Balanced Generalised Tailored Approximation Point Algorithm for Solving Convex Optimisation Mathematical Problems in Bearing Vibration Signal Compressive Sensing

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With the continuous progress of science and technology, the transformation of compressive sensing problems into convex optimization problems has become a hot research topic. In this study, a novel algorithm, the balanced generalized customized proximal point algorithm, is proposed, which integrates the generalized customized proximal point algorithm with the balanced-augmented lagrangian method. Based on this algorithm, a compressive sensing system for bearing fault signals is designed, and the bearing fault signals are compressed by the universal compressive sensing model and the K-singular value decomposition algorithm, Then, the signals are reconstructed using the BG-CPPA. The experimental results showed that the BG-CPPA had a lower number of iterations and computation time compared with the traditional algorithm at different sparsity conditions. The reconstruction effect of the bearing inner ring signal was the best. Specifically, the BG-CPPA reduced the reconstruction error by 33.33% and 20.00%, while reducing the reconstruction time by 32.46% and 52.64%. At compression ratios of 0.3, 0.4, and 0.5, the proposed compressive sensing system reduced the reconstruction error by 35.39%, 44.06%, and 26.76% over the greedy algorithm, respectively. These results confirm the effectiveness of the BG-CPPA in improving the reconstruction accuracy and stability of bearing vibration signals, as well as the potential of the designed compressive sensing system in enhancing the observation efficiency of bearing fault vibration signals.

Povzetek: Predlagan je izboljšan algoritem za konveksno optimizacijo v stisnjenem zaznavanju vibracijskih signalov ležajev, ki zmanjšuje napake rekonstrukcije in izboljšuje stabilnost ter učinkovitost obdelave podatkov.

1 Introduction

Affected by the development of science and technology, various problems in fields such as economy, computer science, and industry can be effectively transformed into optimization problems to solve. The convex optimization problem with linear equality constraints is one of the most common optimization problems, which is widely used in compressive sensing, image processing, machine learning, etc [1-3]. Traditional fault detection of rolling bearings mainly relies on bearing vibration signals. However, the high sampling frequency generated during the operation of rolling bearings can lead to a large amount of data, which have negative impacts on the detection results [4]. Compressive sensing is an abstract mathematical concept. It can effectively reduce the volume of compressed signal data obtained, thereby achieving bearing vibration signal compression and alleviating the drawbacks caused by excessive sampling data [5-6]. Therefore, a vibration signal compressive sensing system for rolling bearings is constructed to improve the accuracy and precision of bearing fault detection. Meanwhile, to improve the effectiveness of this method in practical applications, a convex optimization algorithm is designed based on the Generalized Customized Proximal Point Algorithm (GCPPA) and the Balanced-Augmented Lagrangian Method (B-ALM), namely the Balanced Generalized Customized Proximal Point Algorithm (BG-CPPA). It is expected to improve the compressive sensing reconstruction algorithm and utilize the convex optimization algorithm BG-CPPA to enhance the accuracy and stability of the bearing vibration signal compressive sensing system, promoting the innovation and development of bearing vibration signal processing technology.

The overall structure of the study consists of five sections. The first section summarizes the research achievements and shortcomings of convex optimization algorithms and compressive sensing technology both domestically and internationally. In the second section, a bearing vibration signal compressive sensing system based on the BG-CPPA is designed. In the third section, experiments and analysis are conducted on the proposed BG-CPPA and compressive sensing system. In the fourth section, the proposed BG-CPPA and compressive sensing system are compared with existing methods. In the fifth section, the experimental results are summarized and future research directions are indicated.

2 Related works

In modern signal processing and data analysis, the research on convex optimization algorithms and compressive sensing techniques has become a hot topic. Convex optimization algorithms have received widespread attention due to their high efficiency and stability in solving various mathematical problems, especially in dealing with optimization problems with complex constraints, showing unique advantages [7]. Lu et al. used convex optimization algorithms to re-represent non-convex problems in partially convex optimal control and optimization problems. A convex concave decomposition algorithm was proposed to handle nonlinear equality constraints, which solved the optimal fuel limited thrust spacecraft orbit problem [8]. For the finite time domain robust covariance control problem of partially observable linear systems, Kotsalis developed a computable processing framework for affine control strategy design based on the mean convex quadratic inequality and chance constrained linear inequality, achieving performance specifications in stochastic state control trajectories [9]. He et al. proposed a second-order continuous primal dual dynamical system with a time-dependent positive damping term for separable convex optimization problems with linear equality constraints. The time asymptotic properties of the system were verified through the Lyapunov analysis method. The convergence speed at different damping coefficients was derived [10]. A reliable and efficient trajectory generation method is a fundamental requirement for autonomous power systems. Malyuta et al. proposed a comprehensive tutorial on three trajectory generation methods based on convex optimization. The lossless convexity and two sequence convex programming algorithms ensured continuous convexity to optimize sequence trajectories. The convex optimization was used to generate non-convex trajectories [11]. To solve the collaborative problem where only the cost function of each node and its neighboring point information can be obtained, Liu et al. proposed a continuous time primal dual algorithm for constrained convex optimization problems in time-varying indirect connected graphs. This achieved the optimal solution under global convergence of the average state [12].

In addition, compressive sensing technology is gradually changing traditional signal processing methods due to its high efficiency and low-cost characteristics in signal acquisition and reconstruction. As a key component in mechanical systems, accurate vibration signals analysis is crucial for fault diagnosis and predictive maintenance. By combining convex optimization and compressive sensing techniques, researchers can more effectively process and analyze these complex signals, thereby improving the accuracy and efficiency of fault detection [13]. Chen proposed a compressive sensing method to address the limited data transmission capacity during remote machine condition monitoring, significantly reducing the computational complexity of vehicle fault diagnosis [14]. To cope with high sampling points and high sampling points for acoustic emission signals, Tai et al. proposed a compressive sensing processing framework. The wavelet sparse convolutional network was established to solve diagnosis and evaluation, thereby reducing the signal compression rate while ensuring acoustic reconstruction errors, and reducing the transmission signal data and pressure [15]. To promote energy perception in long-term vibration monitoring systems, Zonzini et al. proposed a model assisted variant based on the compressive sensing method. Sensing nail tied steel beams could retain reconstructed structural parameters even in defective configurations [16]. Although the remote wind turbine status monitoring system has better computing resources, there is data loss. Therefore, Peng et al. proposed a fault-tolerant missing data fault detection method based on compressive sensing. The compressive sensing signal reconstruction algorithm effectively reduced the probability of bearing fault detection data loss for two types of wind turbines [17]. Wang et al. proposed a novel modeling and control strategy for axial hybrid magnetic levitation bearings used in household flywheel energy storage systems to achieve effective monitoring of bearings. A new magnetic flux density feedback control was adopted instead of traditional control, achieving performance consistent with traditional position feedback control strategies [18]. Al-Chaab et al. put forward a medical image security compression system based on compressive sensing principle to solve the medical data security and privacy protection. The image was segmented and encoded using a Gaussian random number sensor matrix. The compression rate of the image size was about 30%, and the least significant bit technique was used to hide the data in the audio file, thereby improving the security and compression efficiency of the data [19].

