Evaluation of the Numerical Method of Differential Formula Based on Optimal Control of SR's Attitude and Motion

Bing Bu
School of Science, Jilin Institute of Chemical Technology, Jilin, Jilin, 132022, China
E-mail: bubing@jlict.edu.cn

Keywords: attitude motion, optimal control, numerical methods of differential formulas

Recieved: April 19, 2024

Due to the complexity of the SR system structure and working environment, various noises and friction torque between the joints of the manipulator sloshing of liquid fuel, etc., uncertainty and external disturbance are inevitable. If these factors are not considered in the control system design, the accuracy and stability of the controlled system will not be guaranteed. In this paper, the robust trajectory tracking control of an SR system with a base attitude controlled under the conditions of uncertainty and external disturbance was studied respectively. In this paper, for the SR system with uncertainty, a robust trajectory tracking controller design method based on the optimal control differential formula of SR attitude motion was proposed, and the uncertainty of the SR system was analyzed. The quasi-linear structure of the system was obtained by analyzing the two uncertainty functions numerically using differential formulas. Combining the robust performance index with the optimal control formula of attitude motion, a robust optimal controller was obtained. Taking the optimal path obtained by optimization as an example, simulations were carried out for the SR system with uncertainty and external disturbance respectively. When N was 160 cycles, the angles of each joint were 41°, 76°, and 10°, so the planning in this paper is feasible. The optimal control scheme of SR attitude motion proposed in this paper not only improved the robustness but also reduced the pose disturbance caused by the motion of the manipulator to the base vehicle.

Povzetek: Raziskava robustnega sledenja trajektorij sistema SR z obvladovanjem negotovosti in zunanjih motenj za izboljšanje natančnosti in stabilnosti kontroliranega sistema.

1 Introduction

Since there is no external force in the space microgravity environment, it means that when the SR (SR) moves to a certain point in space according to different paths, the robot's body posture is different [1]. From the perspective of safety, efficiency, and cost, it is not enough to rely on astronauts to complete a large number of complex space operations. It is essential to replace astronauts to complete extravehicular tasks, which needs to be done by SRs that people study [2]. Therefore, the dynamic coupling problem must be considered, which makes the modeling of SR very complicated. Precise manipulation of the manipulator's activity is required for the SR to accomplish its mission, that is, how to ensure that the end effector of the manipulator reaches the designated position with the designated attitude [3]. Effective motion control depends on correct modeling, appropriate control methods, and effective motion planning algorithms.The following is a summary of this work's contributions

- Introduces a novel method for robust trajectory tracking control of an SR system, considering uncertainties and external disturbances.

- Creates reliable trajectory tracking controllers using the SR attitude motion optimum control difference method.
- Thoroughly analyses the uncertainty of the SR system, identifying key sources and quantifying their effects.
- Derives a robust optimal controller by combining a robust performance index with the optimal control formula.
- Simulation results validate the feasibility of the proposed approach, enhancing robustness and reducing pose disturbance in practical SR applications.

Many challenging problems arise in the study of path optimization and tracking control of SRs. Zhang B believed that non-cooperative targets grasped by SRs would lead to instability in composite systems [4]. Zhu Z considered the influence of uncertain factors on the system and regarded the control error and error rate as the system state [5]. Cheng Z proposed a robust path planning method for space manipulators for satellite attitude adjustment and end effector tasks [6]. Li Z aimed to propose an improved target acquisition control scheme to improve control performance [7]. Hu Y derived the kinetic model using the Lagrangian method [8]. Nevertheless, the study failed to identify the best controlling movement technique or attitude for an SR. It developed the computational method of differentiating equation optimization.

In the study of practical problems in engineering science and technology, it is usually necessary to use numerical methods of differential formulas to deal with them. Novo S gave the application of non-autonomous reaction-diffusion formulas with finite and infinite time delays in the nonlinear reaction terms [9]. Zhang L. studied the exponential stability of traveling wave solutions of differential formulas [10]. Yaozhong H U studied the properties of stochastic differential formulas driven by rough differential formulas [11]. Li R studied strong approximation schemes for solutions of stochastic differential formulas [12]. For mean-square stochastic dynamical systems generated by stochastic delay formulas with stochastic delays, Wu F established the existence of additional pseudorandoms [13]. However, the numerical method of differential formulas proposed by them does not involve the attitude optimization of SRs.Table 1 demonstrates the related work.

Table 1: Depicts the existing works

Study no	Method	Result	Limitations
Zhang B [4]	Modified adaptive sliding mode control algorithm used for Momentum Reduction.	Successful stabilization of a noncooperative target with large inertia. Reduced unknown angular momentum effectively. Good robust performance.	The method assumes knowledge of the initial angular momentum. Limited discussion on real-world implementation challenges.
Zhu Z [5]	Consideration of uncertain factors, treating control error and error rate as system state.	System state representation improves the handling of uncertain factors.	Lack of details regarding specific uncertain factors considered.
Cheng Z [6]	Task-priority reaction null-space control applied for base attitude adjustment and end-effector task.	Singularity robustness improves algorithm performance. Real-time simulation under Linux/RTAI verifies feasibility and reliability.	Limited discussion on the scalability of the algorithm for more complex tasks.
Li Z [7]	Modified target capturing control scheme with delay calibration algorithm and motion predictor.	Experimental validation demonstrates improved control performance in a three-dimensional ground experimental system.	Potential challenges in scaling the system to larger satellites or varying environmental conditions are not addressed.
Hu Y [8]	Disturbance observer designed to estimate uncertainty in tethered SR (TSR) dynamics.	The proposed controller outperforms the dynamic inverse controller in position and attitude tracking of TSR. Feasibility validated through numerical simulation.	Potential challenges in implementing disturbance observers in real-world TSR systems are not addressed.
Novo S [9]	Dynamical study of skew-product semiflows	Relations established between classical concepts and dynamical concepts. Existence of minimal semiflows with a specific dynamical structure derived.	Limited to monotone settings.

