in three cases, and in one case find good solutions in compare to others. Hence, it can be concluded that both the ESGO and EMSGO algorithms achieve improved results.

Table 1: Parameter setting of algorithms compared to the SGO and MSGO algorithms

Sl. No.	Algorithms	Parameters	Values
1	SGO	C	0.2
	MSGO	C SAP	0.2 0.7
2	GWO	Control parameter	[2, 0]
3	AVOA	p1 p2 p3 alpha betha gamma	0.6 0.4 0.6 0.8 0.2 2.5
4	SMA	Parameter	0.03
5	EDO	f= 2*rand-1 a=f^10 b=f^5 c=d*f	
6	DE	F Cr	0.5 0.5
7	KOA	Tc M0 lambda	3 0.1 15
8	LSO	Ps Pe Ph B	0.05 0.6 0.4 0.05
9	MSA	p A a P Alp Pc	0.5 1.0 0.5 2 6 0.2
10	NOA	Alpha Pa2 Prb	0.05 0.2 0.2
11	RSA	Alpha value Beta	0.1 0.1 0.1
12	SWO	TR Cr N_min=20	0.3 0.2 20
13	WOA	Spiral updating probability Shrinking encircling Random search ability	0.5 0.5 0.1
14	ESGO	C	0.2
15	EMSGO	C SAP	0.2 0.7



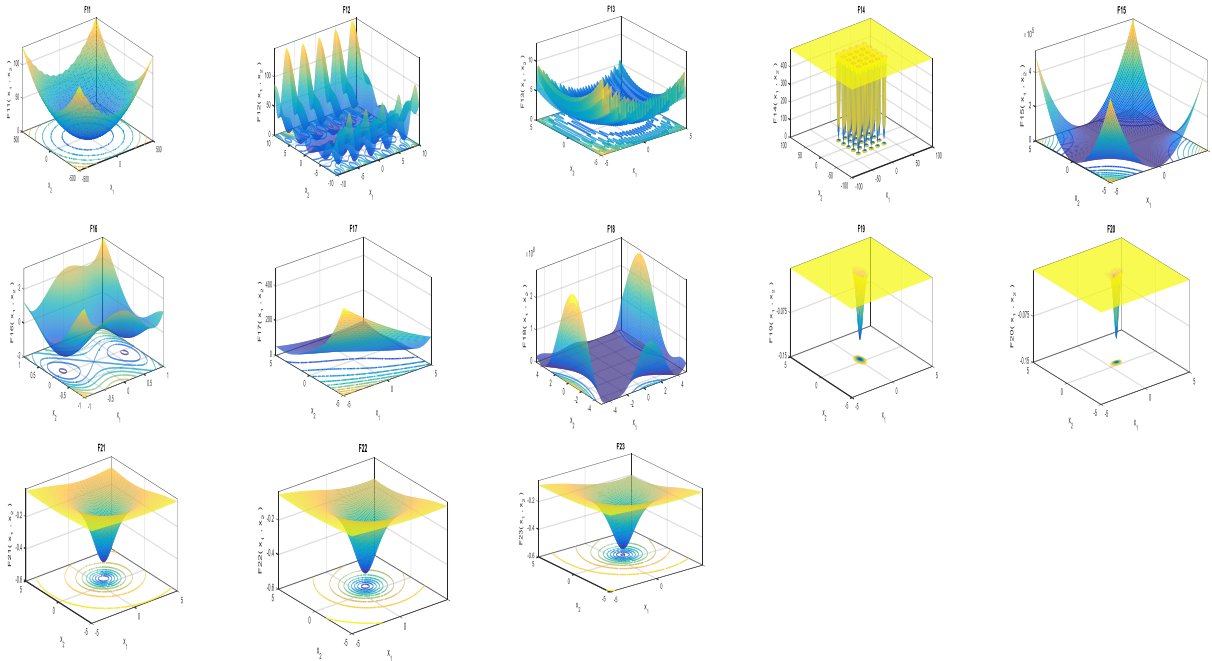


Figure 1: Graphical representation of classical benchmark functions

Table 2: Comparison results of family of SGO algorithms

Algo/Fu nctions		F1	F2	F3	F4	F5	F6
ESGO	BEST	0	4.1803e-199	8.2277e-149	5.9334e-140	3.4173e-09	0
	WORST	0	1.9582e-189	3.3997e-140	7.4641e-135	2.8200e-08	1.8489e-32
	MEAN	0	1.9721e-190	9.1732e-145	1.4414e-135	1.3524e-08	4.9304e-33
	STD	0	0	1.2569e-146	2.4817e-135	8.3790e-09	7.4354e-33
SGO	BEST	9.2725e-206	1.5010e-103	1.2056e-205	1.1175e-103	25.2399	1.4422e-05
	WORST	1.4198e-205	1.8192e-103	2.8262e-205	1.2648e-103	26.5470	7.4685e-04
	MEAN	1.2120e-205	1.6575e-103	2.1541e-205	1.2011e-103	25.9628	1.7258e-04
	STD	0	1.0090e-104	0	4.4263e-105	0.3815	2.4597e-04
EMSG O	BEST	0	1.1289e-259	8.2056e-239	2.5451e-130	0	4.1842e-05
	WORST	0	2.4844e-196	5.7833e-179	2.1033e-125	1.2171e-04	5.3231e-04
	MEAN	0	2.5618e-197	5.7883e-180	2.3576e-126	1.4548e-05	2.4457e-04
	STD	0	0	0	6.5915e-126	3.8171e-05	1.7999e-04
MSGO	BEST	3.0864e-212	2.7434e-106	1.6631e-207	7.0946e-106	0	2.4271e-06
	WORST	1.1362e-205	1.7802e-103	6.3071e-195	4.8986e-104	0.1337	0.0215
	MEAN	4.1136e-206	7.9570e-104	6.3071e-196	1.8810e-104	0.0244	0.0112
	STD	0	5.4012e-104	0	1.5099e-104	0.0518	0.0075
Algo/Fu nctions		F7	F8	F9	F10	F11	F12
ESGO	BEST	2.0781e-05	-1.0832e+04	0	8.8818e-16	0	1.5705e-32
	WORST	8.1608e-05	-7.5157e+03	28.8538	8.8818e-16	0.0123	1.5705e-32
	MEAN	4.3626e-05	-9.0179e+03	17.3123	8.8818e-16	0.0012	1.5705e-32
	STD	2.0376e-05	1.1567e+03	10.2887	0	0.0039	2.8850e-48
SGO	BEST	5.0521e-06	-9.6243e+03	0.0039	'''	0	2.0146e-06
	WORST	2.5723e-04	-5.7770e+03	0.0039	'''	0	4.4800e-05
	MEAN	1.2684e-04	-7.6685e+03	0.0039	'''	0	1.0564e-05
	STD	8.3738e-05	1.4999e+03	0	'''	0	1.2502e-05
EMSG O	BEST	9.1415e-06	-1.2569e+04	0	'''	'''	1.0378e-07
	WORST	6.1383e-05	-1.2569e+04	0	'''	'''	2.9400e-05