Based on the above, current research on convex optimization problems with linear equality constraints mainly focuses on algorithm applications, while there is relatively little research on algorithm improvement. Compressive sensing technology, as a commonly used compression method in signal processing, has received less research from domestic and foreign scholars on its combination with convex optimization algorithms. In this context, a new convex optimization algorithm is proposed by combining GCPP algorithm and B-ALM algorithm. Then, a bearing vibration signal compressive sensing system is designed. Unlike current research methods, an innovative reconstruction algorithm for bearing vibration signals is designed to address the accuracy and stability of signal reconstruction in compressive sensing systems. It is expected to expand the application value of convex optimization algorithms and improve the effective observation of bearing fault vibration signals in compressive sensing systems. The study further summarizes the differences between the existing literature and the proposed method, as shown in Table 1.

Table 1: Comparison between existing literature and proposed method

Reference	Method	Problem domain	Limitations
Lu et al. [8]	Convex-concave decomposition	Nonlinear equality constraints	May not scale well with large datasets
Kotsalis [9]	Affine control strategy	Stochastic state control	Sensitive to specific types of noise
He et al. [10]	Second-order continuous primal-dual dynamical system	Separable convex optimization	Strongly dependent on damping coefficients
Malyuta et al. [11]	Convex optimization-based trajectory generation	Trajectory generation	Limited applicability to non-convex trajectory generation tasks
Liu et al. [12]	Continuous time primal-dual algorithm	Distributed consensus problems	High computational complexity
Chen [14]	Compressive sensing approach	Remote machine condition monitoring	High demand on data transmission capacity
Tai et al. [15]	Wavelet sparse convolutional network	Acoustic emission signal processing	Inefficient for processing high sampling point data
Zonzini et al. [16]	Tilt-degree compressive sensing method	Structural health monitoring	Limited adaptability to defect configurations
Peng et al. [17]	Fault-tolerant method based on compressive sensing	Wind turbine condition monitoring	Sensitive to data loss issues
Wang et al. [18]	Hybrid magnetic bearing control	Flywheel energy storage system	Strong dependency on control strategies
Al-Chaab et al. [19]	CS-based medical image compression system	Medical data security	Limited hiding capability in audio files
This paper	GCPPA and B-ALM	Bearing fault signal compressive sensing	-

3 A compressive sensing system for bearing vibration signals based on BG-CPPA

The compressive sensing problem can be effectively transformed into a linear equality constrained convex optimization problem for solving. Therefore, a BG-CPPA based on GCPPA and B-ALM is proposed to solve the bearing fault vibration signal observation. On this basis, a bearing vibration signal compressive sensing system is designed based on the universal compressive sensing theory model, which is reconstructed using the BG-CPPA.

3.1 BG-CPPA design

Aiming at the low computational accuracy and poor stability of convex optimization algorithms in compressive sensing systems, a BG-CPPA suitable for solving linear equality constrained convex optimization problems is designed by combining GCPPA and B-ALM. The GCPPA is an extension of the Customized Proximal Point Algorithm (CPPA). It has great application value in fields such as image processing, statistics, and compressive sensing. Unlike the drawbacks of the CPPA where convergence efficiency is limited by the relaxation factor, GCPPA eliminates the relaxation step while ensuring convergence efficiency [20]. The specific iterative expression is shown in equation (1).

$$\begin{cases} \gamma^{k+1} = \gamma^k - \frac{\partial}{t}(Ax^k - b) \\ x^{k+1} = \arg\min_{x\in\chi} \left\{ \vartheta(x) + \frac{r}{2} \left\| \begin{matrix} (x - x^k) - \frac{1}{r}A^T \\ \left[(1+\partial)\gamma^{k+1} - \partial\gamma^{k+1}\right] \end{matrix} \right\|^2 \right\} \end{cases} \quad (1)$$

In equation (1), $\vartheta(x)$ represents a convex function and also represents the objective function. x represents a convex optimization problem. χ represents a set. γ represents the Lagrange multiplier. A represents the coefficient matrix. b represent the known vector. r, k, t and ∂ all represent parameters, $r,k,t>0$, and $\partial \in (0.5,1)$. The B-LAM algorithm is an extension of the Augmented Lagrangian Method (ALM). It balances the excessive proportions of the objective function, coefficient matrix, and set in the two sub-problems by reconstructing ALM [21]. The specific iteration is shown in equation (2).

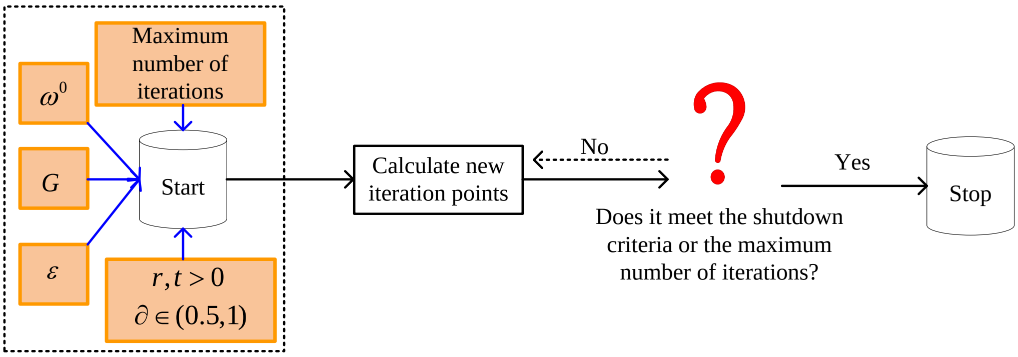


Figure 1: Iterative steps of BG-CPPA

$$\begin{cases} \gamma^{k+1} = \gamma^k - M_0^{-1}[A(2x^{k+1} - x^k) - b] \\ x^{k+1} = \arg\min_{x\in X} \left\{ \vartheta(x) + \frac{r}{2} \left\| x - x^k - \frac{1}{r}A^T\gamma^k \right\|^2 \middle| x \in \chi \right\} \end{cases} \quad (2)$$

In equation (2), M_0 represents a positive definite matrix. The B-LAM is often transformed into an equivalent form in solving convex optimization problems, as shown in equation (3).

$$\begin{cases} \gamma^{k+1} = \gamma^k - M_0^{-1}[A(2x^{k+1} - x^k) - b] \\ x^{k+1} = \mathrm{Prox}_{\vartheta}^{r}(x^k + \frac{1}{r}A^T\gamma^k) \end{cases} \quad (3)$$

However, the GCPPA has limited applicability in solving practical problems. The iterations of the B-ALM is affected by the quantity. The convergence efficiency decreases accordingly, which is not conducive to solving problems related to large-scale data. Therefore, a BG-CPPA is proposed by combining two algorithms. The specific calculation steps are shown in Figure 1.