Zhang L [10]	Fixed point theorems, linearization, eigenvalue functions	Existence and exponential stability of traveling wave solutions of integral differential equations from neuronal networks proven.	Inapplicability of previous methods due to lack of maximum principle or conservation laws.
Yaozhong H U [11]	Time discrete Taylor scheme variations	Incomplete Taylor schemes developed for rough differential equations and stochastic differential equations driven by fractional Brownian motions.	Complexity in analysis due to the sophistication of incomplete Taylor schemes.
Li R [12]	Strong approximation schemes	Strong approximation schemes studied for solutions of stochastic differential formulas.	The study indicates that certain stochastic differential formulas may be limited in applicability to specific equation types or accuracy under particular conditions.
Wu F [13]	Tazumilhin-type technique, mean-square random dynamical system	The existence of a random attractor established for a mean-square random dynamical system generated by stochastic delay equation with random delay.	Limited to stochastic delay equations with drift term dominated by a nondelay component satisfying one-side dissipative Lipschitz condition.

This paper proposed a straight-knee walking planning method in which the active and passive gaits of the robot were arbitrarily mixed. The straight-knee walking gait was generated by predictive control and optimal control, and the passive gait was simulated by the passive dynamic model. The two gaits were proportionally fused and powered by online reinforcement learning to improve energy efficiency. At the same time, the numerical iterative algorithm of inverse kinematics was improved. A method of determining the sample data according to the distance of the singular pose was proposed, which reduced the sample capacity and improved the training effect of the neural network. A complementary algorithm was proposed, which combined neural network prediction and numerical iteration to determine the dominant position of the neural network or numerical iteration by judging the singularity of attitude, improving the accuracy of inverse kinematics, and ensuring the convergence of the algorithm. The maximum position error in the entire motion range was no more than 0.1mm, and the attitude tracking error was less than 0.8mm. It would stop when N=180 cycles. At this time, the angles of each joint are 268°, -90° and 178°.

2 Optimal control method of attitude motion of SR

2.1 Numerical simulation of SR attitude motion

The attitude controller includes control signal processing and torque actuator, jet and magnetic control belong to external torque control, and the flywheel motor realizes internal torque control. The use of angular momentum exchange between various flywheels and stars is the main mode of attitude stability control. There are many forms of coordinate systems, and the coordinate system should be selected according to the specific usage. Since the trajectory of the target aircraft is generally known, the origin of the coordinate system is also known. This situation can be solved by coordinate system transformation.

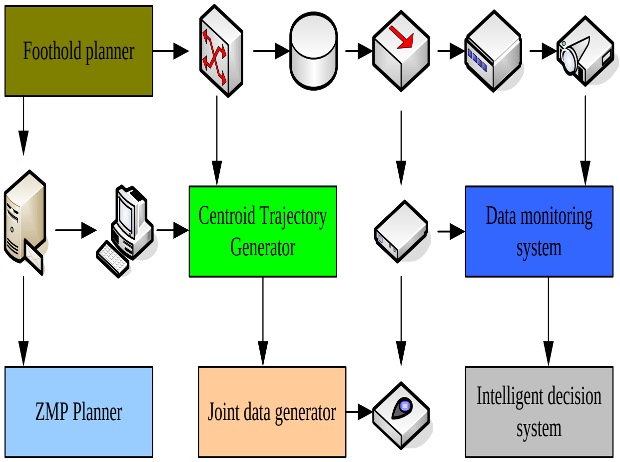


Figure 1: Motion control comprehensive software framework

The conversion can be done as long as the separation among both vehicles' centers of masses and the parameters of the vehicle's orbit are known. The comprehensive software framework of motion control is shown in Figure 1 [14]. The attitude motion control of the SR is to control the motion path of the end effector of the manipulator so that it can move from the initial position to the target position while meeting specific constraints and performance requirements. Usually, the SR attitude motion control problem is described as an optimal control problem that requires specific performance as the objective function and satisfies specific boundary constraints and attitude path constraints at the same time.

The attitude motion control of the trajectory of the end effector of the SR is proposed by using the numerical method of control variables, and the accurate motion trajectory is obtained. In the solution process of this method, the equality boundary constraints are approximated as inequality constraints, because in the control variable parameterization method, the equality boundary constraints may produce higher-order differential formulas (differential formulas with an unknown function whose derivative is higher than first-order). The state trajectory obtained by the control variable parameterization method is more accurate, and it also reduces a lot of solving time, so this method can complete the path planning task with high real-time requirements. The angular momentum of the SR is also conserved (when the resultant external torque is zero, angular motion is preserved). The angular momentum Hof the relative inertial frame is [15]:

$$\mathrm{H} = \sum_{i=0}^{n}(\mathrm{I}_i\omega_i + \mathrm{r}_i) \tag{1}$$

There are many methods for deriving the dynamic formula of the manipulator. Two of the most convenient are the Newton-Euler method and the Lagrange method, which are based on the momentum theorem and the energy theorem, respectively. The Newton-Euler method is a recursive method that is often used in online dynamics calculations. The Lagrangian method uses vector notation to derive the dynamics, and the energy characteristic information of the robot is easily obtained from the derivation process, this energy information is very important for the subsequent stable controller design. Therefore, the Lagrangian method is used in this paper to deduce the dynamic model of the SR. In the process of using the Lagrangian method to deduce the dynamic formula of the SR, the Lagrangian function is first established [16-17]:

$$\mathrm{L} = \mathrm{T} - \mathrm{V} \tag{2}$$

In this paper, a specific initial condition is first obtained, and then the differential equation is solved by numerical integration to generate a periodic positive definite solution of the SR's attitude. The multi-point periodic generation method selects multiple initial conditions to calculate more accurately. The state space of an SR is described as [18]:

$$\begin{cases} x_1 = x_2 \\ x_2 = M^{-1}(x_1)(\tau - C - d) \end{cases} \tag{3}$$

In this paper, the optimal control differential equation of attitude motion is deduced affording to the motion state of the SR, and the workspace and singularity of the SR are analyzed. Workspace and singularity analysis. Taking the specific SR system parameters as an example, the derivation results of the kinematic matrix and dynamic equations are given, as well as the numerical set of differential equations that can reach the optimal control of the workspace and attitude motion.For SRs, on the one hand, the parameters are uncertain due to factors such as the change of the manipulator arm or fuel consumption; on the other hand, the presence of obstacles may be encountered in the process of the manipulator tracking the reference trajectory. Therefore, it is necessary to design an SR trajectory tracking controller that is robust (the ability of a system to survive abnormal and dangerous situations) and can avoid obstacles. The SR can avoid obstacles through path planning. However, obstacle avoidance during path planning would increase the amount of calculation, and the planning time would increase, which cannot achieve the purpose of real-time performance. Only the obstacles in the static environment can be avoided in the task planning. Therefore, this paper considers the obstacle avoidance problem of the SR into the underlying tracking control process, so that the real-time obstacle avoidance can be realized. The operating cycle T of the manipulator is [19]:

$$\mathrm{T} = \sum_{\mathrm{i}=0}^{\mathrm{n}} t_i \tag{4}$$

Differential formulas have a profound application background, and have always been a powerful tool for people to describe and characterize phenomena and laws, and are widely used in various scientific technologies and engineering practices. With the continuous improvement and development of science and the mature progress of technology, human beings have a more and more extensive and profound understanding of the laws of nature and the laws of human society. Some complex phenomena and laws cannot be well described by only using general differential formulas. Therefore, people need new models to solve complex practical problems. There can be impediments in the way of facilitating the robot's end effector's intended course in its workplace. Therefore, during the execution of the task, the end effector must not only move to the target position but also avoid collision with obstacles. This increases the constraints on the movement of the robotic arm. In actuality, sensors can identify obstructions in the SR end-effector's workplace. (such as radar or stereo imaging cameras), describing the space obstacle as an object with a tightening boundary. The performance index function for establishing the optimal control problem of SR attitude motion is [20-21]:

$$\mathrm{J} = \int_{\mathrm{t}_0}^{\mathrm{t}_\mathrm{f}} \mathrm{u}_\mathrm{r} \mathrm{dt} \tag{5}$$

2.2 Numerical control of optimal differential formula for attitude movement of SR

With the attention paid to the uncertainty in the attitude motion model of the actual SR, such as the uncertain parameters, coefficients, source terms, boundary conditions, etc. appearing in the equations, uncertainty quantification has been widely studied by researchers in recent years. close attention. Many practical problems, such as SRs and manipulator trajectories, can be modeled as optimal partial differential equations if the uncertainty is considered. In general, it is much more difficult to obtain the optimal solution of an optimal differential equation than a deterministic equation, so a lot of investigators have completed a lot of work on the numerical solution of the optimal differential equation.

The main tasks performed by SRs include: transporting parts and assembling to build large space stations; maintaining space stations, satellites, and spacecraft; refueling and recycling scrapped or faulty satellites; replacing parts; conducting scientific experiments in a vacuum, clean, microgravity space environment.It is applied to the military field to capture and destroy enemy reconnaissance or communication satellites.To complete complex and diverse space tasks, accurate visual recognition ability and operation ability are indispensable for SRs. SRs are high-tech aerospace equipment that integrates vision, mechanics, electronics, dynamics, and control. The adaptive sliding mode variable structure control method does not contain a sign function, and can dynamically estimate the uncertain moment of inertia and external disturbance, and use the estimated parameters to design the control law so that the performance of the controller is enhanced. Combined with the adaptive control law and the sliding mode variable structure control law, the control law of the SR attitude platform is designed as [22]:

$$\mathrm{u} = \mathrm{MS} - \frac{\rho \mathrm{S}}{\|\mathrm{S}+\sigma\|} \tag{6}$$

Among them, σ is a small positive number.