	MEAN	3.6674e-05	-1.2569e+04	0			8.4505e-06
	STD	2.0307e-05	5.9041e-05	0			1.1504e-05
MSGO	BEST	1.2110e-05	-1.2569e+04	" "	" "	" "	7.5531e-07
	WORST	1.6058e-04	-1.2569e+04				7.5531e-07
	MEAN	8.3206e-05	-1.2569e+04				7.0659e-05
	STD	6.1619e-05	0.0559				1.7495e-04
Algo/Fu nctions		F13	F14	F15	F16	F17	F18
ESGO	BEST	1.3498e-32	0.9980	3.0749e-04	-1.0316	0.3979	3.0000
	WORST	0.0110	0.9980	3.0749e-04	-1.0316	0.3979	3.0000
	MEAN	0.0033	0.9980	3.0749e-04	-1.0316	0.3979	3.0000
	STD	0.0053	0	1.0537e-19	0	0	1.0978e-15
SGO	BEST	3.2121e-05	0.9980	3.0749e-04	" "	" "	3.0000
	WORST	0.0979	0.9980	3.1132e-04			3.0000
	MEAN	0.0110	0.9980	3.0867e-04			3.0000
	STD	0.0307	7.4015e-17	1.3701e-06			5.1279e-16
EMSG O	BEST	1.0838e-09	0.9980	3.0749e-04	" "	" "	3.0000
	WORST	9.8844e-05	0.9980	3.0749e-04			3.0000
	MEAN	2.5227e-05	0.9980	3.0749e-04			3.0000
	STD	3.6575e-05	0	1.0537e-20			0
MSGO	BEST	4.0152e-06	" "	3.0749e-04	" "	" "	3.0000
	WORST	0.0110		7.7817e-04			3.0000
	MEAN	0.0024		5.0234e-04			3.0000
	STD	0.0037		2.0656e-04			2.1251e-16
Algo/Fu nctions		F19	F20	F21	F22	F23	
ESGO	BEST	-3.8628	-3.3220	-10.1532	-10.4029	-10.5364	
	WORST	-3.8628	-3.2031	-10.1532	-10.4029	-10.5364	
	MEAN	-3.8628	-3.2982	-10.1532	-10.4029	-10.5364	
	STD	9.3622e-16	0.0501	1.3240e-15	1.6748e-15	1.32149e-15	
SGO	BEST	" "	-3.3220	-5.0552	-10.4029	-5.1756	
	WORST		-3.2031	-5.0552	-5.0877	-5.1285	
	MEAN		-3.2863	-5.0552	-5.6192	-5.1332	
	STD		0.0574	0	1.6808	0.0149	
EMSG O	BEST	" "	-3.3220	-10.1532	-10.4029	-10.5364	
	WORST		-3.3220	-10.1532	-10.4029	-10.5364	
	MEAN		-3.3220	-10.1532	-10.4029	-10.5364	
	STD		9.2038e-09	0	1.8724e-15	2.7773e-15	
MSGO	BEST	" "	-3.3220	-10.1532	-10.4029	-10.5364	
	WORST		-3.2031	-10.1532	-10.4029	-10.5364	
	MEAN		-3.2863	-10.1532	-10.4029	-10.5364	
	STD		0.0574	1.3240e-15	1.6748e-15	2.1349e-15	

Experiment 2: The performance comparison with state-of-the-art metaheuristics algorithms

Based on the results obtained from Experiment 1, it is evident that ESGO and EMSGO exhibit superior performance in terms of fitness function evaluation when compared to other algorithms. Consequently, in this

experiment, ESGO and EMSGO are subjected to a comprehensive comparison with the remaining twelve algorithms to validate their performance. The experiment is repeated 30 times, with the statistical results—including the BEST, WORST, MEAN, and SD of fitness solutions presented in Table 3. This rigorous analysis is designed to ensure stability and establish statistical significance, with the most remarkable results highlighted in bold in the

table, and the symbol "||" indicating that its value is equivalent to the value in the preceding column. Table 4 reports the p-values derived from the WRS test [65] at a significance level of 5% for ESGO(E) versus other approaches and EMSGO(EM) versus other approaches. When p-values fall below 0.05, it indicates a rejection of

the null hypothesis, while "N" signifies that the input values are similar. Additionally, in Table 4, "-" indicates that the performance of other approaches is inferior, "+" signifies it is superior, and "S" suggests a similar performance when compared to ESGO and EMSGO.

Table 3: Comparison Results of ESGO, EMSGO and other algorithms

Algo/Function		F1	F2	F3	F4	F5	F6
ESGO	BEST	0	5.2836e-199	3.5401e-46	3.1051e-142	7.4975e-10	0
	WORST	0	3.1359e-190	2.6954e-42	1.9962e-134	2.4696e-08	0
	MEAN	0	3.7878e-191	8.1478e-43	2.7076e-135	8.9214e-09	0
	SD	0	0	1.0752e-42	6.1875e-135	9.9187e-09	0
EMSGO	BEST	0	6.9340e-263	2.4211e-243	7.1047e-133	0	9.7104e-07
	WORST	0	1.7585e-199	3.1882e-186	4.3710e-125	0.0032	3.8264e-04
	MEAN	0	1.8564e-200	3.1982e-187	8.3185e-126	4.2652e-04	1.0457e-04
	SD	0	0	0	1.6478e-125	0.0010	1.4912e-04
AVOA	BEST	7.7025	0.0334	53.4364	0.0490	30.7586	37.9652
	WORST	2.3803e+04	72.2977	6.9949e+04	67.0993	8.7707e+07	2.6602e+04
	MEAN	8.9613e+03	46.2646	3.8489e+04	38.0639	2.9522e+07	8.2425e+03
	SD	1.0486e+04	26.2007	2.6043e+04	26.1463	3.8485e+07	1.1002e+04
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
DE	BEST	4.4059e+04	163.1432	8.4222e+04	80.8065	1.9217e+08	4.2435e+04
	WORST	6.5836e+04	1.7386e+10	1.0999e+05	89.0112	2.3023e+08	6.2838e+04
	MEAN	5.8571e+04	3.4780e+09	9.4515e+04	85.4072	2.0369e+08	5.6172e+04
	SD	6.4844e+03	5.8684e+09	8.7802e+03	2.5891	1.1840e+07	6.4762e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
EDO	BEST	1.4834e-102	5.0862e-56	0	1.5668e-51	28.7129	0.5508
	WORST	8.0060e-85	1.2305e-40	5.2972e-76	3.9187e-40	28.7434	1.1080
	MEAN	1.0280e-85	1.2404e-41	5.5843e-77	3.9239e-41	28.7295	0.8615
	SD	2.5035e-85	3.8879e-41	1.6674e-76	1.2390e-40	0.0105	0.1757
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	0.0028	" "	1.7265e-04	1.8267e-04
GWO	BEST	1.1432e+04	45.8397	1.9213e+04	45.0504	1.1085e+07	1.0877e+04
	WORST	1.9109e+04	1.0544e+03	4.3169e+04	56.7561	2.6838e+07	1.8265e+04
	MEAN	1.4651e+04	209.3304	3.1801e+04	51.0770	1.7175e+07	1.5388e+04
	SD	2.7088e+03	310.0702	7.1282e+03	4.0340	5.5991e+06	2.7820e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
KOA	BEST	5.4244e+04	2.5173e+06	6.4049e+04	77.5024	1.6814e+08	4.2841e+04
	WORST	6.5658e+04	4.3553e+10	1.2905e+05	86.0408	2.2523e+08	6.7207e+04
	MEAN	6.0795e+04	1.3480e+10	9.6211e+04	83.4309	2.0486e+08	5.8797e+04
	SD	3.7316e+03	1.5055e+10	2.6865e+04	2.6637	1.9144e+07	8.3548e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
LSO	BEST	0	0	0	0	29	7.5000
	WORST	0	0	0	0	29	7.5000
	MEAN	0	0	0	0	29	7.5000
	SD	0	0	0	0	0	0
	P-value(E)	NaN	6.3864e-05	6.3864e-05	6.3864e-05	6.3864e-05	1.5938e-05
	P-value(EM)	0.3681	" "	" "	" "	5.9363e-05	6.3864e-05
MSA	BEST	3.8432e+04	1.0814e+05	4.1720e+04	69.9315	8.1597e+07	4.0168e+04
	WORST	3.8432e+04	1.0814e+05	4.1720e+04	69.9315	8.1597e+07	4.0168e+04
	MEAN	4.3015e+04	2.0010e+06	5.2453e+04	72.9467	1.0748e+08	4.4521e+04