Firstly, the initial point, positive definite matrix, termination condition, and other parameters are set. A new iteration point is calculated based on the maximum number of iterations. The iterative of the BG-CPPA is shown in equation (4).

$$\begin{cases} \gamma^{k+1} = \gamma^k - \frac{a}{t}(Ax^k - b) \\ x^{k+1} = \arg\min_{x\in X} \{\vartheta(x) + \\ \frac{1}{2}\left\|(rA^TA + G)(x - x^k) - A^T\left[(1+\partial)\gamma^{k+1} - \partial\gamma^k\right]\right\|^2 \middle| x \in \chi \} \end{cases} \quad (4)$$

In equation (4), G stands for the positive definite matrix, which can be used to ensure the convexity of the objective function, thus ensuring the convergence and uniqueness of the optimization algorithm. The shutdown criterion condition is shown in equation (5).

$$\begin{cases} \max\left\{\left\|x^k - x^{k+1}\right\|, \left\|\gamma^k - \gamma^{k+1}\right\|\right\} \le \varepsilon \\ or \\ iteration_{\max} < k \end{cases} \quad (5)$$

In equation (5), ε represents the termination condition. $iteration_{\max}$ represents the maximum number of iterations. The first-order optimality condition for the convergence iteration of the proposed BG-CPPA is shown in equation (6).

$$\begin{cases} \vartheta(x)-\vartheta(x^{k+1})+(x-x^{k+1})^T \begin{Bmatrix} (rA^TA+G)(x^{k+1}-x^k) \\ -A^T\left[(1+\partial)\gamma^{k+1}-\partial\gamma^k\right] \end{Bmatrix} \geq 0 \\ (\gamma-\gamma^{k+1})^T\left\{\partial(Ax^k-b)+t(\gamma^{k+1}-\gamma^k)\right\} \geq 0 \end{cases} \quad (6)$$

According to the optimality condition equation, the equivalent form of its inequality is transformed into a compact form of inequality, as shown in equation (7).

$$\begin{aligned} &\vartheta(x)-\vartheta(x^{k+1})+(\omega-\omega^{k+1})^T[F(\omega^{k+1})+Q(\omega^{k+1}-\omega^k)] \\ &+\frac{t(\partial-1)}{\partial}(\gamma-\gamma^{k+1})^T(\gamma^{k+1}-\gamma^{k+2}) \geq 0 \end{aligned} \quad (7)$$

In equation (7), Q represents a positive definite matrix. ω represents the iteration point. $F(\omega)$ denotes the affine monotone function, as shown in equation (8).

$$F(\omega)=\begin{pmatrix} -A^T\gamma \\ Ax-b \end{pmatrix} \quad (8)$$

The lemma of algorithms is closely related to their properties. Therefore, two lemmas are further proposed as reference equations for subsequent performance verification of the BG-CPPA. Lemma 1: The solution of the first-order optimality condition inequality for convex optimization problems is generated based on the sequence generated by the BG-CPPA, as shown in equation (9).

$$\left(\omega^*-\omega^{k+1}\right)^T Q(\omega^{k+1}-\omega^k) \geq t(1-\frac{1}{\partial})(\gamma^*-\gamma^{k+1})^T(\gamma^{k+1}-\gamma^{k+2}) \quad (9)$$

Lemma 2: The sequences calculated according to the iterative equation of the BG-CPPA satisfy the inequality shown in equation (10).

$$\begin{aligned} &\left\|x^k-x^{k+1}\right\|_D^2+t(2-\frac{1}{\partial})\left\|\gamma^{k+1}-\gamma^{k+2}\right\|^2 \\ &\leq \left\{\left\|\omega^*-\omega^{k+1}\right\|_Q^2+t(1-\frac{1}{\partial})\left\|\gamma^*-\gamma^{k+1}\right\|^2-\left\|\omega^*-\omega^{k+1}\right\|_Q^2 \right. \\ &\left. -t(1-\frac{1}{\partial})\left\|\gamma^*-\gamma^{k+1}\right\|^2\right\} \end{aligned} \quad (10)$$

According to Lemma 1 and Lemma 2, the convergence property of the BG-CPPA can be further effectively verified, thereby achieving broader application value. Therefore, the process of BG-CPPA to achieve convergence is shown in Table 2.

3.2 A compressive sensing system design for bearing vibration signals based on BG-CPPA

Compressive sensing technology obtains signals in actual engineering construction through under-sampling based on signal sparsity and non correlation. Combined with optimization algorithms, the data are reconstructed to directly collect natural signal data through compression. The sparse signal is shown in equation (11).

$$y=\Phi\sigma \quad (11)$$

In equation (11), y represents the sampled value or observed data. Φ represents a matrix that multiplies the number of linear measurements by the length of the original signal. σ represents a sparse signal. In the sensing process of sparse targets, the signal itself contains fewer non-zero elements, i.e., the target finite coefficient signal contains only several non-zero elements. The specific sparse signal compressive sensing routine theory model is shown in Figure 2.

However, in practical applications, the sampled signal data do not exhibit sparsity. Sparse features often need to be formed through changes in a certain transformation domain. Therefore, combined with the mathematical expression equation of basic compressive sensing, a universal compressive sensing mathematical model is developed, as shown in Figure 3.

Compared with the sparse signal compressive sensing theory model, the target signal in the universal compressive sensing model is a vector with sparse features that has a finite length and a discrete distribution in the spatiotemporal dimension [22-23]. Therefore, the bearing vibration signal compressive sensing system based on the BG-CPPA proposed in the study follows a universal compressive sensing model, forming an over-complete dictionary in dictionary learning. It combined with the optimal measurement matrix to capture bearing vibration signal data. The specific implementation process is shown in Figure 4.