Dynamics is one of the important research fields of SRs. The research and control platform must first establish differential formulas. The joint torque required for the free-floating SR to complete the predetermined motion is calculated by the dynamic formula. In this paper, the feasibility of the joint moment is calculated by the dynamic formula. Workspace and singularity (singularity is the discontinuity of the function or the non-existence of derivatives, and the point that exhibits singularity is called singularity) analysis are also an important part of the basic theoretical research of free-floating SRs. For a given target point, it is first necessary to determine whether it is within the working space of the robot. Singular poses make the manipulator lose one or more degrees of freedom, and the influence of singular poses should be considered in the trajectory planning process. When the manipulator is in a non-singular pose, different joint angular motion combinations make the manipulator end effector move in different directions. These different motion directions are combined with vectors, and then reversely solved to combine different joint angle motions, to achieve the purpose of trajectory planning. To meet the requirement of zero acceleration at the beginning and end points in the numerical method of differential formulas, it is assumed that [23]:

$$\mathrm{a} = \sin\frac{2\pi \mathrm{t}}{\mathrm{T}} \tag{7}$$

The so-called nonholonomic system refers to a mechanical system with constraints expressed by differential equations, and the numerical methods of differential equations cannot be integrated. The characteristic of this type of mechanical system is that its optimal control of attitude motion and configuration can be performed by control actuators with fewer independent generalized coordinates than independent generalized coordinates. Nonholonomic constraints only restrict the motion of the system, not the configuration of the system directly. Its physical characteristic is that even if the manipulator's joints return to their initial configuration after a series of motions, the robot carrier's posture may be different from its initial posture. Numerical methods for nonholonomic motion and attitude planning belong to infinite-dimensional optimal control problems. The velocity formula is obtained:

$$v = \frac{AT}{2\pi}\cos\frac{2\pi t}{T} \tag{8}$$

According to the form and performance of SR, its development stage is:

- **Close-range remote-control stage:** the operator performs remote control operations on the space manipulator that is very close to him in a basic control mode, and the manipulator strictly executes the operator's commands.
- **Long-distance remote-control stage:** an SR is a platform equipped with a manipulator on a spacecraft, which can perform a given task while flying in orbit. Due to the long distance, time delay and how to solve the problem of image pre-display due to communication delay must be considered.
- **Autonomous control stage:** the autonomous SR can perform a given task autonomously while flying freely in the orbit without being controlled by the operator. This is the most advanced stage of the development of SRs, and it is still mainly in the research stage.Due to factors such as communication delay and operator fatigue, SRs need more autonomy than any other type of robot [24-26].

In engineering practice, the most important thing is to obtain the real solution to the specific problem. However, due to the complexity of the real problem, it is difficult to obtain a real solution to the optimal control problem in most cases. Therefore, in recent years, there have been many numerical methods for solving the optimal control problem of attitude motion.For example, finite element method, finite difference method, spectral method, finite volume method, adaptive method, and so on have been widely used in the numerical solution of optimal control problems.For a certain non-singular point P in the task space, it is necessary to advance the tiny vectors dP, dP_1, and dP_2 to be irrelevant. According to the vector synthesis rule, DP must be linearly represented by $dP_1$and dP_2, namely:

$$dP = dP_1 + dP_2 \tag{9}$$

There are two basic requirements to achieve the tracking control of SR with a differential formula numerical method: the stability of the closed-loop platform and the tracking of a given reference model. Therefore, the solution to this problem can be separated into two parts, namely the design of the stabilization controller and the tracking controller (trace controllers are applications and tools for managing trace sessions). State feedback is used to stabilize the platform, and a feedforward tracking compensator is designed to track a given reference model, namely:

$$K = u + x_m \tag{10}$$

Among them, the state feedback gain matrix is K [26].

The execution framework of online foothold adjustment and motion control differential formula is shown in Figure 2. The content of the online foothold adjustment and motion control algorithm can be roughly summarized as follows: the given path trajectory is transformed into the reference foothold path of the SR, and its description is transformed into the trajectory of the reference center of mass of the SR. The state of the optimal reference point of the robot is adjusted online. As a result, the input of the control platform can better adapt to the current motion state of the SR and realize the stable walking control of the SR. The best reference point obtained is used as the input of the control platform and is further optimized into a reference input curve. Finally, by predicting the control effect and applying it to the table-cart model of the SR, the trajectory of the smooth center of mass motion state of the robot is obtained. The dynamic effect of the online planning and execution of the motion control of the SR can be gained by dynamically executing the above process in each control cycle that controls the actual walking of the SR. Since the real-time motion state of the robot and the follow-up effect of the path to be tracked are fully considered in the process of foothold adjustment, the final planning and execution effect not only realizes the good following of the original planning path but also makes the robot maintain a good stability during the execution process. In addition, since the online planning directly analyzes and solves the various states of the robot from the bottom of the model, the optimal solution of the robot's attitude control by the differential formula numerical method under different conditions can be obtained. Compared with the general search for the best motion pattern from the "executable motion library", it not only improves the time performance but also makes the robot's motion more flexible [27].

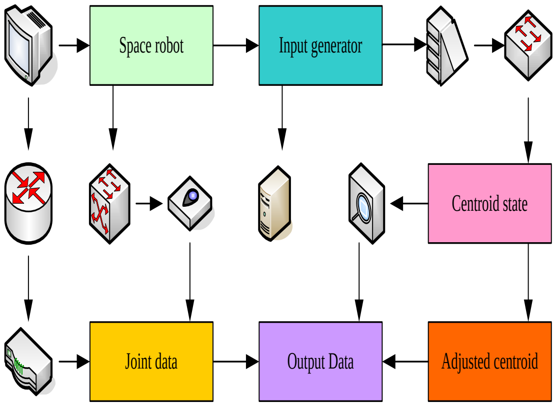


Figure 2: Execution framework for online foothold adjustment and motion control differential formulas