	SD	2.6972e+03	2.2635e+06	5.9061e+03	1.8691	1.6092e+07	2.4725e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
NOA	BEST	4.8660e+04	2.9131e+08	7.3451e+04	77.6661	1.4000e+08	5.1567e+04
	WORST	6.6244e+04	8.0625e+10	1.3115e+05	86.9414	2.2977e+08	6.3968e+04
	MEAN	5.7156e+04	2.3403e+10	1.0086e+05	83.1270	1.8809e+08	5.8477e+04
	SD	6.0233e+03	2.7126e+10	2.0704e+04	3.2065	3.0255e+07	3.7992e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	8.7450e-05	" "	" "	" "	1.7265e-04	1.8267e-04
RSA	BEST	0	0	0	0	29	7.5000
	WORST	0	0	0	0	29	7.5000
	MEAN	0	0	0	0	29	7.5000
	SD	0	0	0	0	0	0
	P-value(E)	NaN	6.3864e-05	6.3864e-05	6.3864e-05	6.3864e-05	1.5938e-05
	P-value(EM)	0.3681	" "	" "	" "	" "	" "
SMA	BEST	4.1063e-05	0.0080	0.0037	0.1079	28.9931	7.1919
	WORST	0.1126	0.5269	292.4747	0.3502	30.1790	9.2486
	MEAN	0.0402	0.1431	71.5265	0.1975	29.4534	7.7576
	SD	0.0342	0.1830	98.6191	0.0770	0.3974	0.6508
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8165e-04	6.3864e-05
	P-value(EM)	" "	" "	" "	" "	" "	1.8267e-04
SWO	BEST	4.8980e+04	3.6923e+05	5.6283e+04	78.8348	1.6983e+08	5.3525e+04
	WORST	6.3486e+04	7.6522e+10	1.0584e+05	84.7210	2.3214e+08	6.6370e+04
	MEAN	5.8234e+04	1.3004e+10	8.6366e+04	82.0490	1.9669e+08	5.8365e+04
	SD	4.7426e+03	2.4953e+10	1.4348e+04	2.0315	2.2956e+07	4.1333e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	" "	" "	" "	" "	" "	1.8267e-04
WOA	BEST	1.0763e+04	37.4285	7.9057e+04	53.3947	4.6152e+06	8.5596e+03
	WORST	2.2247e+04	171.652	1.5291e+05	86.0943	6.9715e+07	3.7200e+04
	MEAN	1.6084e+04	80.5780	1.1317e+05	76.7905	2.7538e+07	1.8897e+04
	SD	4.4502e+03	42.3134	2.3395e+04	10.7489	2.0558e+07	8.2834e+03
	P-value(E)	6.3864e-05	1.8267e-04	1.8267e-04	1.8267e-04	1.8267e-04	6.3864e-05
	P-value(EM)	" "	" "	" "	" "	" "	1.8267e-04
Algo/Function		F7	F8	F9	F10	F11	F12
ESGO	BEST	4.3625e-05	-1.1089e+04	0	8.8818e-16	0	1.5705e-32
	WORST	1.3583e-04	-7.9700e+03	23.8790	8.8818e-16	0	1.6109e-32
	MEAN	6.9865e-05	-8.8463e+03	12.1385	8.8818e-16	0	1.5802e-32
	SD	2.9670e-05	962.9930	9.0560	0	0	1.5117e-34
EMSGO	BEST	2.3125e-05	-1.2569e+04	0	8.8818e-16	0	7.0940e-12
	WORST	6.3363e-05	-1.2569e+04	0	8.8818e-16	0	9.1524e-06
	MEAN	4.3487e-05	-1.2569e+04	0	8.8818e-16	0	1.8881e-06
	SD	1.3955e-05	7.2809e-04	0	0	0	2.9886e-06
AVOA	BEST	0.2208	-4.1571e+03	55.3583	1.1428	1.0421	0.2444
	WORST	23.8537	-3.2544e+03	325.4436	12.2769	229.6541	5.5755e+07
	MEAN	11.7494	-3.6179e+03	217.0793	6.9852	44.9105	5.7285e+06
	SD	9.9263	256.5975	108.4945	3.8592	89.3992	1.7581e+07
	P-value(E)	1.8267e-04	1.8267e-04	6.3864e-05	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	" "	" "	" "	" "
DE	BEST	66.7799	-3.3978e+03	335.9169	20.4348	462.1846	2.8041e+08
	WORST	108.1115	-2.3055e+03	415.4900	20.6292	588.1518	5.4382e+08
	MEAN	93.8855	-2.7284e+03	398.0476	20.5596	516.8565	4.0688e+08
	SD	12.6976	315.6260	22.9089	0.0620	37.1096	8.8929e+07
	P-value(E)	1.8267e-04	1.8267e-04	1.7861e-04	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	6.3864e-05	" "	" "	1.8267e-04
EDO	BEST	5.6818e-05	-1.2566e+04	0	8.8818e-16	0	0.0239

	WORST	7.1180e-04	-1.2352e+04	0	8.8818e-16	0	0.096
	MEAN	2.8708e-04	-1.2489e+04	0	8.8818e-16	0	0.064
	SD	2.2037e-04	69.8756	0	0	0	0.024
	P-value(E)	0.0017	1.8267e-04	0.00	NaN	NaN	1.4939e-04
	P-value(EM)	4.3964e-04	" "	NaN	NaN	NaN	1.8267e-04
GWO	BEST	3.9947	-3.8122e+03	231.6198	16.0005	106.2397	1.0871e+06
	WORST	11.1918	-2.2596e+03	311.4545	18.2003	185.9527	4.0395e+07
	MEAN	6.2076	-2.8373e+03	272.2285	17.4764	147.1419	1.5626e+07
	SD	2.4455	574.3139	30.7015	0.6500	28.2347	1.4349e+07
	P-value(E)	1.8267e-04	1.8267e-04	1.7861e-04	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	1.8267e-04	1.8267e-04	1.1067e-04	6.3864e-05	6.3864e-05	1.8267e-04
KOA	BEST	77.6168	-5.4177e+03	367.0005	19.9668	387.9851	3.1296e+08
	WORST	112.4230	-5.4177e+03	424.7784	19.9668	575.4793	5.0309e+08
	MEAN	97.2831	-5.4177e+03	399.6969	19.9668	516.5733	4.3850e+08
	SD	12.8711	9.5869e-13	22.5140	0	54.2910	6.1302e+07
	P-value(E)	1.8267e-04	6.3864e-05	1.7861e-04	1.5938e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	1.1067e-04	" "	" "	1.8267e-04
LSO	BEST	0.0258	-5.0565e+03	0	8.8818e-16	0	1.669
	WORST	0.1386	-2.3825e+03	0	8.8818e-16	0	1.669
	MEAN	0.0685	-3.1907e+03	0	8.8818e-16	0	1.669
	SD	0.0412	1.0299e+03	0	0	0	2.3406e-16
	P-value(E)	1.8267e-04	1.8165e-04	0.0022	NaN	NaN	4.9177e-05
	P-value(EM)	" "	" "	NaN	NaN	NaN	6.3864e-05
MSA	BEST	45.5181	-4.4134e+03	336.7071	19.3659	366.8070	9.5920e+07
	WORST	45.5181	-4.4134e+03	336.7071	19.3659	366.8070	9.5920e+07
	MEAN	51.7201	-4.0591e+03	346.9462	19.7710	398.6249	1.7816e+08
	SD	4.7908	195.7146	6.9562	0.1759	26.6722	3.8232e+07
	P-value(E)	1.8267e-04	1.8267e-04	1.7861e-04	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	1.1067e-04	" "	" "	1.8267e-04
NOA	BEST	79.4873	-5.4177e+03	364.2811	19.9668	471.3946	2.5340e+08
	WORST	120.1875	-5.4177e+03	434.5688	19.9668	592.1890	5.7029e+08
	MEAN	103.4432	-5.4177e+03	403.0857	19.9668	565.1993	4.2632e+08
	SD	14.0245	9.5869e-13	25.9011	0	35.3447	1.1130e+08
	P-value(E)	1.8267e-04	6.3864e-05	1.7861e-04	1.5938e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	1.1067e-04	" "	" "	1.8267e-04
RSA	BEST	7.1089e-04	-3.0950e+03	0	8.8818e-16	0	1.6690
	WORST	0.0103	-1.9350e+03	0	8.8818e-16	0	1.6690
	MEAN	0.0047	-2.3866e+03	0	8.8818e-16	0	1.6690
	SD	0.0031	401.2004	0	0	0	2.3406e-16
	P-value(E)	1.8267e-04	1.8267e-04	0.0022	NaN	NaN	4.9177e-05
	P-value(EM)	" "	" "	NaN	NaN	NaN	6.3864e-05
SMA	BEST	4.2446e-04	-1.2555e+04	0.0027	0.0020	0.0036	0.831
	WORST	0.0740	-3.7818e+03	26.9253	0.0524	0.0880	1.662
	MEAN	0.0293	-7.2608e+03	4.1987	0.0282	0.0487	1.254
	SD	0.0217	3.8479e+03	8.7322	0.0167	0.0305	0.287
	P-value(E)	1.8267e-04	0.3847	0.3840	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	1.8267e-04	1.1067e-04	" "	" "	1.8267e-04
SWO	BEST	66.0314	-3.3444e+03	375.6053	20.1243	423.7847	2.1526e+08
	WORST	102.7668	-2.6099e+03	429.9510	20.5447	550.9944	4.6015e+08
	MEAN	90.2103	-2.9264e+03	407.7917	20.3921	507.9397	3.4502e+08
	SD	11.9828	225.8055	17.5389	0.1530	47.6517	6.2470e+07
	P-value(E)	1.8267e-04	1.8267e-04	1.7861e-04	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	" "	1.1067e-04	" "	" "	1.8267e-04
WOA	BEST	4.0683	-8.4432e+03	241.8468	12.7928	66.1645	1.1880e+06
	WORST	18.1893	-4.7413e+03	340.4939	17.8355	215.8781	6.0313e+07
	MEAN	11.1807	-6.3963e+03	306.6318	15.5953	155.9841	2.5707e+07
	SD	4.4140	1.2397e+03	34.3787	1.6976	51.5082	2.2675e+07