Table 2: Convergence process of BG-CPPA

Pseudocode of # BG-CPPA
# Initialization
x_k = initial_point_x
y_k = initial_point_y
G = positive_definite_matrix
epsilon = termination_condition

```
max_iterations = maximum_number_of_iterations
r = positive_real_number
t = positive_real_number
alpha = alpha_parameter_in_range_0_5_to_1
# Iteration process
for k in range(max_iterations):
# Calculate new iteration point x_{k+1}
x_k_plus_1 = argmin_x {theta(x) + 0.5 * ||x - y_k||_G^2 + r * ||Ax - b||^2}
# Calculate new iteration point y_{k+1}
y_k_plus_1 = x_k_plus_1 + alpha * (A * x_k_plus_1 - b)
# Convergence Check
if ||x_k_plus_1 - x_k|| <= epsilon or ||A * x_k_plus_1 - b|| <= epsilon:
print ("Algorithm has converged.")
break
else:
x_k = x_k_plus_1
y_k = y_k_plus_1
# Output the solution
print ("Solution:", x_k_plus_1)
```

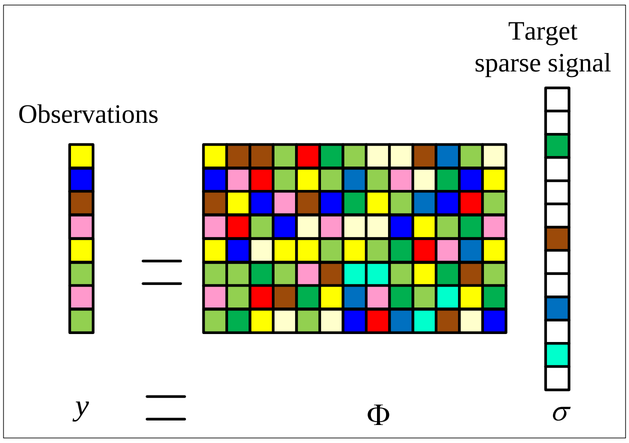


Figure 2: Theory model of sparse signal compressive sensing

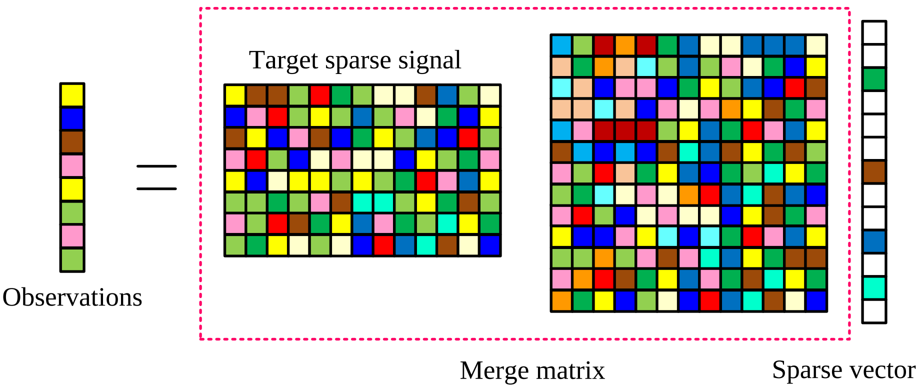


Figure 3: Universal compressive sensing theory model

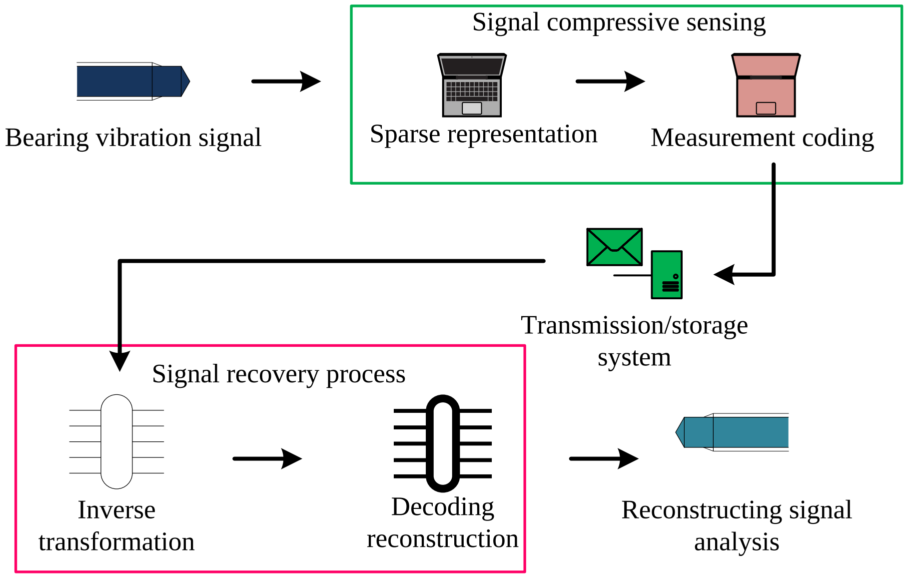


Figure 4: Implementation process of compressive sensing system for bearing vibration signals

After compression detection, the BG-CPPA is used to reconstruct and restore the original bearing vibration signal. This compressive sensing system is used to compress signals under under-sampling conditions. Therefore, the negative impact of large-scale sampled signal data on transmission and storage systems can be mitigated. Meanwhile, considering the limitations of compressive sensing systems in signal usage, actual signal data have potential sparsity. Therefore, sparse matrices are used to represent the collected data signals, as shown in equation (12).

$$\sigma = \sum_{i=1}^{N} \varphi_i \theta_i = \Psi \theta \qquad (12)$$

In equation (12), N represents the length of the original signal. φ_i represents the coefficient atom. θ represents a sparse vector in a sparse matrix. Ψ represents the sparse basis. Sparse vectors have potential sparse features in sparse bases. Then, the K-Singular Value Decomposition (K-SVD) algorithm is used to construct an over-complete dictionary internally associated with vibration signals. The core of the K-SVD mainly includes two parts: iteratively updating the dictionary and sparse encoding [24-25]. The process of iteratively updating the dictionary is often accompanied by significant deviations. Therefore, under the sparsity control, the overall error is reduced by optimizing and updating each column of atoms. The error matrix is shown in equation (13).

$$Err = Y - \sum_{i \neq \kappa} d_i \sigma_i \qquad (13)$$

In equation (13), Err represents the error matrix. Y represents the atomic matrix of the bearing vibration signal. κ represents the sparsity. d represents the error value. The mathematical expression of sparse encoding is shown in equation (14).

$$P, O = \arg\min \left\| Y - PO \right\|_2^2 \qquad (14)$$

In equation (14), P represents the original over-complete dictionary. O represents a sparse matrix. According to the constructed bearing vibration signal compressive sensing system, the termination condition of the BG-CPPA in numerical experiments is shown in equation (15).

$$\max\left\{ \left\| x^k - x^{k+1} \right\|, \left\| \gamma^k - \gamma^{k+1} \right\| \right\} \leq \varepsilon \qquad (15)$$

The algorithm parameter selection for decompressive sensing problem is shown in equation (16).

$$\begin{cases} G = (r \cdot \rho(A^T A) + 0.01) I_n - r A^T A \\ \partial = 0.95 \\ r = 0.1 \\ t = \dfrac{\partial^2 + 0.15}{r} \end{cases} \qquad (16)$$

In equation (16), I_n represents the identity matrix of n dimension. ρ represents the number of repeated experiments.