2.3 Numerical optimal control results of srattitude and motion differential formulas

According to the task requirements, the SR has a high-precision pointing in the task space. The attitude parameter description of attitude is the basis of dynamic modeling and motion control of SR.The parameter characteristics of the SR's attitude are given by the numerical method of the SR's differential formula.Regarding the CPU (Central Processing Unit) calculation time, error, and other indicators, the results of comparing the adaptive pseudo-spectral method with the single-segment pseudo-spectral method and the multi-segment pseudo-spectral method are shown in Table 2. Since the multi-segment pseudo-spectral method(Fourier transform is used to solve partial differential formulas, and the terms containing partial derivatives are transformed into Fourier forward and inverse transforms)and the adaptive pseudo-spectral method both automatically adjust the number of nodes according to the results during the simulation process, the number of nodes obtained in the simulation is not the same. However, the comparison of the three methods can still be seen from the results in the table. The computation time of the multi-segment pseudo-spectral method is 66% higher than that of the single-segment pseudo-spectral method, and it has higher accuracy than the single-segment pseudo-spectral method; the adaptive pseudo-spectral method meets the error required by the simulation. Although the error is slightly higher than that of the multi-segment pseudo-spectral method, the number of nodes is less than that of the multi-segment pseudo-spectral method, and the computational efficiency is significantly improved. In tasks with high real-time requirements, the adaptive pseudospectral method should be adopted.

In this article, an SR path optimization strategy based on the numerical method of differential formulas and the improved numerical method of differential formulas was proposed.Considering the state constraints and control constraints, for the numerical method of differential formulas, the time segment and Jacobian matrix are changed into a sparse matrix, which is beneficial to the fast solution of nonlinear programming algorithms. Further, a node selection method based on the relative error distribution evaluation function is proposed, and the adaptive pseudospectral method is improved, which significantly improves the computational efficiency, and meets the real-time requirements for the path optimization of the attitude-controlled SR.

Table 2: Adaptive pseudo spectral method compared to single-segment pseudo spectral method and multi-segment pseudo spectral method

Method used	Number of points	CPU time (s)
single-segment pseudospectral method	30	300
single-segment pseudospectral method	40	500
Multi-segment pseudospectral method	40	170
Adaptive Pseudospectral method	30	10

The advantages of SRs are that they save valuable non-renewable control fuel and extend satellite life, but the problem of optimal motion planning and control becomes more difficult. The motion of the manipulator affects the attitude of the base, so that the communication equipment or other on-board equipment cannot work normally, and the change of the attitude of the base will affect the positioning accuracy of the end effector of the manipulator. The research on the numerical method of differential equations for optimal trajectory planning of SR carriers without disturbance of attitude is an important part of this problem.It is stipulated that when the error between the claw pose and the transition pose is less than 10mm (three-axis composite error), and the angle between the corresponding axes is less than 5°, and the transition target pose is replaced with the target pose. In Figure 3, the X direction is the direction close to the target satellite. There would be small fluctuations in the process of stopping, at most about 15mm in the direction of the target satellite, and finally stop at the target pose. Of course, this error is permissible according to the tolerance of the gripper. In the other direction, the fluctuations are very small. If it is considered to end the planning when the error is ±1mm and ±1° (if no special instructions are given, this error refers to the composite error of the three-axis position and the corresponding axis angle), the planning would stop at N=180 cycles, about 40s. At this time, the angles of each joint are 268°, -90°, and 178°. Of course, if it has been stopped without judgment, the joints would naturally stop rotating due to the small fluctuation speed of each joint at the end.The motion planning of the plane three-degree-of-freedom robot is shown in Figure 3.

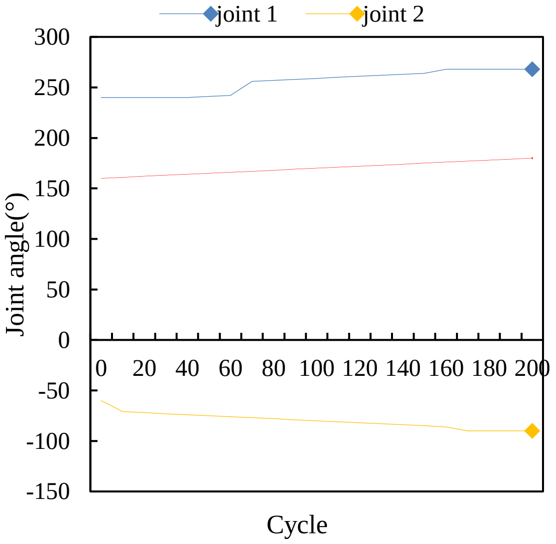


Figure 3: Planar 3DOF robot motion planning

In this paper, the research on the trajectory planning of SRs without disturbance of attitude is carried out based on the numerical method of differential equations of optimal control, so that the free SR can complete the predetermined task under the condition of ensuring the normal operation of communication equipment and other space-borne equipment, and solve the problem of free-floating SR. The optimal planning problem of non-holonomic pose motion in the undisturbed state of carrier attitude is an important part of optimal motion planning and optimal control of SR.

The tracking error of joint 1 in no-load mode is shown in Figure 4. Among them, the solid line and the dotted line represent the results of the robust controller and the PD (Proportional Derivative) controller, respectively. Under this working condition, the dynamic response and tracking error of the two controllers are not much different. This is because the nonlinear coupling of the manipulator is weak under this working condition, and the parameter setting of the PD controller is reasonable so that the PD controller achieves a good control effect.