	P-value(E)	1.8267e-04	5.8284e-04	1.7861e-04	6.3864e-05	6.3864e-05	1.4939e-04
	P-value(EM)	" "	1.8267e-04	1.1067e-04	" "	" "	1.8267e-04
Algo/F unction s		F13	F14	F15	F16	F17	F18
ESGO	BEST	1.3498e-32	0.9980	3.0749e-04	-1.0316	0.3979	3.000
	WORST	0.0548	0.9980	3.0749e-04	-1.0316	0.3979	0
	MEAN	0.0077	0.9980	3.0749e-04	-1.0316	0.3979	0
	SD	0.0172	0	9.7310e-20	0	0	9.3622e-16
EMSG O	BEST	1.4482e-09	0.9980	3.0749e-04	-1.0316	0.3979	3.000
	WORST	9.3515e-05	0.9980	3.0749e-04	-1.0316	0.3979	0
	MEAN	1.8852e-05	0.9980	3.0749e-04	-1.0316	0.3979	0
	SD	3.3044e-05	0	9.7310e-20	0	0	0
AVOA	BEST	3.4455	3.3015	0.0018	-1.0316	0.3982	3.002
	WORST	2.2399e+08	25.6376	0.0510	-0.9123	0.4404	9
	MEAN	7.5618e+07	12.5651	0.0205	-0.9929	0.4142	5
	SD	8.7433e+07	7.4563	0.0159	0.0441	0.0166	2
E D	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
	BEST	6.7444e+08	3.0014	0.0042	-1.0215	0.4034	4
	WORST	1.0352e+09	15.5151	0.0256	-0.7381	0.6127	26
E D	MEAN	8.9934e+08	9.6993	0.0181	-0.8525	0.4704	7
	SD	1.1373e+08	4.4249	0.0080	0.0957	0.0748	5
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
EDO	BEST	0.1964	0.9980	6.0609e-04	-1.0316	0.3979	3.000
	WORST	0.5513	0.9982	0.0013	-1.0316	0.3979	0
	MEAN	0.3684	0.9981	8.3949e-04	-1.0316	0.3979	7
	SD	0.1092	8.7514e-05	1.9852e-04	1.3264e-06	1.4829e-05	2
GWO	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
	BEST	1.8154e+07	2.0230	0.0038	-1.0308	0.4006	4
	WORST	1.1507e+08	12.9125	0.0413	-0.9536	0.6533	3
KOA	MEAN	5.8232e+07	7.1274	0.0164	-1.0038	0.4841	1
	SD	2.9227e+07	3.8136	0.0125	0.0263	0.0904	1
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
KOA	BEST	7.2168e+08	2.9821	0.0049	-1.0276	0.4016	7
	WORST	1.0797e+09	27.8353	0.1063	-0.6278	0.5907	46
	MEAN	9.2260e+08	12.2474	0.0525	-0.8687	0.4792	06
	SD	1.1321e+08	8.1667	0.0338	0.1568	0.0626	4
KOA	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05

LSO	BEST	3	4.0339	0.0102	-1.0139	0.4016	5.845
	WORST	3	12.6705	0.1022	-0.3175	0.5907	37.65
	MEAN	3	9.8962	0.0532	-0.6705	0.4792	16.87
	SD	0	3.7132	0.0282	0.2216	0.0626	11.80
	P-value(E)	6.1582e-05	4.9177e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	6.3864e-05	" "	1.8267e-04	" "	" "	6.3864e-05
MSA	BEST	2.8700e+08	0.9980	0.0030	-1.0298	0.4016	3.025
	WORST	2.8700e+08	0.9980	0.0030	-1.0298	0.5907	3.025
	MEAN	3.8465e+08	1.2827	0.0049	-1.0233	0.4792	3.153
	SD	6.9371e+07	0.3251	0.0015	0.0078	0.0626	0.099
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
NOA	BEST	7.6419e+08	7.9004	0.0138	-0.9764	0.4016	3.861
	WORST	9.5384e+08	64.6598	0.1132	-0.1700	0.5907	31.75
	MEAN	8.4808e+08	28.3656	0.0656	-0.5786	0.4792	11.91
	SD	6.8994e+07	21.7623	0.0336	0.3114	0.0626	9.195
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
RSA	BEST	3	9.2110	0.0090	-0.4096	0.5116	6.6847
	WORST	3	12.6705	0.1484	0	0.5907	123.1639
	MEAN	3	12.3246	0.0922	-0.0815	0.4522	40.6273
	SD	0	1.0940	0.0590	0.1429	0.0626	31.6910
	P-value(E)	6.1582e-05	2.4282e-05	1.6494e-04	4.1717e-05	6.3864e-05	1.0997e-04
	P-value(EM)	6.3864e-05	" "	1.7265e-04	" "	" "	6.3864e-05
SMA	BEST	2.9824	0.9980	7.1102e-04	-1.0315	0.4016	3.000
	WORST	3.2405	9.8039	0.0143	-1.0209	0.5907	3.360
	MEAN	3.1123	4.8996	0.0057	-1.0291	0.4792	3.082
	SD	0.0930	3.2198	0.0042	0.0038	0.0626	0.122
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	3.2984e-04	" "	" "	6.3864e-05
SWO	BEST	5.4627e+08	1.0171	0.0193	-1.0283	0.4116	3.371
	WORST	1.0179e+09	22.0117	0.0942	-0.0493	0.5957	23.61
	MEAN	7.5225e+08	8.7381	0.0611	-0.7510	0.4892	10.25
	SD	1.3204e+08	5.9854	0.0224	0.3141	0.0726	6.967
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
WOA	BEST	1.0576e+08	1.1509	0.0018	-1.0308	0.4016	3.001
	WORST	3.9234e+08	20.1571	0.0514	-0.5923	0.5907	33.11
	MEAN	2.2762e+08	11.2657	0.0180	-0.9283	0.4792	10.45
	SD	9.3427e+07	6.4055	0.0175	0.1413	0.0626	11.98
	P-value(E)	1.7761e-04	6.3864e-05	1.7462e-04	6.3864e-05	6.3864e-05	1.0997e-04
	P-value(EM)	1.8267e-04	" "	1.8267e-04	" "	" "	6.3864e-05
		F19	F20	F21	F22	F23	
ESGO	BEST	-3.8628	-3.3220	-10.1532	-10.4029	-10.5364	
	WORST	-3.8628	-3.2031	-10.1532	-10.4029	-10.5364	
	MEAN	-3.8628	-3.3101	-10.1532	-10.4029	-10.5364	