4 Verification analysis of bearing vibration signal compressive sensing system based on BG-CPPA

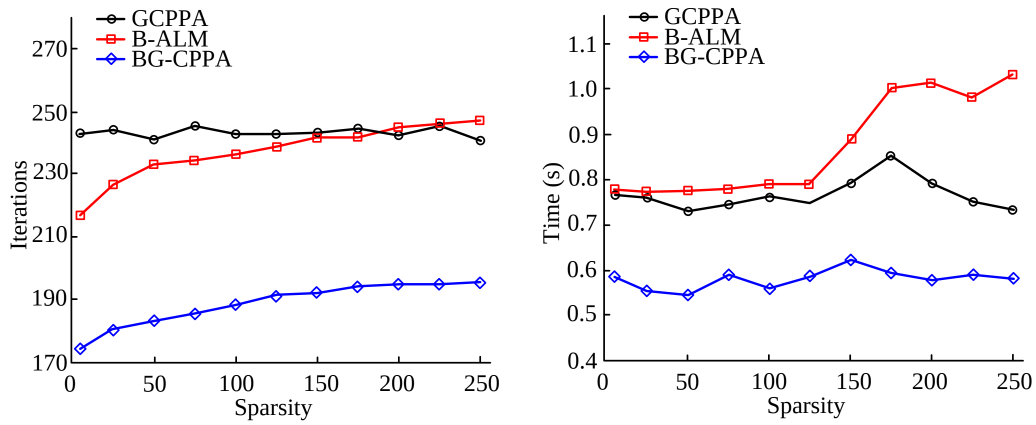
The convex optimization algorithm BG-CPPA and the compressive sensing system are used for sparse signal simulation experiments in response to the potential sparsity characteristics of bearing fault vibration signals. The algorithm is validated in terms of function performance comparison, simulated signal reconstruction effect, and reconstructed Signal-To-Noise Ratio (SNR) at different compression ratios. Based on wavelet threshold function denoising, the fault signal observation verification of the compressive sensing system is carried out.

4.1 BG-CPPA validation analysis

To verify the effectiveness of the proposed BG-CPPA in solving convex optimization problems such as compressive sensing, the algorithm performance is verified by combining function performance comparison, one-dimensional simulation signal reconstruction efficiency, and reconstruction SNR of bearing fault signals at different compression ratios.

The BG-CPPA takes the value of 0.95 for ∂, 0.1 for r, and $t = \frac{\partial^2 + 0.15}{r}$. The number of data training samples is 4096 and the number of tests is 1024, with sparsity $k \in \{1, 2, \ldots, 250\}$. Different sparsities are selected and the average value is calculated by repeating the experiment 10 times.

By controlling the increase in sparsity, the comparison results of BG-CPPA, GCPPA, and B-LAM at termination conditions of 10-8 are shown in Figure 5.



(a) Comparison of iteration times of three algorithms at different sparsity levels

(b) Comparison of computation time for three algorithms at different sparsity levels

Figure 5: Comparison of algorithm performance at different sparsity variations

From Figure 5 (a), the B-ALM exhibited significant fluctuations with increasing sparsity. The iterations of BG-CPPA and GCPPA were relatively more stable. The number of iterations of BG-CPPA was lower than the other two methods in the whole sparsity change, which shows that the algorithm can maintain consistent performance and better robustness when dealing with data with different sparsity. The sparsity of detection algorithm is low, which is very important for fault detection system. In bearing fault detection, the reliability is directly related to the accuracy and timeliness of fault detection, thus affecting the maintenance and operation safety of equipment. Figure 5 (b) shows the computation time of three algorithms at different sparsity levels. The calculation time of BG-CPPA was less affected by sparsity, which fluctuated around 0.60s. The calculation time of B-ALM was greatly affected by sparsity. When the sparsity was greater than 150, the calculation time increased by 12.13% -30.47%. This shows that BG-CPPA can maintain high efficiency at different sparsity. In contrast, the calculation time of B-ALM algorithm increases significantly with the increase of sparsity, which indicates that it is inefficient when dealing with high sparsity data. Overall, the BG-CPPA performs significantly better than the other two algorithms when the termination condition is 10-8.

The BG-CPPA is to transform the compressive sensing signal of bearing vibration into a convex optimization problem for reconstruction. Therefore, to further verify the performance of the BG-CPPA, a reconstruction verification is conducted using simulated signals. Considering that the signal data obtained in practical applications often contains external noise disturbances, an additional 15 decibel Gaussian white noise is added to the simulated signal for performance testing. 15dB is a common SNR level. After repeated experiments, it is found that 15 dB is a balance point between algorithm performance and computing resources, which can ensure sufficient performance and avoid excessive computing burden. The comparison results of the three methods are shown in Figure 6.

From the signal reconstruction results of the three algorithms, the three convex optimization algorithms were not sensitive to the signal sparsity. There was no distortion in the reconstruction results. This indicates that when the convex optimization algorithm performs reconstruction, it can automatically predict sparsity and accurately recover the original signal. Overall, the GCPPA shown in Figure 6 (a) has the worst simulation effect, followed by Figure 6 (b). The simulation effect of the BG-CPPA proposed in the study is superior among the three algorithms. The average reconstruction time, reconstruction error, SNR and Peak Signal-to-Noise Ratio (PSNR) comparisons of the three methods are shown in Table 3.