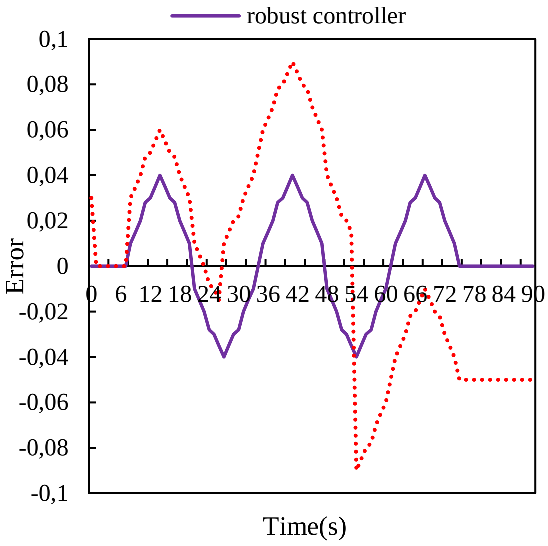


Figure 4: Tracking error of joint 1 in no-load mode

SR is a complex system with multiple inputs multiple outputs nonlinear strong coupling. The uncertainty of capturing target load and fuel consumption at the same time leads to the inability to obtain accurate kinematics and dynamic parameters of SR. Therefore, to meet the high-precision trajectory tracking, the controller must be able to have good control performance when the control object changes.The current method for tracking and controlling the space trajectory of SR tasks only designs the control law to meet the requirements of system stability but does not analyze the time domain performance index of the control system. Figure 5 illustrates that even when the inverse solution does not exist at the target pose, the differential formula numerical method based on the optimal control of SR attitude and motion would also converge to the closest point to the target pose, and the numerical method based on differential equations for optimal control of SR attitude motion uses different initial parameters with slightly different closest points. It can be seen from the speed curve that the robotic arm would eventually stop at this point, and the error of the two poses is smaller than the tolerance of the gripper. When N=160 cycles, the angles of each joint are 41°, 76°, and 10°, so the planning is feasible. The motion planning of the SR is shown in Figure 5.

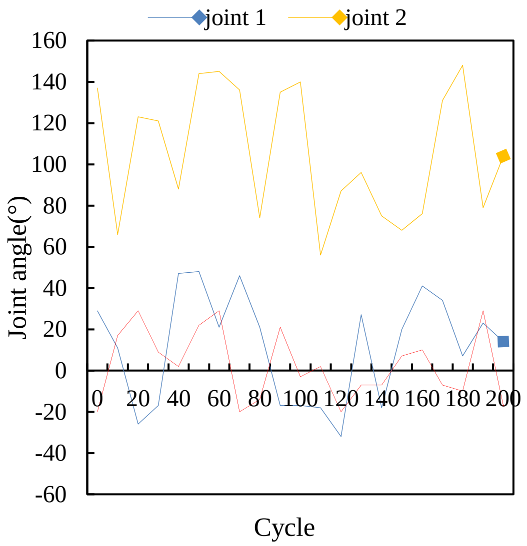


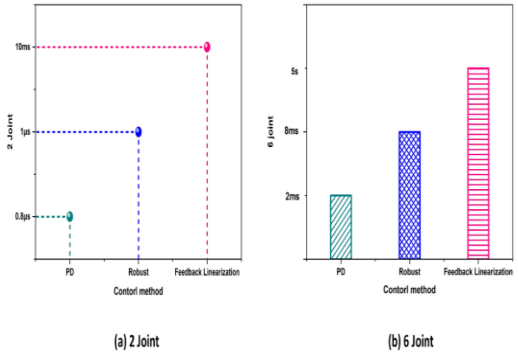
Figure 5: SR motion planning

Dynamic modeling is the basis for studying the trajectory planning, control, and ground digital simulation of SRs.In the zero gravity environment of space, it is a typical unrooted tree system for the whole system.It is necessary to study its kinematics and dynamics model to better guide the design of the robot platform. To meet the requirements of real-time digital simulation on the ground, for multi-degree-of-freedom SR systems, it is necessary to carry out efficient dynamic modeling research that is convenient for computer programming to improve the efficiency and stability of dynamic simulation. It also has important theoretical and practical significance. This paper compares the real-time performance of two-joint and six-joint manipulators using a PD controller, robust controller, and feedback linear controller, respectively. Table 3 and Figure 6 show the calculation time of different controllers when they do a feedback control (computer CPU main frequency 2.2GHZ). Usually, the time interval for an actual controller to perform a feedback control is several tens of milliseconds, so the feedback linearization method cannot be used on a multi-joint space manipulator (one feedback calculation for a six-joint manipulator can take up to4 seconds). The effect of the PD controller is not ideal, and it cannot meet the task needs of the nonlinear strongly coupled space manipulator.

Comprehensive consideration of the robust controller not only has good real-time performance but also has good robustness because the control performance is less affected by system structural parameters and disturbance torque.

Table 3: The calculation time of different controllers when they do a feedback control (computer CPU main frequency 2.2GHZ)

Control method	2 joints	6 joints
PD	0.8μs	2ms
Robust	1μs	8ms
Feedback Linearization	10ms	5s



(a) 2 Joint (b) 6 Joint

Figure 6: The calculation time of different controllers

The simulation time in this paper is set to 25 seconds, and the interactive simulation is used. It is found that the tracking and control performance of the SR trajectory has been greatly improved by using the robust adaptive PD control method. Compared with the self-tuning PID (Proportion Integral Derivative) control, the dynamic performance has been greatly improved; although the fluctuation is relatively large in the initial stage, error-free tracking can be achieved. Robust adaptive PD control can achieve undifferentiated tracking of the trajectory, with faster convergence speed and less disturbance to the attitude of the base, which is a good method. Figure 7 shows the position tracking situation of the differential formula numerical method based on the optimal control of the attitude motion of the SR.