	SD	9.3622e-16	0.0376	0	1.8724e-15	5.3864e-15
EMSG O	BEST	-3.8628	-3.3220	-10.1532	-10.4029	-10.5364
	WORST	-3.8628	-3.3220	-10.1532	-10.4029	-10.5364
	MEAN	-3.8628	-3.3220	-10.1532	-10.4029	-10.5364
	SD	0	9.2038e-09	0	0	5.3864e-15
AVOA	BEST	-3.8554	-3.0677	-6.3022	-9.1029	-4.4976
	WORST	-3.6830	-2.3662	-1.2040	-1.5697	-1.4449
	MEAN	-3.8016	-2.7888	-2.5218	-3.6354	-2.4322
	SD	0.0532	0.2461	1.6158	2.2237	1.0535
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	6.3864e-05	6.3864e-05
	P-value(EM)	" "	1.8165e-04	" "	" "	" "
E D	BEST	-3.8529	-2.9290	-2.3594	-2.2949	-3.2228
	WORST	-3.6123	-1.9355	-0.7682	-0.7686	-1.0506
	MEAN	-3.7591	-2.4405	-1.2893	-1.1888	-1.5033
	SD	0.0866	0.3006	0.5691	0.4318	0.6396
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	6.3864e-05	1.6118e-04
	P-value(EM)	" "	1.8165e-04	" "	1.5932e-04	" "
EDO	BEST	-3.8627	-3.2807	-10.0372	-10.1394	-10.3280
	WORST	-3.8624	-3.1114	-9.0513	-6.6129	-5.8198
	MEAN	-3.8626	-3.1647	-9.4825	-8.8969	-8.5269
	SD	1.2376e-04	0.0475	0.3170	1.0859	1.5640
	P-value(E)	6.3864e-05	1.7865e-04	6.3864e-05	6.3864e-05	4.3745e-04
	P-value(EM)	6.3864e-05	4.3745e-04	6.3864e-05	6.3864e-05	4.3745e-04
GWO	BEST	-3.8598	-3.2761	-6.6657	-5.0855	-2.8311
	WORST	-3.7899	-2.7823	-0.9199	-1.8350	-1.3021
	MEAN	-3.8323	-3.0315	-1.9021	-3.3878	-2.1339
	SD	0.0275	0.1266	1.7055	1.2214	0.4926
	P-value(E)	6.3864e-05	1.7865e-04	6.3864e-05	6.3864e-05	1.6118e-04
	P-value(EM)	" "	4.3745e-04	" "	0.0297	1.6118e-04
KOA	BEST	-3.7779	-2.8099	-1.0715	-1.9384	-1.5148
	WORST	-3.5197	-2.0402	-0.4657	-0.6809	-0.8724
	MEAN	-3.6068	-2.4278	-0.7505	-1.0839	-1.1098
	SD	0.0958	0.2350	0.2215	0.3508	0.1946
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	6.3864e-05	1.6118e-04
	P-value(EM)	" "	1.8165e-04	6.3864e-05	1.5932e-04	1.6118e-04
LSO	BEST	-3.7985	-2.7576	-1.7209	-1.3938	-3.1473
	WORST	-3.5779	-1.8099	-0.6040	-0.6387	-0.8197
	MEAN	-3.7193	-2.3107	-1.0016	-0.9915	-1.4593
	SD	0.0750	0.3213	0.4116	0.2900	0.7417
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	6.3864e-05	1.6118e-04
	P-value(EM)	" "	1.8165e-04	" "	1.5932e-04	" "
MSA	BEST	-3.8605	-3.2122	-5.3730	-4.7984	-5.0588
	WORST	-3.8605	-3.2122	-5.3730	-4.7984	-5.0588
	MEAN	-3.8554	-3.0574	-3.1027	-3.3813	-5.0588
	SD	0.0055	0.0765	0.9302	0.9535	1.3777e-14
	P-value(E)	6.3864e-05	1.7865e-04	6.3864e-05	6.3864e-05	2.9377e-04
	P-value(EM)	" "	4.3745e-04	" "	" "	" "
NOA	BEST	-3.8433	-2.5874	-1.1580	-1.5230	-2.4136
	WORST	-3.5174	-1.8432	-0.5307	-0.7422	-0.8937
	MEAN	-3.7123	-2.1747	-0.7900	-1.1315	-1.2139
	SD	0.1111	0.3111	0.2374	0.2992	0.4667
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	1.5932e-04	1.6118e-04
	P-value(EM)	" "	1.8165e-04	" "	6.3864e-05	" "
RSA	BEST	-3.8094	-2.4156	-0.8657	-1.2773	-1.1469
	WORST	-3.0148	-0.9463	-0.2834	-0.4220	-0.6601
	MEAN	-3.4383	-1.6450	-0.4905	-0.6030	-0.9586
	SD	0.2788	0.5156	0.2049	0.2477	0.1349
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	1.5932e-04	1.6118e-04
	P-value(EM)	" "	1.8165e-04	" "	6.3864e-05	" "
SMA	BEST	-3.8623	-3.1346	-10.0208	-9.3983	-8.2808
	WORST	-3.8341	-2.2623	-2.4945	-1.9579	-1.0896
	MEAN	-3.8514	-2.7689	-4.5584	-4.6781	-3.7356
	SD	0.0078	0.3421	2.1547	2.6561	2.1286
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	6.3864e-05	3.8932e-04
	P-value(EM)	" "	1.8165e-04	" "	" "	" "

SWO	BEST	-3.8379	-2.7585	-1.4098	-1.9429	-3.4058
	WORST	-3.6279	-1.8461	-0.6324	-0.7866	-0.9884
	MEAN	-3.7513	-2.2405	-0.9346	-1.1783	-1.5817
	SD	0.0579	0.2779	0.3051	0.3436	0.6943
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	1.5932e-04	1.6118e-04
	P-value(EM)	' '	1.8165e-04	' '	6.3864e-05	' '
WOA	BEST	-3.8622	-2.9763	-6.2016	-3.4370	-4.4309
	WORST	-3.6581	-2.1432	-1.4378	-1.0475	-1.6685
	MEAN	-3.7568	-2.7168	-3.2018	-2.2719	-2.7508
	SD	0.0746	0.2455	1.3968	0.6886	1.0772
	P-value(E)	6.3864e-05	1.3093e-04	6.3864e-05	3.8932e-04	2.9377e-04
	P-value(EM)	' '	1.8165e-04	' '	6.3864e-05	' '

Table 4: WRS test results on Table 3

	F1		F2		F3		F4		F5		F6		F7		F8		F9		F10		F11		F12	
	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM
AVOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
EDO	-	-	-	-	+	+	-	-	-	-	-	-	-	-	-	-	+	N	N	N	N	N	N	-
GWO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LSO	N	N	+	+	+	+	+	+	-	-	-	-	-	-	-	+	N	N	N	N	N	N	-	-
MSA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
NOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RSA	N	N	+	+	+	+	+	+	-	-	-	-	-	-	-	+	N	N	N	N	N	N	-	-
SMA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	S	-	-	-	-	-	-	-	-	-
SWO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
WOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	F13		F1		F15		F16		F17		F18		F19		F20		F21		F22		F23			
	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM	E	EM		
AVOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
EDO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
GWO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
KOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
LSO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
MSA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
NOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
RSA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SMA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
SWO	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
WOA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Total no of '+'=10, Total no of 'S'=1, Total no of 'N'=8, Total no of '-'=257 (For ESGO algorithm)
 Total no of '+'=7, Total no of 'S'=0, Total no of 'N'=11, Total no of '-'=258 (For EMSGO algorithm)
 Here, E represents ESGO and EM represents EMSGO algorithm. “-”, “+”, and “S” denote that the performance of other approaches is worse, better, and similar to ESGO and EMSGO respectively.