In Table 3, the BG-CPPA had the lowest reconstruction error and reconstruction time among the three algorithms. In terms of reconstruction error, BG-CPPA decreased by 32.46% and 52.64% compared with GCPPA and B-ALM, respectively. In terms of reconstruction time, the BG-CPPA reduced 33.33% and 20.00% respectively compared with the two algorithms. From the reconstruction results, it is demonstrating that the BG-CPPA has significant advantages in reconstruction speed, noise resistance, and reconstruction accuracy, making it more effective in solving convex optimization problems in compressive sensing technology. From the SNR and PSNR of the three algorithms, the SNR and PSNR of the proposed method are better than the other two methods. This indicates that the reconstructed signal quality of BG-CPPA is better, with less difference from the original signal and higher recovery accuracy. The reconstruction SNR results of three algorithms at different compression ratios are shown in Figure 7. From Figure 7, the reconstruction SNR of the three algorithms increased with the increase of the compression ratio. The reconstruction SNR of BG-CPPA had the best increase in amplitude and speed compared with the other two algorithms. This indicates that BG-CPPA has advantages in reconstruction processing, which has the best reconstruction effect. The performance of three algorithms in reconstruction testing is shown in Figure 8.

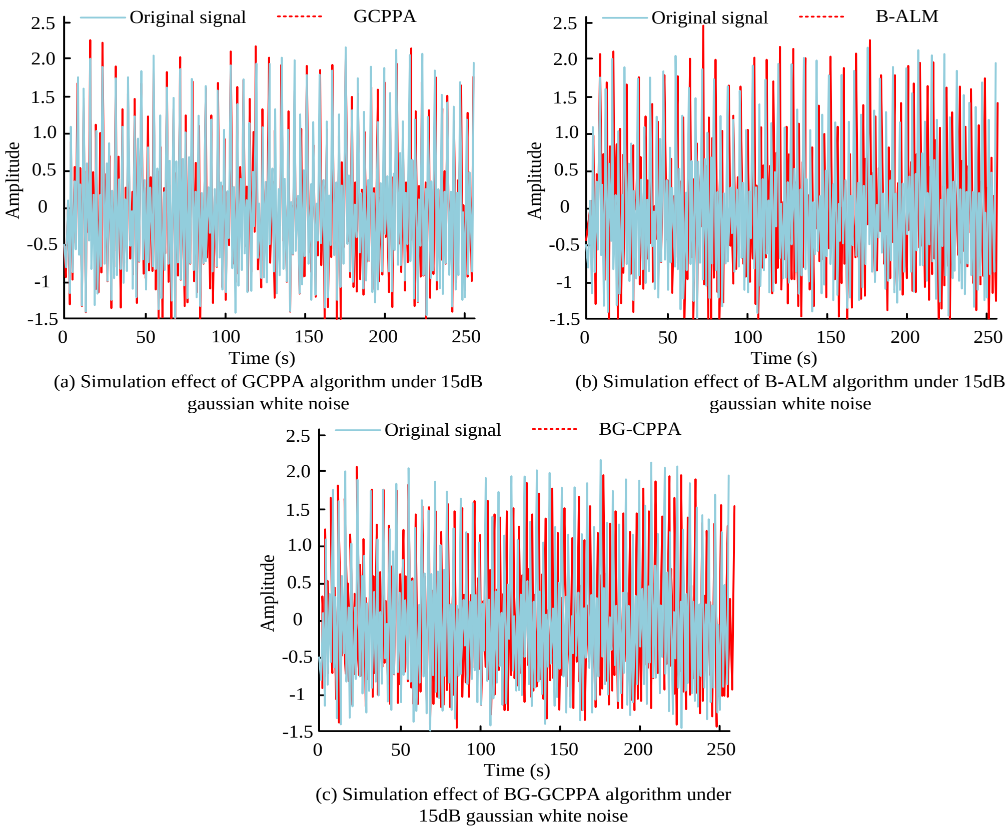


Figure 6: Comparison of reconstruction effects at 15 dB Gaussian white noise

Table 3: Comparison of analog signal reconstruction performance at 15dB Gaussian white noise

Targets	GCPPA	B-ALM	BG-CPPA
Err	3.05	4.35	2.06
Time (s)	0.06	0.05	0.04
SNR (dB)	21.50	28.13	29.76
PSNR (dB)	22.04	28.97	30.53