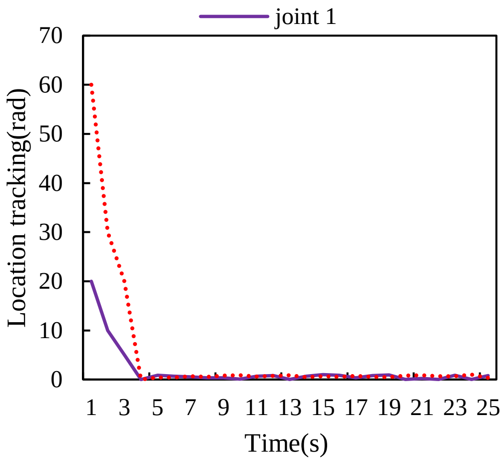


Figure 7: Position tracking situation based on differential formula numerical method for optimal control of SR attitude and motion

This paper mainly discusses the optimal control problem of the differential equation of attitude motion of SR and gives the numerical solution and prior error estimation of state variables, adjoint state variables, and control variables.To make up for the poor tracking control of traditional PID, this paper uses fuzzy self-tuning technology to adjust parameters online to greatly enhance the robot trajectory tracking ability. To overcome the dynamic uncertainty of the SR, an adaptive control is introduced to compensate for it, and a robust adaptive PD control is designed considering the disturbance with known supremum. Compared with the fuzzy PID control, the dynamic and static characteristics of the system have been greatly improved, and the disturbance to the base vehicle during the movement of the end of the space manipulator is analyzed, and the disturbance to the base of the vehicle becomes very small.

μ is an adjustment coefficient, which can generally take a value of around 1 in point-to-point motion planning. In the linear trajectory assumed in this paper, the changes of the end position relative to the initial position are -1.2m and -0.3m, respectively, and the end attitude has rotated 60° around its x-axis relative to the initial attitude. In this simulation, μ=50 is taken. Because the distance between the target poses and the claw pose is always small in planning, it is suggested that μ should be about the order of 10 in Cartesian planning. If μ still takes 0.8, although it has little effect on the tracking accuracy of the position, it would cause the tracking lag of the attitude, and there is a certain attitude error. The position error in the entire motion range is very small, the maximum is not more than 0.1mm, and the attitude tracking error is below 0.8mm. When N=61, the gripper has already reached the end of the linear trajectory, and the residual velocity of the algorithm is reduced to 0 after several cycles. If higher attitude tracking accuracy is obtained, the value of μ can be increased and the number of planned cycles can be adjusted. The linear trajectory motion planning in Cartesian space is shown in Figure 8. As long as the target pose is in its workspace, the algorithm can be used to successfully find a path for the paw to reach the target pose, which further illustrates the generality of the algorithm. When the target pose is unreachable, the algorithm converges to a point near the target pose, which shows that the algorithm still has good performance. The general method of applying the algorithm to continuous trajectory planning in Cartesian space is described, and the algorithm is not affected by singular points.

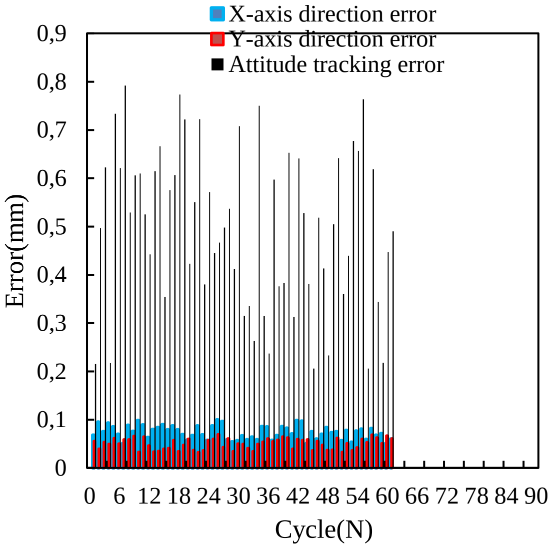


Figure 8: Linear trajectory motion planning in Cartesian space

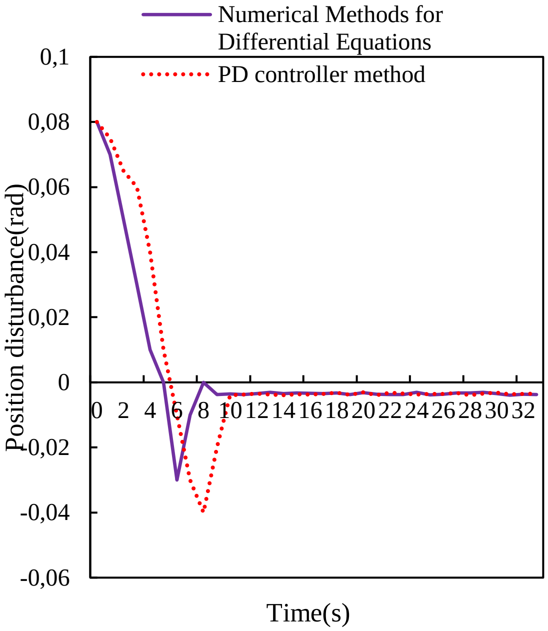


Figure 9: Response of SR body pose motion

In the working condition (with load) mode, when the joint motion adopts the differential formula numerical and the PD controller method respectively, the response of the SR body posture motion is shown in Figure 9. When the PD controller is used, the dynamic response quality of the excessive joint motion is poor, and the control torque is large, so the interference with the posture motion is also large. On the contrary, when the differential formula numerical method designed in this paper is used for trajectory tracking control, the trajectory tracking control effect is not only very good but also has little interference with the body posture.