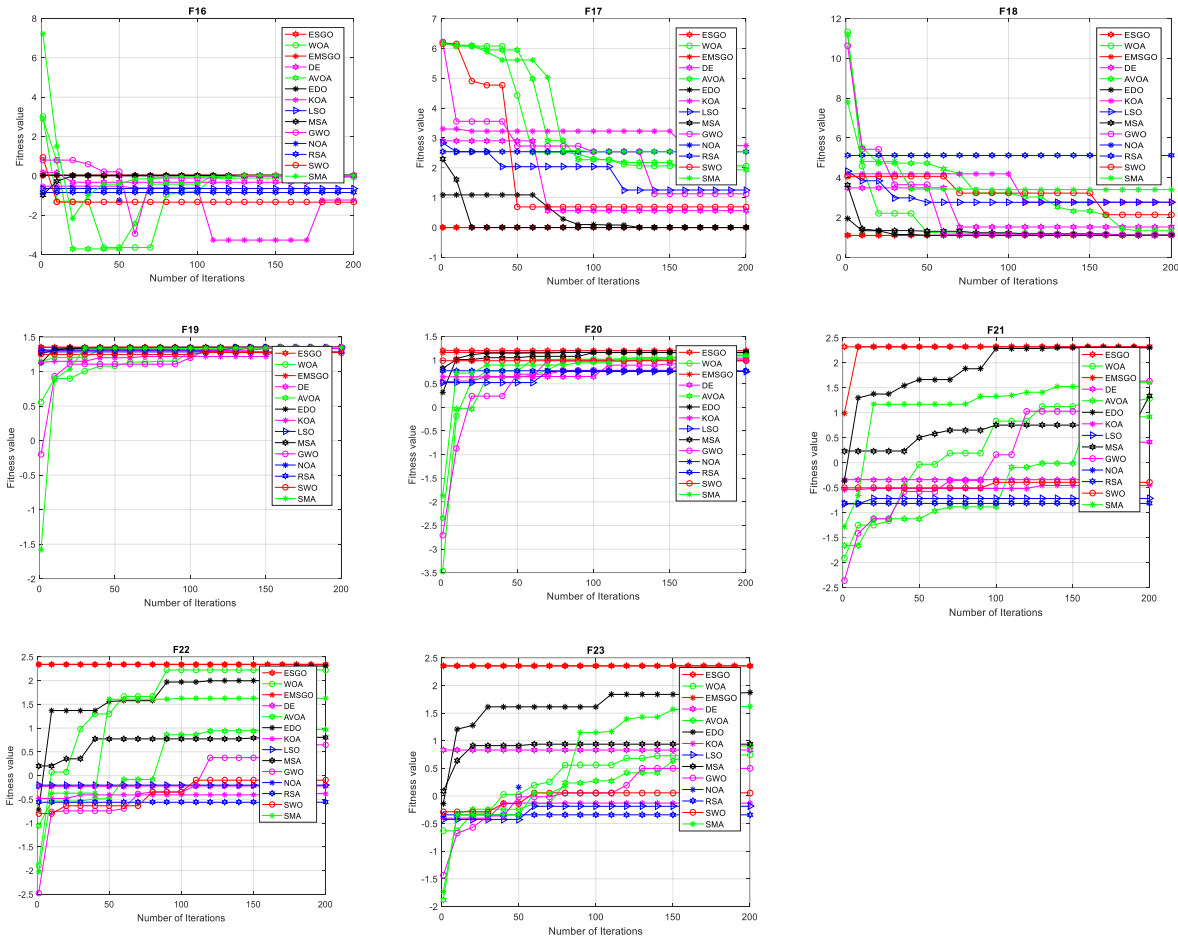


Figure 3: Convergence characteristics of algorithms

Discussion

In this experimental evaluation, the effectiveness of ESGO and EMSGO in navigating, exploiting, and avoiding local minima across a diverse range of benchmark functions, including both unimodal and multimodal functions, was examined.

Unimodal functions, characterized by a singular global optimum, serve as a measure of an algorithm's exploitation capability. The results presented in Table 3 for unimodal test functions (F1-F7) demonstrate the superior performance of ESGO and EMSGO, surpassing most other algorithms in all evaluated functions. These findings underscore the proficiency of ESGO and EMSGO in exploitation, showcasing their capacity to efficiently converge towards and exploit the optimal solution. This efficacy is attributed to the incorporation of the self-awareness probability (SAP) parameter.

Multimodal test functions, featuring multiple local optima that escalate with dimensionality, provide a platform for assessing an algorithm's exploration ability. Functions F8 through F23 represent multimodal scenarios. As indicated in Table 3, ESGO and EMSGO exhibit remarkable exploration capabilities, surpassing other methods. Across multimodal functions, ESGO and EMSGO not only achieve optimal solutions but also outperform all compared algorithms, demonstrating competitiveness

with high-performance optimizers. The exploration prowess of ESGO and EMSGO can be attributed to the distinctive phases of optimization and the self-introspection parameter C.

Based on the WRS test results in Table 3, For the ESGO algorithm:

- It performs worse than EDO for F3 and F9, LSO for F2, F3, F4, and F9, and RSA for F2, F3, F4, and F9.
- It matches the performance of EDO for F10 and F11, LSO for F1, F10, and F11, as well as RSA for F1, F10, and F11.
- ESGO outperforms other algorithms in all other functions.
- It consistently surpasses AVOA, DE, KOA, MSA, NOA, SMA, and SWO across all twenty-three functions.

For the EMSGO algorithm:

- It is inferior to EDO for F3, LSO for F2, F3, F4, and RSA for F2, F3, and F4.
- It matches the performance of EDO for F9, F10, and F11, LSO for F1, F9, F10, F11, and RSA for F1, F9, F10, and F11.

- EMSGO outshines other algorithms in the remaining functions.

It consistently outperforms AVOA, DE, KOA, MSA, NOA, SMA, and SWO for all twenty-three functions.

As depicted in Table 4, the ESGO algorithm surpasses AVOA, DE, EDO, GWO, KOA, LSO, MSA, NOA, RSA, SMA, SWO, and WOA in 23 cases out of 23, 23, 19, 23, 23, 16, 23, 23, 16, 22, 23, and 23 cases, respectively. Conversely, the ESGO algorithm performs less effectively than AVOA, DE, EDO, GWO, KOA, LSO, MSA, NOA, RSA, SMA, SWO, and WOA in zero, zero, two, zero, zero, four, zero, zero, four, one, zero, and zero cases, respectively. Additionally, the ESGO algorithm exhibits equivalence with EDO in two cases and with LSO and RSA in three and three cases, respectively. In summary, out of 276 instances, ESGO achieves equivalent results in 8 cases, the same solution in one case, a worse solution in 10 cases, and superior outcomes in 257 cases compared to other algorithms.

Similarly, as illustrated by Table 4, the EMSGO algorithm outperforms AVOA, DE, EDO, GWO, KOA, LSO, MSA, NOA, RSA, SMA, SWO, and WOA in 23 cases out of 23, 23, 19, 23, 23, 16, 23, 23, 16, 22, 23, and 23 cases, respectively. Conversely, the EMSGO algorithm performs less effectively than AVOA, DE, EDO, GWO, KOA, LSO, MSA, NOA, RSA, SMA, SWO, and WOA in zero, zero, one, zero, zero, three, zero, zero, three, one, zero, and zero cases, respectively. Additionally, the EMSGO algorithm exhibits equivalence with EDO in three cases and with LSO and RSA in four and four cases, respectively. In summary, out of 276 instances, EMSGO

achieves equivalent results in 11 cases, the same solution in zero cases, a worse solution in 7 cases, and superior outcomes in 258 cases compared to other algorithms.

In conclusion, both ESGO and EMSGO demonstrate outstanding performance when addressing unimodal and multimodal functions.

Experiment 3: The performance comparison with well-known improved and hybrid metaheuristics algorithms

In this section, the same set of 23 optimization functions used in Experiments 1 and 2 is employed. The objective is to perform a comparative analysis between the simulation results obtained from ESGO, EMSGO, and the findings previously reported in reference [66] for two prominent optimization algorithms: the dynamic Harris Hawks Optimization with a mutation mechanism (DHHO/M) [67], and the Harris Hawks Optimization incorporating genetic operators such as crossover and mutation (HHOCM) [68]. Additionally, the analysis is extended to compare the simulation results from reference [69] for three other prominent optimization techniques: the Exponential Crow Search Algorithm (ECSA), the Power Crow Search Algorithm (PCSA), and the S-shaped Crow Search Algorithm (SCSA). Furthermore, the simulation results of a prominent hybrid algorithm, the Improved Hybrid Aquila Optimizer (IHAO) [70], and the Harris Hawks Optimization (HHO) [71] algorithm IHAOHHO [71] are also investigated.