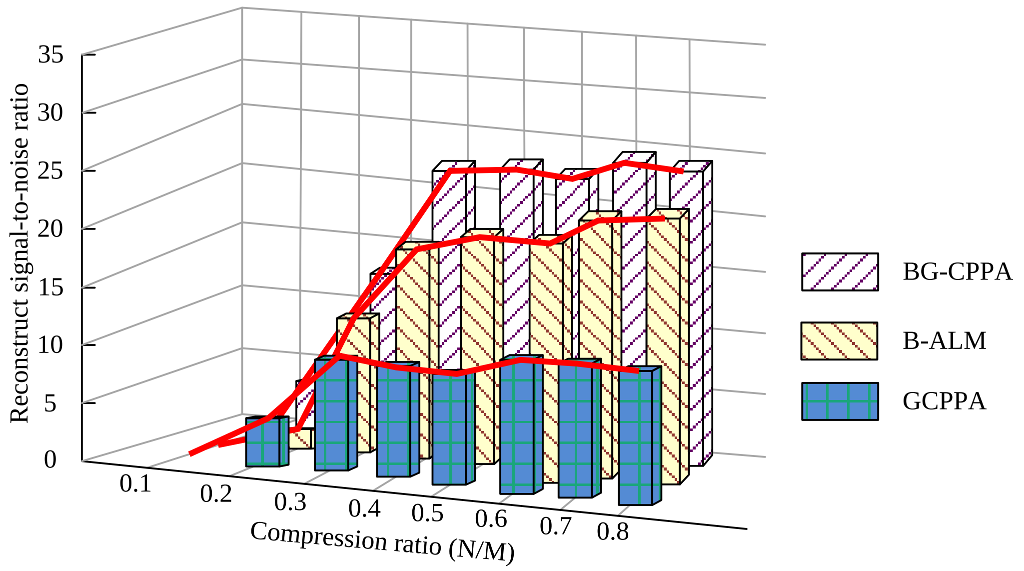


Figure 7: Reconstruction signal-to-noise ratio at different compression ratios

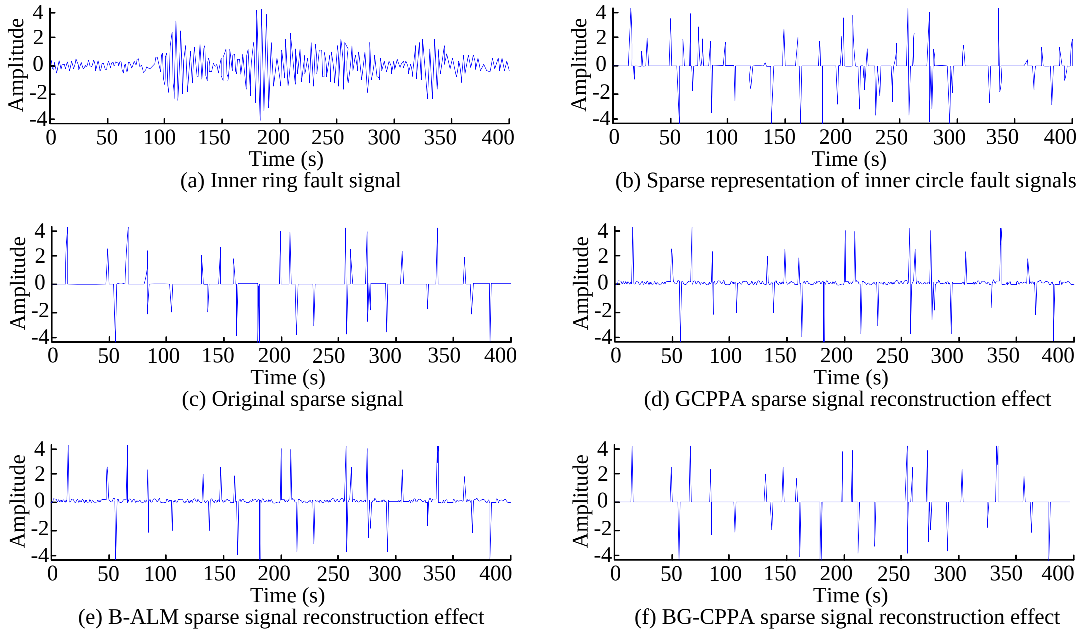


Figure 8: Comparison of three algorithms for reconstructing sparse signal of bearing inner ring fault

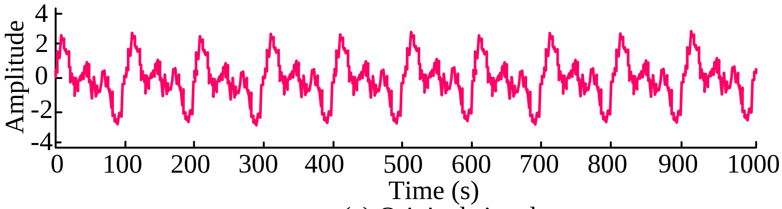
From Figures 8 (a) and 8 (b), the trained over-complete dictionary showed that the bearing vibration signal had potential sparsity. The sparsity effect was superior, which met the experimental requirements of the proposed compressive sensing system. The sparse representation signal reconstruction effects of the three algorithms at a compression ratio of 0.5 are shown in Figures 8 (d) - (f). Figure 8 (c) shows the original sparse signal. The reconstruction effects of the three algorithms showed that the BG-CPPA had the best reconstruction effect. The convex optimization algorithm BG-CPPA performed the best in reconstructing sparse signals. It is suitable as a reconstruction algorithm for the compressive sensing system of bearing vibration signals, thereby improving the reconstruction processing performance of the compressive sensing system.

4.2 Verification analysis of compressive sensing system for bearing vibration signals

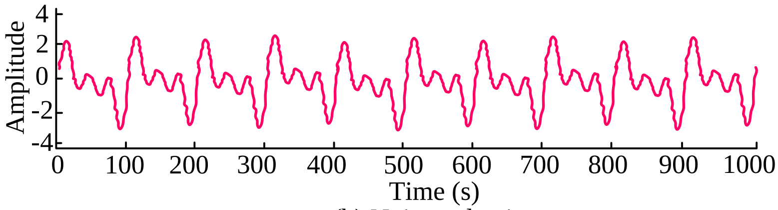
To further verify the superiority of the convex optimization algorithm BG-CPPA in the bearing vibration signal compressive sensing system, the proposed bearing vibration signal compressive sensing system is analyzed. The vibration signals generated by bearing failure operation are complex and diverse, with a large amount of noise. Therefore, the wavelet threshold function is used to denoise it. The pre- and post noise reduction effects of bearing vibration signals are shown in Figure 9.

By comparing before and after signal denoising, the signal in Figure 9 (a) was affected by noise. The threshold function showed more fluctuations, making it impossible to directly compress and sense the signal data. Figure 9 (b) shows the threshold function after denoising. The overall signal was smoother after denoising. On this basis, the compressive sensing system is validated for bearing fault signal compression. The testing and reconstruction results of the bearing fault inner ring signal at a compression ratio of 0.5 are shown in Figure 10.

From Figure 10 (a) and Figure 10 (b), the improved K-SVD sparsely represented the bearing fault signal, fully demonstrating its potential sparse features. Based on the Gaussian random observation matrix with a compression ratio of 0.5 in Figure 10 (c), sparse signals were compressed through matrix compression to achieve signal data compression. Figure 10 (d) shows the reconstruction effect of the proposed convex optimization algorithm. It effectively reconstructed and recovered bearing signals. Meanwhile, the Greedy Algorithm (GA) is introduced to compare the reconstruction performance of bearing inner ring fault signals with three methods at different compression ratios. The specific comparison data is shown in Table 4.



(a) Original signal



(b) Noise reduction

Figure 9: Simulation analysis of noise reduction effect on noise signals

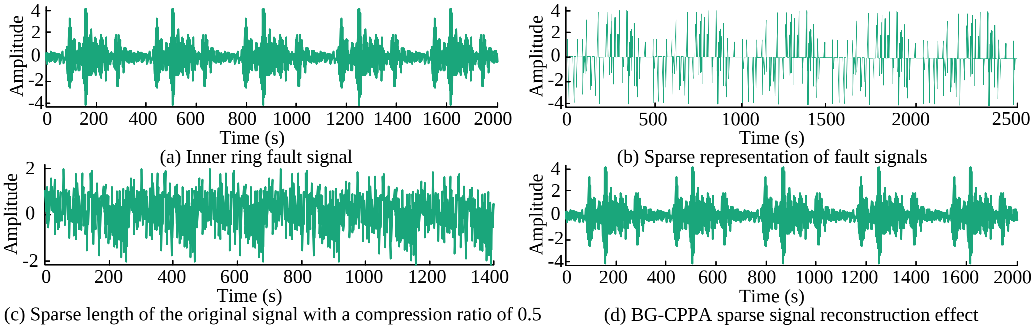


Figure 10: Signal reconstruction effect of compressive sensing system at 0.5 compression ratio

Table 4: Comparison of reconstruction performance for bearing fault inner ring signal at different compression ratios