In this article, a numerical parameterization design method of differential formulas based on optimal control of SR attitude motion is proposed, which comprehensively optimizes several performance indicators and improves numerical stability. The method is divided into two parts: a state feedback controller and a feedforward tracking controller. By using the free parameters in the controller design, the performance indicators of tracking error, robustness, and control energy are comprehensively optimized. It is further considered that most of the existing control methods first convert the SR system into a first-order matrix model, but it is essentially a second-order matrix nonlinear model. Directly relying on the second-order matrix nonlinear model of the SR, the matrix inversion operation is avoided. The numerical simulation of differential formulas is aimed at the situation of SR joint space trajectory tracking, task space trajectory tracking, and dual-arm point-to-point tracking [28].

3 Sensitivity analysis

A sensitivity analysis was conducted. This analysis involved systematically varying each parameter within a reasonable range and observing the resulting performance metrics, such as tracking error or control effort. By quantifying how changes in parameters affect the system's behavior, the robustness and effectiveness of the chosen parameter values were evaluated, ensuring the controller's reliability under different operating conditions and uncertainties. Figure 10 shows the result of the sensitivity analysis.

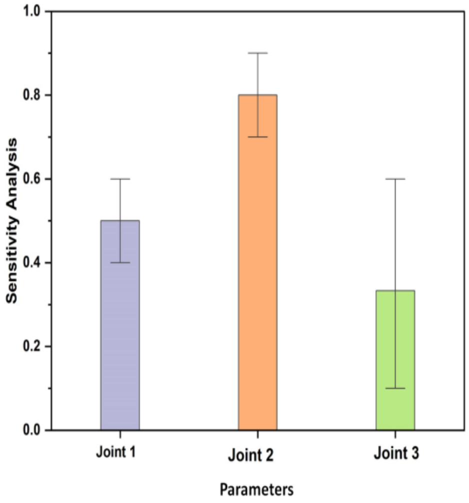


Figure 10: Sensitivity analysis

Table 4: parameter provides a list of the robust trajectory tracking controller's specifications

Parameter	**Initial value**	**Sensitivity Analysis Range**	**Performance metric (tracking error)**
Gain for Joint 1	0.5	0.4 to 0.6	0.01 to 0.05
Gain for Joint 2	0.8	0.7 to 0.9	0.01 to 0.06
Gain for Joint 3	0.6	0.1 to 0.3	0.008 to 0.0012

Table 4 contains the parameter that provides a list of the robust trajectory tracking controller's specifications. The parameter's original value selected during the design phase is represented by the initial value. The parameter's range of variation during sensitivity analysis is indicated by the term of sensitivity analysis range. Performance Metric refers to the performance metric that was obtained when the parameter was changed within the given range, such as tracking error.

4 Discussion

An important real-world application of Zhang B [4] was lacking uncertainty. Particularly focusing on focus SR attitude motion, Zhu Z [5] handled unknown aspects. Singularity robust route planning control and explicit exploration of applicability uncertainty SR systems are provided by study Cheng Z [6]. Li Z [7] presented an updated target-capturing control technique with an emphasis on robust trajectory tracking presence uncertainty. Hu Y [8] explicitly addressed trajectory tracking uncertainty by designing a disturbance observer tethered SR. The dynamical features of SR control are examined in Novo S [9]. Zhang L [10] covered linearization methods, fixed point theorems, and significant, immediately relevant suggested problems. Yaozhong H U [11] and Li R [12] center on time-discrete approaches and powerful approximation techniques that are specially designed for robust trajectory tracking in unpredictable space conditions. The mean-square random dynamical system approach, which established the application of SR control uncertainty, was presented in Wu F [13]. The suggested approach for SR attitude motion incorporated a thorough investigation of trajectory tracking uncertainty, to address the shortcomings found in previous research. A comprehensive solution was provided by combining ideas from several fields to provide coordinated stabilization, unknown variables consideration, singularity robust path design, and approach. Robust performance in uncertain space settings was ensured by strategies for disturbance observation that carefully examined dynamical characteristics. To connect the gap between theoretical developments and practical implementations in SRics, the method offered a mean-square random dynamical system framework that was modified to SR control uncertainty.

5 Conclusion

The research on motion control of SR (space manipulator) is the research hotspot of space application technology. SRs can replace astronauts to complete various complex and dangerous tasks, which can not only prolong the service life of target aircraft such as orbiting satellites and space stations but also greatly reduce the economic cost of space missions and improve economic benefits. In this paper, the numerical method of differential formulas was studied, and the robust optimal control strategy was used to complete the real-time tracking of the dynamic model of the SR. A differential formula numerical method based on the optimal control of SR attitude motion was designed, which can reduce the joint control torque and improve the trajectory tracking accuracy, thereby improving the on-orbit service life of the SR. Due to insufficient capacity and insufficient research time, there are still many problems in the research. The main task of future work is to find a suitable nonlinear programming method to replace the method of subdividing the angle range of each joint first and then traversing it, which may achieve the same effect as the speed-level method.

Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of interest

The authors declare no conflicts of interest.

Funding statement

This study didn't receive any funding in any form.

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