Table 5: Simulation results for ESGO, EMSGO, DHHO/M, HHOCM, IHAOHHO, ECSA, PCSA, and SCSA

function		ESGO	EMSGO	DHHO/M	HHOCM	IHAOHHO	ECSA	PCSA	SCSA
F1	BEST	0	0						
	MEAN	0	0	1.97e-95	0	3.37e-253	7.62e-28	1.41e-32	8.36e-35
	STD	0	0	6.74e-95	0	0	1.02e-27	1.87e-32	1.33e-34
F2	BEST	0	0						
	MEAN	0	0	1.326e-48	1.22e-203	1.56e-127	2.13e-11	1.02e-11	3.14e-12
	STD	0	0	6.07e-48	0	8.53e-127	2.13e-11	1.78e-11	7.20e-12
F3	BEST	2.20e-122	0						
	MEAN	1.46e-116	0	7.67e-70	0	2.74e-199	1.35e-22	2.27e-24	1.0e-24
	STD	2.37e-117	0	4.20e-69	0	0	5.36e-22	5.43e-24	4.37e-24
F4	BEST	0	0						
	MEAN	0	0	3.96e-43	4.55e-197	2.22e-129	2.87e-13	7.64e-13	4.20e-13
	STD	0	0	2.16e-42	0	1.11e-128	4.35e-13	1.28e-12	6.27e-12
F5	BEST	6.34e-21	0						
	MEAN	7.11e-20	0	6.70e-03	3.14e-02	5.39e-04	7.97e-01	1.25	1.32
	STD	8.47e-20	0	9.58e-03	5.02e-02	2.27e-03	1.62	1.83	1.91
F6	BEST	0	0						
	MEAN	0	0	7.39e-05	3.13e-04	3.59e-06	5.81e-28	7.08e-33	0
	STD	0	0	1.09e-04	3.83e-04	7.73e-06	1.08e-27	9.91e-33	0
F7	BEST	1.56e-05	4.25e-07						
	MEAN	2.53e-05	5.29e-06	1.58e-04	1.62e-04	9.53e-05	4.72e-04	4.50e-04	4.88e-05
	STD	1.18e-05	3.23e-06	1.44e-04	1.76e-04	7.67e-05	2.72e-04	2.98e-04	3.31e-05
F8	BEST	-1.05e+04	-1.26e+04						
	MEAN	-9.25e+03	-1.26e+04	-1.26e+04	-1.26e+04	-1.26e+04	-2.45e+03	-2.66e+03	-2.81e+03
	STD	7.86e+02	0	5.43e+02	5.50e+01	1.82e-01	3.53e+02	3.09e+02	3.76e+02
F9	BEST	0.00e+00	0						

	MEAN	12.5341	0	0	0	0	2.60	4.44	3.70
	STD	7.2981	0	0	0	0	7.06	6.00	7.08
F1	BEST	8.88e-16	8.88e-16						
0	MEAN	8.88e-16	8.88e-16	8.88e-16	8.88e-16	8.88e-16	1.11e-01	1.09e-01	0.91e-01
	STD	0	0	0	0	0	2.20e-01	3.02e-01	1.36e-01
F1	BEST	0	0						
1	MEAN	0	0	0	0	0	2.68e-02	1.23e-02	1.00e-02
	STD	0	0	0	0	0	2.86e-02	3.48e-02	3.19e-02
F1	BEST	1.59e-32	1.57e-32						
2	MEAN	1.59e-32	1.57e-32	8.53e-06	1.57e-05	2.70e-07	2.23e-08	9.04e-08	8.11e-11
	STD	2.30e-34	2.89e-48	1.00e-05	2.27e-05	4.42e-07	1.47e-07	.76e-07	1.49e-10
F1	BEST	2.34e-32	1.35e-32						
3	MEAN	2.34e-32	1.35e-32	9.52e-05	2.76e-04	3.02e-06	1.11e-04	1.05e-02	1.25e-02
	STD	2.21e-34	2.10e-48	1.10e-04	3.96e-04	5.08e-06	1.93e-04	2.12e-02	2.00e-02
F1	BEST	9.98e-01	9.98e-01						
4	MEAN	9.98e-01	9.98e-01	1.29	1.16	1.59	9.98e-01	9.98e-01	9.98e-01
	STD	0	0	9.40e-01	5.27e-01	9.25e-01	3.43e-02	9.95e-02	5.64e-02
F1	BEST	3.07e-04	3.07e-04						
5	MEAN	3.99e-04	3.07e-04	4.58e-04	5.61e-04	4.42e-04	3.27e-03	4.41e-03	1.23e-03
	STD	2.90e-04	2.64e-16	3.01e-04	4.29e-04	3.46e-04	5.61e-03	8.11e-03	3.64e-03
F1	BEST	-1.0316	-1.0316						
6	MEAN	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
	STD	0	7.40e-17	5.75e-11	3.87e-09	3.19e-08	5.77e-16	6.19e-16	6.42e-16
F1	BEST								
7	MEAN	3.97e-01	3.97e-01	3.98e-01	3.98e-01	3.98e-01	3.97e-01	3.97e-01	3.97e-01
	STD	0	0	3.99e-06	1.65e-06	4.04e-05	0	0	0
F1	BEST	3.0000	3.0000						
8	MEAN	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
	STD	1.20e-15	6.62e-16	7.53e-08	1.55e-08	3.32e-06	2.16e-15	2.13e-15	2.04e-15
F1	BEST	-3.8628	-3.8628						
9	MEAN	-3.86e+00	-3.86	-3.86	-3.86	-3.83	-3.86	3.86	3.86
	STD	9.3622e-16	0	3.09e-03	5.16e-04	7.1972e-02	2.61e-15	2.42e-15	2.37e-15
F2	BEST	-3.32e+00	-3.32						
0	MEAN	-3.32e+00	-3.32	-3.11	-3.26	-3.08	-3.27	-3.27	-3.27
	STD	1.7813e-03	2.23e-11	8.39e-02	6.78e-02	1.19e-01	4.79e-02	5.29e-02	5.92e-02
F2	BEST	-1.02e+01	-1.02e+01						
1	MEAN	-1.02e+01	-1.02e+01	-1.00	-5.06	-1.02	-2.05	-6.72	-6.57
	STD	1.32e-15	0.00e+00	1.26e-01	2.69e-04	1.45e-03	3.31	3.38	3.69
F2	BEST	-1.04e+01	-1.04e+01						
2	MEAN	-1.04e+01	-1.04e+01	-1.02e+01	-5.09	-1.04e+01	-5.43	-3.96	-4.61
	STD	1.24e-13	1.67e-15	1.87e-01	1.06e-04	6.66e-04	3.52	3.80	3.53
F2	BEST	-1.05e+01	-1.05e+01						
3	MEAN	-1.05e+01	-1.05e+01	-1.04e+01	-5.13	-1.05e+01	-6.00	-5.50	-4.55
	STD	1.32e-15	2.78e-17	1.54e-01	1.78e-04	6.67e-04	3.67	3.70	3.84

Table 6: Results of Friedman’s Test on Table 5

Functions	ESGO	EMSGO	DHHO/M	HHOCM	IHAOHHO	ECSA	PCSA	SCSA	
F1	2	2	5	2	4	8	7	6	
F2	1.5	1.5	5	3	4	8	7	6	
F3	4	1.5	5	1.5	3	8	7	6	
F4	1.5	1.5	5	3	4	6	8	7	
F5	2	1	5	4	3	6	7	8	
F6	2	2	7	8	6	5	4	2	
F7	2	1	5	6	4	8	7	3	
F8	5	2.5	2.5	2.5	2.5	8	7	6	
F9	8	1	3	4	2	5	7	6	
F10	3	3	3	3	3	8	7	6	
F11	3	3	3	3	3	8	7	6	
F12	2	1	7	8	6	4	5	3	
F13	2	1	4	5	3	7	6	8	
F14	1.5	1.5	7	6	8	3	5	4	
F15	2	1	4	5	3	7	8	6	

F16	1	2	6	7	8	5	3	4	
F17	3	3	7	6	8	3	3	3	
F18	2	1	7	6	8	5	4	3	
F19	2	1	7	6	8	5	4	3	
F20	2	1	7	6	8	3	4	5	
F21	2	1	4	7	3	8	6	5	
F22	2	1	4	6	3	5	8	7	
F23	2	1	4	7	3	5	6	8	
Sum of ranks	57.5	35.5	116.5	115	107.5	138	137	121	
Average of ranks	2.5	1.543478	5.065217	5	4.673913	6	5.956522	5.26087	
Sum of ranks squared	191.75	66.25	642.25	657.5	611.25	900	873	709	