Algorithm	**M/N=0.3**			**M/N=0.4**			**M/N=0.5**		
	Err	**Reconstruct signal-to-noise ratio (dB)**	**Time (s)**	**Err**	**Reconstruct signal-to-noise ratio (dB)**	**Time (s)**	**Err**	**Reconstruct signal-to-noise ratio (dB)**	**Time (s)**
GCPPA	21.34	0.14	3.76	9.21	3.81	4.37	6.99	11.32	3.83
B-ALM	19.27	0.33	2.15	8.76	5.66	2.28	6.34	13.54	2.67
BG-CPPA	9.22	0.73	0.29	4.45	13.18	0.54	3.01	18.04	0.79
GA	14.27	0.46	1.70	5.49	23.56	1.04	4.11	17.43	1.28

From Table 4, the reconstruction performance of BG-CPPA was significantly better than GCPPA and B-ALM at different compression ratios. At a compression ratio of 0.4, the reconstruction SNR of the GA increased by 10.38dB compared with BG-CPPA. However, at compression ratios of 0.3 and 0.5, the BG-CPPA performed better. In terms of the running time, BG-CPPA showed better ability. It had the lowest average signal reconstruction time among all algorithms at three compression ratios. The BG-CPPA reduced time consumption by 82.94%, 48.08%, and 38.28% compared with the GA at three compression ratios, respectively. As the compression ratio increases, the BG-CPPA requires more time to reconstruct the SNR and signal, while the Err decreases. This indicates that BG-CPPA can effectively improve the signal reconstruction accuracy and efficiency of the bearing vibration signal compressive sensing system. It also confirms the effectiveness and reliability of the proposed compressive sensing system in bearing fault vibration signals and compression processing. The mean, Standard Deviation (SD) and 95% confidence intervals of the reconstruction errors of the bearing fault signals at different compression ratios are shown in Table 5.

From Table 5, the mean value of the reconstruction error decreased with the increase of the compression ratio, which indicates that the compressive sensing system can still maintain high reconstruction accuracy at higher compression ratios. Meanwhile, the smaller standard deviation and confidence interval width indicate the consistency and reliability of the experimental results.

Table 5: Means, standard deviations and 95% confidence intervals of reconstruction errors (Err) for bearing fault signals

Compression ratio	**Mean**	**SD**	**95% confidence interval**
0.3	9.22	0.73	(8.76, 9.68)
0.4	4.45	0.29	(4.16, 4.74)
0.5	3.01	0.54	(2.47, 3.55)

4.3 Time complexity analysis

In the BG-CPPA, the most time-consuming part is the iterative process, especially solving the optimization problem and updating the iteration points in each iteration. The time complexity of each iteration is assumed to be O(f), where f represents the computational complexity of a single iteration. The initialization step has a time complexity of O(1), and the algorithm performs up to N iterations with a time complexity of O(f) per iteration. Therefore, the time complexity of the entire iterative process is O(N·f). A convergence check is performed after each iteration, which also has a time complexity of O(1). Therefore, the overall time complexity of the BG-CPPA is O(N·f). Although the time complexity of the BG-CPPA is proportional to the number of iterations, experimental verification has shown that the algorithm can maintain a low number of iterations at different sparsity levels. This indicates that the BG-CPPA has high efficiency and good scalability in handling compressive sensing problems.

5 Discussion

The BG-CPPA introduced in this study outperforms existing methods in the compressive sensing of bearing fault signals, particularly in stability, efficiency, and error reduction. It exhibits robustness across varying data sparsity, maintaining low iteration counts and computation time with minimal fluctuations, indicating its effectiveness in handling data size changes and adapting to practical uncertainties. The computation time of B-ALM increases with the increase of sparsity, while BG-CPPA remains efficient. Due to its optimized computation and fast convergence, the average reconstruction time of BG-CPPA is only 0.04s, significantly faster than GCPPA and B-ALM. In addition, the reconstruction SNR of BG-CPPA is superior to other algorithms, especially when the compression ratio increases. The accuracy in representing and recovering signal sparsity through the K-SVD-based over-complete dictionary contributes to its superior reconstruction precision.

The novelty of the BG-CPPA is that it combines the advantages of GCPPA and B-ALM while overcoming the limitations of both in practical applications. Although GCPPA eliminates relaxation steps and improves convergence efficiency, its applicability is limited when dealing with large-scale data problems. Although the B-ALM algorithm balances the objective function, coefficient matrix, and set proportion by reconstructing ALM, its iteration times are affected by the size of the problem, and the convergence efficiency is correspondingly reduced. The B-CPCA not only improves the applicability and convergence efficiency of the algorithm by combining the advantages of both, but also significantly improves the computational efficiency and reconstruction accuracy of the algorithm through optimizing the iterative process.

6 Conclusion

A convex optimization algorithm BG-CPPA combining GCPPA and B-ALM is proposed to address the poor CS accuracy and reconstruction effect of bearing fault vibration signals. Therefore, a bearing vibration signal compressive sensing system combining universal compressive sensing model and K-SVD over-complete dictionary is designed. The results showed that the BG-CPPA had fewer iterations and time than traditional algorithms at different sparsity levels. Compared with GCPPA and B-ALM, BG-CPPA reduced the reconstruction error ratio by 32.46% and 52.64%, respectively. At different compression ratios, the reconstruction performance of BG-CPPA was superior to the other two algorithms. According to the BG-CPPA, the compressive sensing system had the best reconstruction effect compared with different reconstruction algorithms. Compared with GA, BG-CPPA reduced reconstruction time by 82.94%, 48.08%, and 38.28% respectively at compression ratios of 0.3, 0.4, and 0.5. The results indicate that the proposed convex optimization algorithm BG-CPPA has great application value in solving linear equality constrained convex optimization problems. The compressive sensing system based on BG-CPPA has certain feasibility, which can effectively improve the reconstruction accuracy and effectiveness of bearing signal compressive sensing. However, the convex optimization algorithm proposed in the study only solves the compressive sensing technology. It has not been validated in other fields. It has certain limitations. Therefore, future research work will be carried out in the following areas:

Further explore the potential applications of BG-CPPA in other fields, such as image processing and machine learning.

Optimize the BG-CPPA to improve its performance in large-scale data processing.

Investigate the application of BG-CPPA in real-time systems, such as online monitoring and fault diagnosis.

Develop more efficient sparse representation and reconstruction techniques to enhance signal processing.

Explore the combination of BG-CPPA with other optimization techniques, such as deep learning or evolutionary algorithms.

Track and evaluate the performance of the BG-CPPA in real-world applications over time to gather feedback and guide future improvements.

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