The assessment methodology and parameters employed for the ESGO and EMSGO algorithms closely adhere to the protocols outlined in Experiment 1. To ensure a fair comparison with the results from other algorithms, the dimension size was standardized to $D = 30$ for all functions, excluding fixed-dimensional benchmark functions. Additionally, the maximum number of fitness function evaluations was set to $\text{Max_FEs} = 15,000$, and 30 independent runs were conducted. Table 5 displays function values in terms of BEST, MEAN, and STD for the 23 classical benchmark functions, as discussed earlier. The BEST function value is provided exclusively for ESGO and EMSGO, as it is not reported for other algorithms in the imported papers. Bold font in the table denotes the most outstanding function value for each function. Evidently, EMSGO outperforms all listed algorithms in Table 5, exhibiting the lowest MEAN function value with the lowest STD for 22 out of the 23 functions. In addition, ESGO performs second-best in Table 5, obtaining the lowest STD for eight of the 23 functions and the lowest MEAN function value. Out of the 23 classical benchmark functions, HHOCM comes in third place with the lowest MEAN function value and the lowest STD for five of them

For each of the 23 classical benchmark functions listed in Table 5, Friedman's test [73], a nonparametric statistical test, was used to identify the lowest MEAN function value with the lowest STD. The findings of Friedman's test are shown in Table 6, revealing statistical insights into the ranks of ESGO, EMSGO, DHHO/M, HHOCM, IHAOHHO, ECSA, PCSA, and SCSA. The top-ranking algorithm is indicated in bold in this table. The algorithms are EMSGO, ESGO, IHAOHHO, HHOCM, DHHO/M, SCSA, PCSA, and ECSA, in that order of ranking. This suggests that out of all the algorithms that were looked at, EMSGO performs the best.

4.2 Process Synthesis and design problems of chemical and mechanical engineering

Real-world optimization problems are extremely difficult to solve due to the incredible complexity of objective functions and the profusion of nonlinear nonconvex equality and inequality constraints. This research focuses on a carefully chosen set of 26 limited problems taken from the mechanical and chemical engineering areas. In addition to the exhaustively documented 19 mechanical engineering problems in [64], the compilation includes seven process synthesis and design problems from chemical engineering. In these cases, the number of decision variables ranges from 2 to 30, the equality constraints from 0 to 4, and the inequality constraints from 1 to 86. See [64] for a detailed discussion of the problems and thorough formula definitions.

In the experimental phase, the parameter Max_FEs (maximum number of fitness function evaluations) is defined as:

$$\text{Max_FEs} = \begin{cases} 1 \times 10^5, & \text{if } D \leq 10 \\ 2 \times 10^5, & \text{if } 10 < D \leq 30 \\ 4 \times 10^5, & \text{if } 30 < D \leq 50 \\ 8 \times 10^5, & \text{if } 50 < D \leq 150 \\ 10^6, & \text{if } 150 < D \end{cases}$$

With a fixed population size of 50, D here stands for the dimension (number of decision variables) for algorithms. Based on the learnings from Experiment 1, additional factors were chosen, and Table 7 provides an overview of the results. For constraint management, Deb's guidelines [74] are adopted, using a criterion that accepts impractical solutions if they show small violations, from 0.01 in the first iteration to 0.001 in the last. In problems pertaining to process synthesis, design, and optimization in mechanical engineering, this approach is very helpful because the global minimum frequently coincides with or is located close to the edge of the viable design space. Candidate solutions tend to gravitate towards these boundaries when this approach is used, increasing the likelihood of obtaining the global minimum [75]

Table 7: Details of 26 real-world constrained optimization problem,

Name	F(x)	ESGO	EMSGO
P8 Process synthesis problem	2.0000000000e+00	2.0000e+00(R)	2.0000e+00(R)
P9 Process synthesis and design problem	2.5576545740e+00	2.5577e+00(R)	2.5577e+00(R)
P10 Process flow sheeting problem	1.0765430833e+00	1.0765e+00(R)	1.0765e+00(R)
P11 Two-reactor Problem	9.9238463653e+01	9.9238e+01(R)	9.9238e+01(R)
P12 Process synthesis problem	2.9248305537e+00	2.9248e+00(R)	2.9248e+00(R)
P13 Process design Problem	2.6887000000e+04	2.6887e+04(R)	2.6887e+04(R)
P14 Multi-product batch plant	5.3638942722e+04	5.8477e+04	5.9484e+04
Mechanical engineering problems			
P15 Weight Minimization of a Speed Reducer	2.9944244658e+03	2.9944e+03(R)	2.9944e+03(R)
P16 Optimal Design of Industrial refrigeration System	3.2213000814e-02	3.2213e-02(R)	3.2213e-02(R)
P17 Tension/compression spring design (case 1)	1.2665232788e-02	1.2665e-02(R)	1.2669e-02
P18 Pressure vessel design	5.8853327736e+03	6.0597e+03	6.3708e+03
P19 Welded beam design	1.6702177263e+00	1.6702e+00(R)	1.6702e+00(R)
P20 Three-bar truss design problem	2.6389584338e+02	2.6390e+02(R)	2.6390e+02(R)
P21 Multiple disk clutch brake design problem	2.3524245790e-01	2.3524e-01(R)	2.3524e-01(R)
P22 Planetary gear train design optimization problem	5.2576870748e-01	5.3000e-01	5.3319e-01
P23 Step-cone pulley problem	1.6069868725e+01	1.6070e+01(R)	1.6226e+01
P24 Robot gripper problem	2.5287918415e+00	2.5288e+00(R)	2.5288e+00(R)
P25 Hydro-static thrust bearing design problem	1.6254428092e+03	1.6348e+03	1.6475e+03
P26 Four-stage gear box problem	3.5359231973e+01	3.5359e+01(R)	3.5359e+01(R)
P27 10-bar truss design	5.2445076066e+02	5.2453e+02	5.2548e+02
P28 Rolling element bearing	1.4614135715e+04	1.6958e+04	1.6958e+04
P29 Gas Transmission Compressor Design	2.9648954173e+06	2.9649e+06(R)	2.9649e+06(R)
P30 Tension/compression spring design (case 2)	2.6138840583e+00	2.6586e+00	2.6586e+00
P31 Gear train design Problem	0.0000000000e+00	0(R)	1.4840e-26
P32 Himmelblau's Function	-3.0665538672e+04	-3.0666e+04(R)	-3.0666e+04(R)
P33 Topology Optimization	2.6393464970e+00	2.6393e+00(R)	2.6393e+00(R)

Discussion

The optimal solution is reached by the ESGO in 19 cases, whereas the ESGO is reached in 16 cases, according to Table 7. In two instances, the results are the same for both, while in five instances, ESGO receives a better result than EMSGO.

Overall discussion

From the entire experiment, it was discovered that ESGO performs better for real-world optimization problems, while EMSGO is more effective for handling classical optimization problems.

5 Conclusion

This paper introduces an innovative adaptation of the Social Group Optimization algorithm, namely Enhanced Social Group Optimization (ESGO) and Enhanced Modified Social Group Optimization (EMSGO). The Improving Phase of the SGO algorithm has been tailored to incorporate the concept of "hone." To evaluate the effectiveness of ESGO and EMSGO, extensive experiments were conducted across 23 benchmark functions, comparing their performance against twelve other optimization techniques and six recently introduced improved/hybrid algorithms. The test outcomes were rigorously assessed using Wilcoxon's rank test and Friedman's test, revealing that both ESGO and EMSGO significantly outperform the compared algorithms. Furthermore, ESGO and EMSGO were applied to address 26 real-world optimization problems. ESGO successfully identified optimal solutions in 19 cases, while EMSGO achieved optimal solutions in 16 cases. The comprehensive experimentation indicates that ESGO excels in tackling real-world optimization problems, whereas EMSGO demonstrates superior performance in classical optimization challenges. Consequently, it is concluded that while these algorithms exhibit exceptional proficiency in classical optimization scenarios, their performance may vary when applied to real-world problems. Future research will explore their applicability in image processing, industry, neural networks, text analysis, and data mining as part of addressing real-world optimization challenges.

Author contributions

I am a single author of this manuscript.

Data availability

The datasets generated and/or analyzed during the current study and code are available from the corresponding author upon reasonable request.

Declarations

Conflict of Interest: The author declares that she has no conflict of interest in the publication of this paper.

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